Introduction to the Parton Model and Pertrubative QCD

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- The Parton Model and Deep-inelastic Scattering
- From the Parton Model to QCD
- Factorization and Evolution

The Context of QCD: "Fundamental Interactions"

- **Electromagnetic**
- **+ Weak Interactions $\Rightarrow$ Electroweak**
- **+ Strong Interactions (QCD) $\Rightarrow$ Standard Model**
- **+ . . . = Gravity and the rest?**
- **QCD: A theory "off to a good start". Think of . . .**
  - $\vec{F}_{12} = -GM_1M_2\hat{r}/R^2 \Rightarrow$ **elliptical orbits . . . 3-body problem . . .**
  - $L_{\rm QCD} = \bar{q}\ \not{D} q - (1/4)F^2 \Rightarrow$ **asymptotic freedom . . . confinement . . .**

The Parton Model and Deep-inelastic Scattering

- **1. Nucleons to Quarks**

- **2. DIS: Structure Functions and Scaling**

- **3. Getting at the Quark Distributions**

- **4. Extensions**

1. From Nucleons to Quarks

- **Nucleons, pions and isospin:**

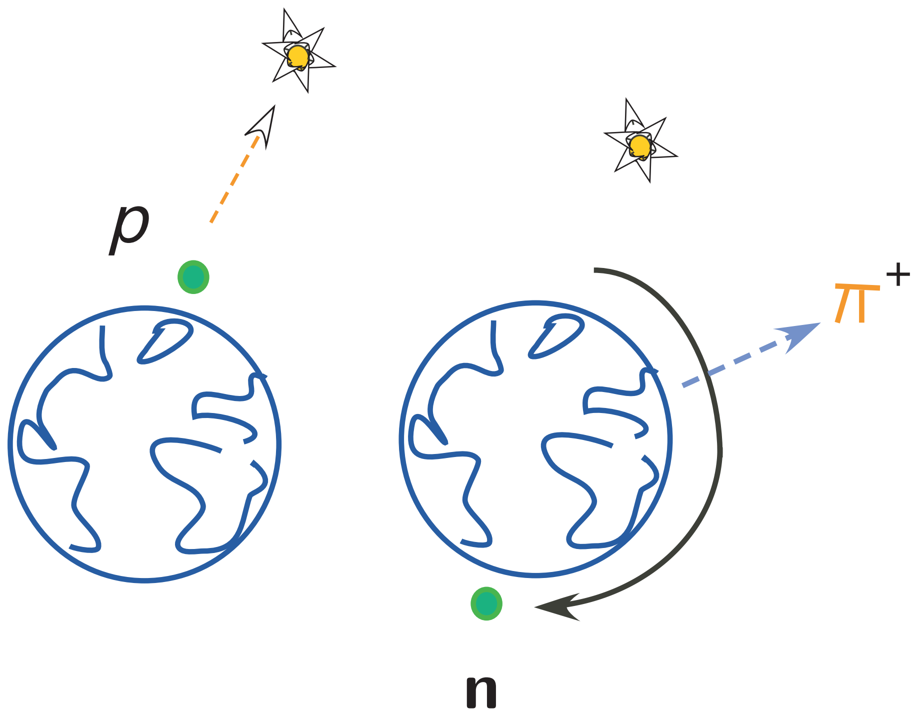
$$\begin{pmatrix} p \\ n \end{pmatrix}$$

  - **p: m=938.3 MeV,** $S = 1/2$**,** $I_3 = 1/2$
  - **n: m=939.6 MeV,** $S = 1/2$**,** $I_3 = -1/2$

$$\begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$$

  - $\pi^\pm$**: m=139.6 MeV,** $S = 0$**,** $I_3 = \pm 1$
  - π^0**: m=135.0 MeV,** $S = 0$**,** $I_3 = 0$

- **Isospin space . . .**
- **With a "north star" set by electroweak interactions:**



Analog: the rotation group (more specifically, $SU(2)$).

- **'Modern': π, N common substructure:** $quarks$
  - **Gell Mann, Zweig 1964**

- **spin $S = 1/2$,**
  isospin doublet (u, d) & singlet (s)
  with approximately equal masses (s heavier);

$$\begin{pmatrix} u \ (Q = 2e/3, I_3 = 1/2) \\ d \ (Q = -e/3, I_3 = -1/2) \\ s \ (Q = -e/3, I_3 = 0) \end{pmatrix}$$

$$\pi^+ = (u\bar{d}) \ , \quad \pi^- = (\bar{u}d) \ , \quad \pi^0 = \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right) \ ,$$

$$p = (uud) \ , \quad n = (udd) \ , \quad K^+ = (u\bar{s}) \ldots$$

- **Requirement for N:** **symmetric spin/isospin wave function (!)**

- μ_p/μ_n = **-3/2 (good to %)**

- **and now, six: 3 'light' (u, d, s), 3 'heavy': (c, b, t)**

Aside: quarks in the standard model: SU(3) $\times$ SU(2)$_L$ $\times$ U(1)

- Quark and lepton fields: L(eft) and R(ight)
  - $\psi = \psi^{(L)} + \psi^{(R)} = \frac{1}{2}(1-\gamma_5)\psi + \frac{1}{2}(1+\gamma_5)\psi$; $\psi = q, \ell$
  - Helicity: spin along $\vec{p}$ (R=right handed) or opposite (L=left handed) in solutions to Dirac equation
  - $\psi^{(L)}$: expanded only in L particle solutions to Dirac eqn. R antiparticle solutions
  - $\psi^{(R)}$: only R particle solutions, L antiparticle
  - An essential feature: L and R have different interactions in general!

– **L quarks come in "weak $SU(2)$" = "weak isospin" pairs:**

$$q_i^{(L)} = \left(\ u_i, d_i' = V_{ij} dj\ \right) \qquad u_i^{(R)},\ d_i^{(R)}$$

$$(u, d') \qquad (c, s') \qquad (t, b')$$

$$\ell_i^{(L)} = \left(\ \nu_i, e_i\ \right) \qquad e_i^{(R)},\ \nu_i^{(R)}$$

$$(\nu_e, e) \qquad (\nu_\mu, \mu) \qquad (\nu_\tau, \tau)$$

– V_{ij} **is the "CKM" matrix**

- **Weak vector bosons: electroweak gauge groups**
  - **SU(2): three vector bosons B_i, coupling g**
  - **U(1); one vector boson C, coupling g'**

  - **The physical bosons:**

$$W^{\pm} = B_1 \pm i B_2$$

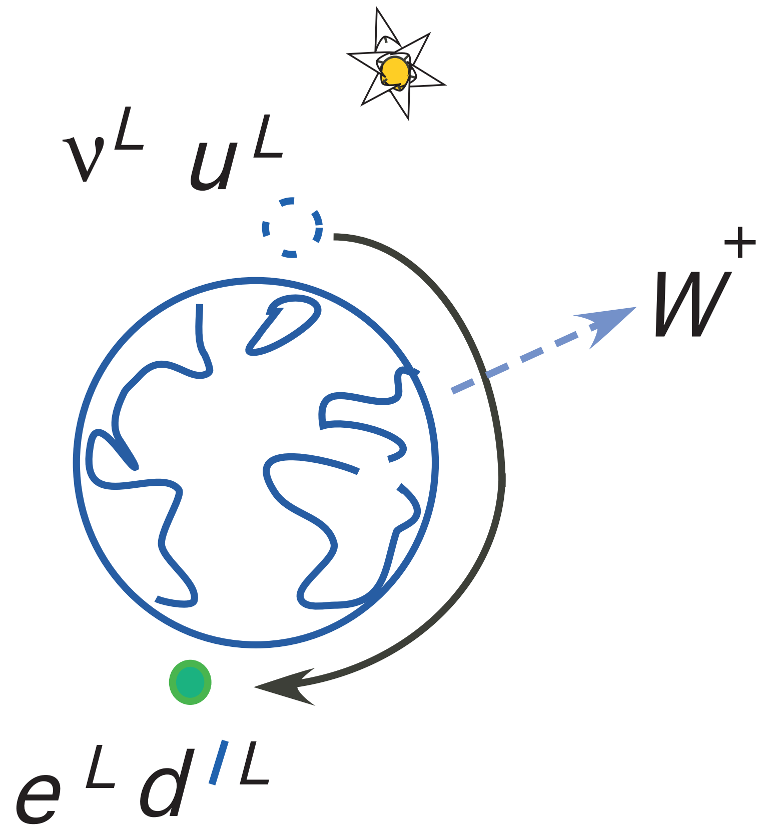
$$Z = -C \sin\theta_W + B_3 \cos\theta_W$$

$$\gamma \equiv A = C \cos\theta_W + B_3 \sin\theta_W$$

$$\sin\theta_W = g'/\sqrt{g^2 + g'^2} \qquad M_W = M_Z/\cos\theta_W$$

$$e = gg'/\sqrt{g^2 + g'^2} \qquad M_W \sim g/\sqrt{G_F}$$

- **Weak isospin space: connecting u with d'**



- **Only left handed fields move around this globe.**

– **The interactions of quarks and leptons with the photon, W, Z**

$$\mathcal{L}_{\mathrm{EW}}^{(fermion)} = \sum_{\text{all }\psi} \bar{\psi}\,(i\not\partial - e\lambda_\psi\,\not A - (gm_\psi 2M_W)h)\,\psi$$

$$-(g/\sqrt{2}) \sum_{q_i, e_i} \bar{\psi}^{(L)}\left(\sigma^+ \not W^+ + \sigma^- \not W^-\right)\psi^{(L)}$$

$$-(g/2\cos\theta_W) \sum_{\text{all }\psi} \bar{\psi}\,(v_f - a_f\gamma_5)\,\not Z\;\psi$$

– **Interactions with the Higgs $h \propto$ mass**

– **Interactions with W are through ψ_L's only**

– **Neutrino Z exchange sensitive to $\sin^2\theta_W$, even at low energy. Observation made it clear by early 1970's that $M_W \sim g/\sqrt{G_F}$ is large (need for colliders)**

- **Symmetry violations in the standard model**
  - W**'s interact through** $\psi^{(L)}$ **only,** $\psi = q, \ell$**.**
  - **Left-handed quarks, leptons; right-handed antiquarks, leptons.**
  - **Parity (P) exchanges L/R; Charge conjugation (C) exchanges particles, antiparticles.**
  - **CP combination OK** $L \rightarrow R \rightarrow L$ **if all else equal, but it's not (quite). Complex phases in CKM** $V \rightarrow$ **CP violation.**

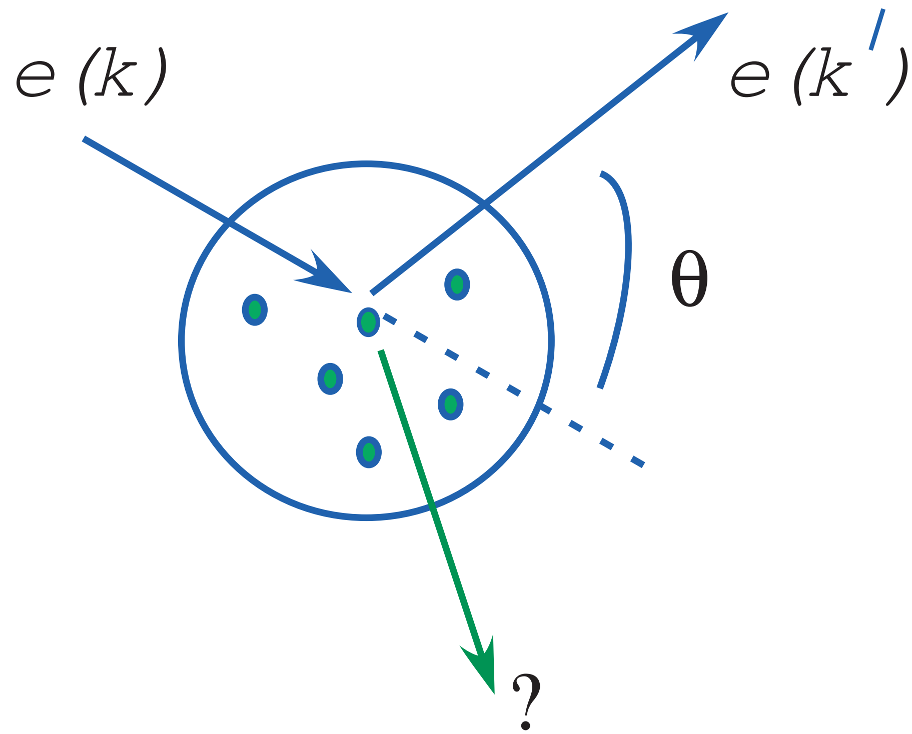
- **Quarks as Partons: “Seeing” Quarks**

  No isolated fractional charges seen (“confinement.”)

  Can such a particle be detected? (SLAC 1969)

  Look closer: do high energy electrons bounce off anything hard? (Rutherford ‘prime’.)

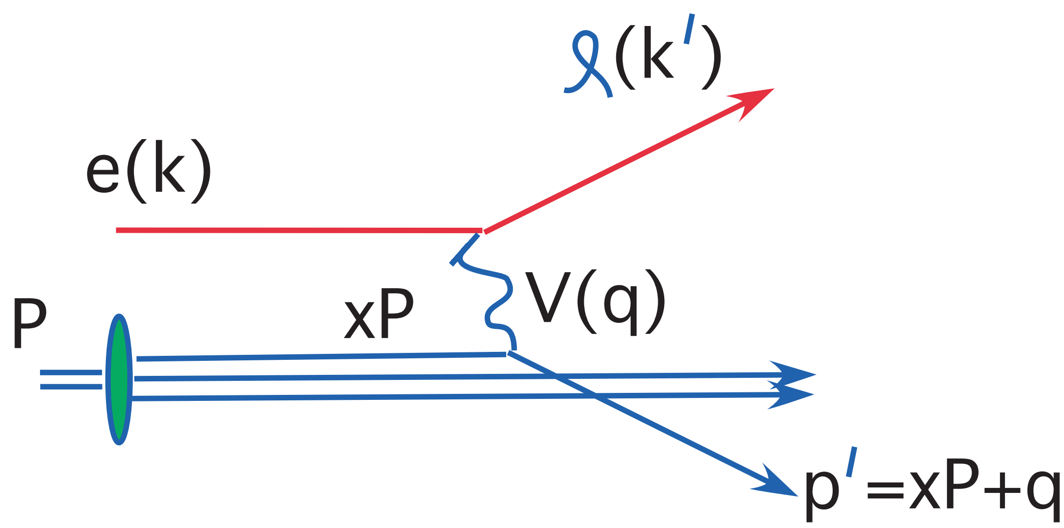
- **So look for:**



"Point-like' constituents.

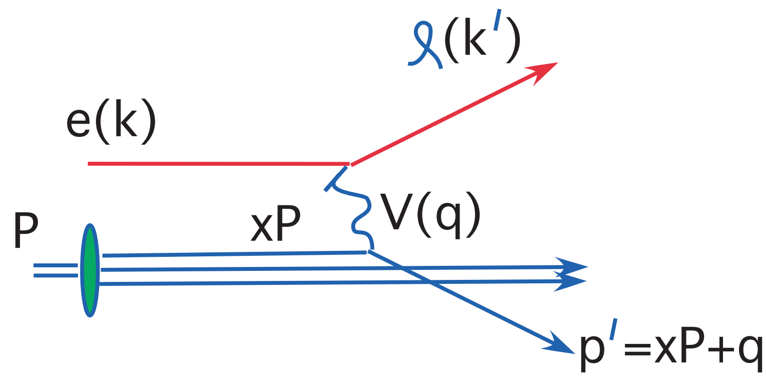
The angular distribution: information about the constituents.

Kinematics ($e + N(P) \to \ell + X$)



- $V = \gamma, Z_0 \Rightarrow \ell = e$ **"neutral current" (NC).**

- $V = W^-(e^-, \nu_e)$, $V = W^+(e^+, \bar{\nu}_e)$ **"charged current" (CC).**



$Q^2 = -q^2 = -(k - k')^2$ **momentum transfer.**

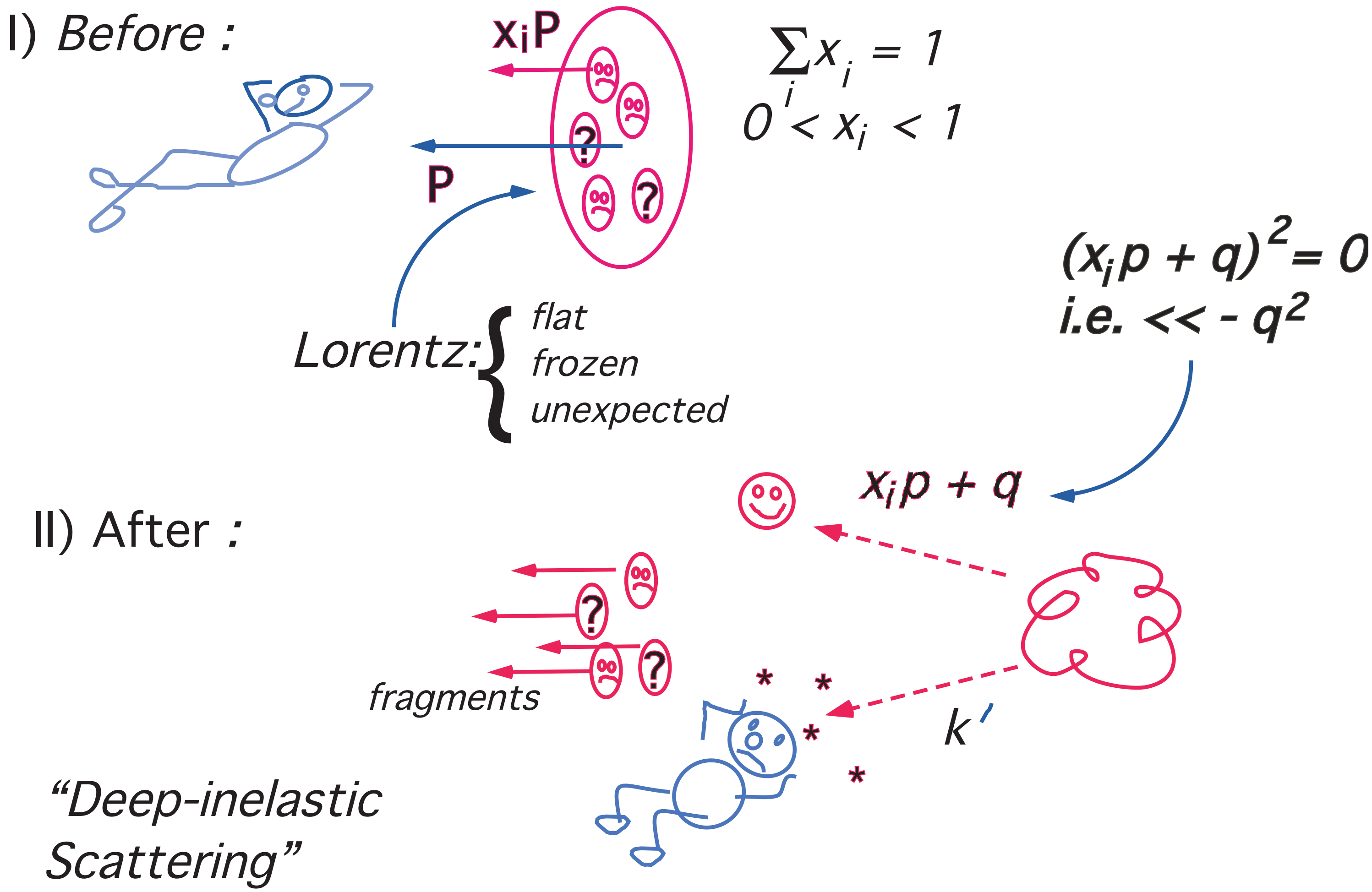
$x \equiv \frac{Q^2}{2p\cdot q}$ **momentum fraction (from** $p'^2 = (xp + q)^2 = 0$**).**

$y = \frac{p\cdot q}{p\cdot k}$ **fractional energy transfer.**

$W^2 = (p + q)^2 = \frac{Q^2}{x}(1 - x)$ **squared final-state mass of hadrons.**

$$xy = \frac{Q^2}{S}$$

Parton Interpretation (Feynman 1969, 72)
Look in the electron's rest frame . . .



– **Basic Parton Model Relation**

$$\sigma_{\rm eh}(p,q) = \sum_{\text{partons } a} \int_0^1 d\xi\, \hat{\sigma}^{\rm el}_{ea}(\xi p, q)\ \phi_{a/h}(\xi)$$

– **where:** σ_{eh} **is the cross section for**
$e(k) + h(p) \to e(k' = k - q) + X(p + q)$

– **and** $\hat{\sigma}^{\rm el}_{ea}(xp, q)$ **is the elastic cross section for**
$e(k) + a(\xi p) \to e(k' - q) + a(\xi p + q)$ **which sets**
$(\xi p + q)^2 = 0 \to \xi = -q^2/2p \cdot q \equiv x$.

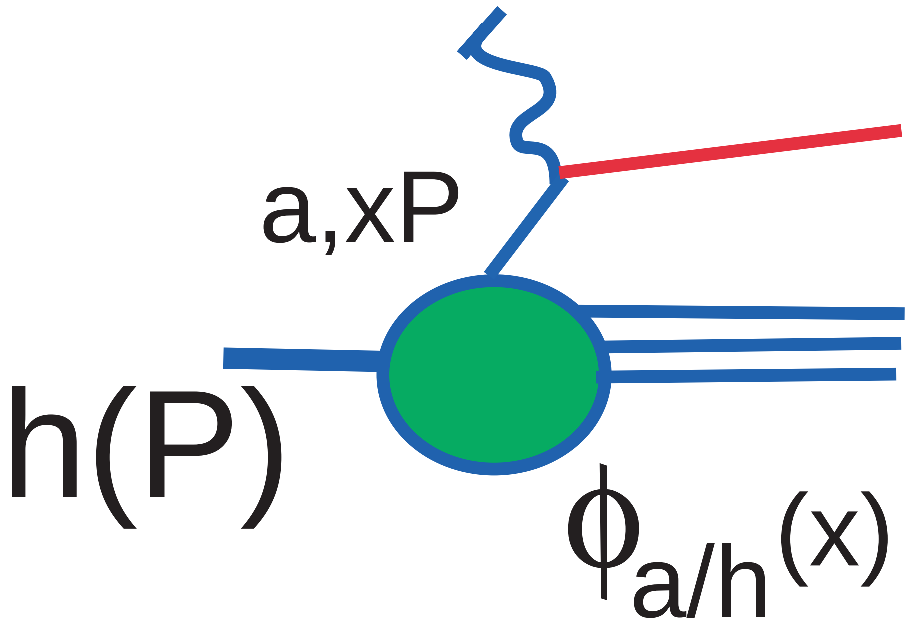
– **and** $\phi_{a/h}(x)$ **is the distribution of parton a in hadron h,**
the "probability for a parton of type a
to have momentum xp**".**

– **in words:** *Hadronic INELASTIC cross section is the sum of convolutions of partonic ELASTIC cross sections with the hadron's parton distributions.*

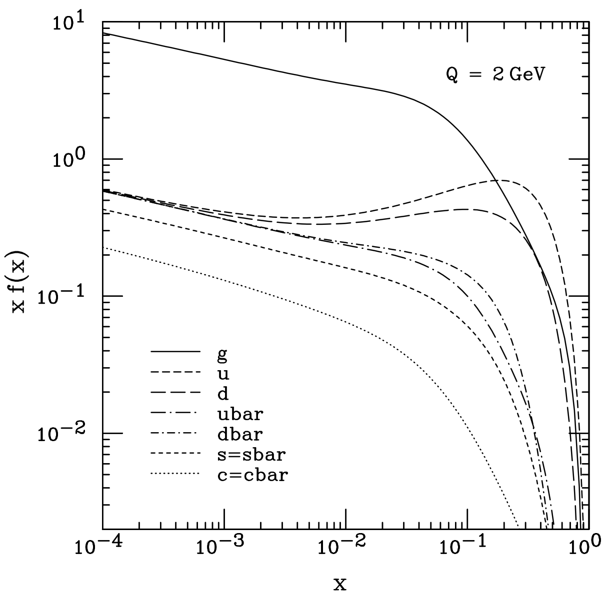
– **The nontrivial assumption:** *quantum mechanical incoherence* **of large-q scattering and the partonic distributions. Multiply probabilities without adding amplitudes.**

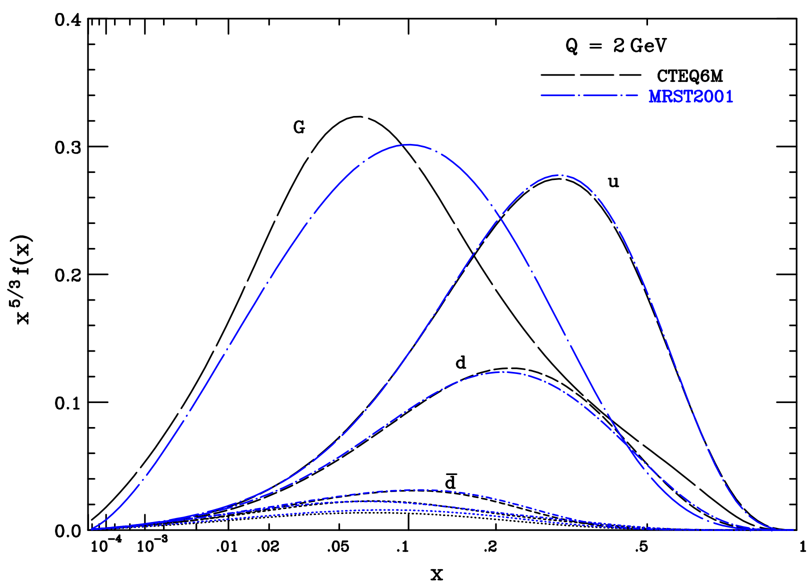
– **Heuristic justification: the binding of the nucleon involves long-time processes that do not interfere with the short-distance scattering. Later we'll see how this works in QCD.**

The familiar picture



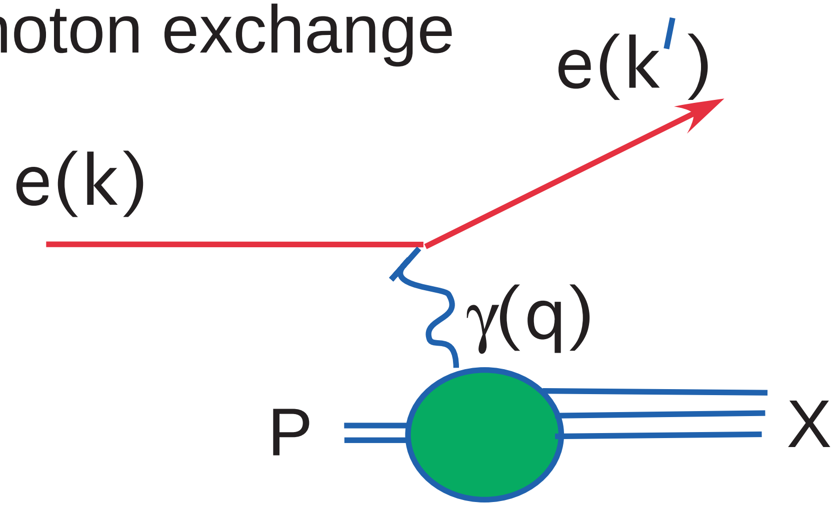
- **Two modern parton distribution sets at moderate momentum transfer (note different weightings with x):**





2. DIS: Structure Functions and Scaling

Photon exchange



$$A_{e+N\to e+X}(\lambda,\lambda',\sigma;q) \;=\; \bar{u}_{\lambda'}(k')(-ie\gamma_\mu)u_\lambda(k)$$
$$\times\frac{-ig^{\mu\mu'}}{q^2}$$
$$\times\langle X|\, eJ^{\mathrm{EM}}_{\mu'}(0)\, |p,\sigma\rangle$$

$$d\sigma_{\mathrm{DIS}} = \frac{1}{2^2}\frac{1}{2s}\frac{d^3k'}{(2\pi)^3 2\omega_{k'}} \sum_X \sum_{\lambda,\lambda',\sigma} |A|^2 \; (2\pi)^4 \, \delta^4(p_X + k' - p - k)$$

In $|A|^2$, separate the known leptonic part from the "unknown" hadronic part:

- **The leptonic tensor:**

$$L^{\mu\nu} = \frac{e^2}{8\pi^2} \sum_{\lambda,\lambda'} (\bar{u}_{\lambda'}(k')\gamma^\mu u_\lambda(k))^* \, (\bar{u}_{\lambda'}(k')\gamma^\nu u_\lambda(k))$$

$$= \frac{e^2}{2\pi^2} \left(k^\mu k'^{\,\nu} + k'^{\,\mu} k^\nu - g^{\mu\nu} k \cdot k' \right)$$

- **The hadronic tensor:**

$$W_{\mu\nu} = \frac{1}{8\pi} \sum_{\sigma,X} \langle X|J_\mu|p,\sigma\rangle^* \langle X|J_\nu|p,\sigma\rangle$$

- **And the cross section:**

$$2\omega_{k'}\frac{d\sigma}{d^3k'} = \frac{1}{s(q^2)^2}\, L^{\mu\nu}W_{\mu\nu}$$

- $W_{\mu\nu}$ **has sixteen components,**
  but known properties of the strong interactions constrain $W_{\mu\nu}$ **. . .**

- **An example: current conservation**

$$
\begin{aligned}
&\partial^\mu J_\mu^{\mathrm{EM}}(x) = 0 \\
&\qquad \Rightarrow \langle X| \, \partial^\mu J_\mu^{\mathrm{EM}}(x) \, |p\rangle = 0 \\
&\qquad \Rightarrow (p_X - p)^\mu \langle X| \, J_\mu^{\mathrm{EM}}(x) \, |p\rangle = 0 \\
&\qquad \Rightarrow q^\mu W_{\mu\nu} = 0
\end{aligned}
$$

- **With parity, time-reversal, etc . . .**

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1(x, Q^2)$$
$$+ \left(p_\mu - q_\mu \frac{p \cdot q}{q^2}\right) \left(p_\nu - q_\nu \frac{p \cdot q}{q^2}\right) W_2(x, Q^2)$$

- **Often given in terms of the dimensionless structure functions**

$$F_1 = W_1 \qquad F_2 = p \cdot q W_2$$

- **Note that if there is no other massive scale the F's cannot depend on Q except indirectly through x.**

- **Structure functions in the Parton Model: The Callan-Gross Relation**

From the "basic parton model formula":

$$\frac{d\sigma_{eh}}{d^3k'} = \int d\xi \; \frac{d\sigma^{\rm el}_{eq}(\xi)}{d^3k'} \; \phi_{q/h}(\xi) \qquad (1)$$

Can treat a quark of "flavor" f just like any hadron and get

$$\omega_{k'} \; \frac{d\sigma^{\rm el}_{ef}(\xi)}{d^3k'} = \frac{1}{2(\xi s)Q^4} \; L^{\mu\nu} W^{ef}_{\mu\nu}(k + \xi p \longrightarrow k' + p')$$

Let the charge of f be e_f. Exercise 1: Compute $W_{\mu\nu}^{ef}$ to find:

$$W_{\mu\nu}^{ef} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\,\delta\left(1-\frac{x}{\xi}\right)\frac{e_f^2}{2}$$
$$+\left(\xi p_\mu - q_\mu\,\frac{\xi p\cdot q}{q^2}\right)\,\left(\xi p_\nu - q_\nu\,\frac{\xi p\cdot q}{q^2}\right)\,\delta\left(1-\frac{x}{\xi}\right)\frac{e_f^2}{\xi p\cdot q}$$

Ex. 2: by substituting in (1), find the Callan-Gross relation,

$$F_2(x) = \sum_{\mathrm{quarks}\,f} e_f^2 x\,\phi_{f/p}(x) = 2xF_1(x)$$

And Ex. 3: that this relation is quite different for scalar quarks.

- The Callan-Gross relation shows the compatibility of the quark and parton models.

- In addition: parton model structure functions are independent of Q^2, a property called "scaling". With massless partons, there is no other massive scale. Then the F's must be Q-independent; see above.

- Approximate properties of the kinematic region explored by SLAC in late 1960's – early 1970's.

- Explore corrections to this picture in QCD "evolution".

- **Structure Functions and Photon Polarizations**

In the P rest frame can take

$$q^\mu = \left(\nu; 0, 0, \sqrt{Q^2 + \nu^2}\right) , \qquad \nu \equiv \frac{p \cdot q}{m_p}$$

In this frame, the possible photon polarizations $(\epsilon \cdot q = 0)$**:**

$$\epsilon_R(q) = \frac{1}{\sqrt{2}}(0; 1, -i, 0)$$

$$\epsilon_L(q) = \frac{1}{\sqrt{2}}(0; 1, i, 0)$$

$$\epsilon_{\text{long}}(q) = \frac{1}{Q}\left(\sqrt{Q^2 + \nu^2}, 0, 0, \nu\right)$$

- **Alternative Expansion**

$$W^{\mu\nu} = \sum_{\lambda=L,R,long} \epsilon_\lambda^{\mu *}(q)\epsilon_\lambda^\nu(q)\, F_\lambda(x,Q^2)$$

- **For photon exchange (Exercise 4):**

$$F_{L,R}^{\gamma e} = F_1$$

$$F_{\text{long}} = \frac{F_2}{2x} - F_1$$

- **So F_{long} vanishes in the parton model by the C-G relation.**

- **Generalizations: neutrinos and polarization**

- **Neutrinos: flavor of the "struck" quark is changed when a $W^{\pm}$ is exchanged. For W^{+}, a d is transformed into a linear combination of u, c, t, determined by CKM matrix (and momentum conservation).**

- **Z exchange leaves flavor unchanged but still violates parity.**

- **The Vh structure functions for $= W^+, W^-, Z$:**

$$W_{\mu\nu}^{(Vh)} - \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1^{(Vh)}(x, Q^2)$$
$$+ \left(p_\mu - q_\mu \frac{p \cdot q}{q^2}\right) \left(p_\nu - q_\nu \frac{p \cdot q}{q^2}\right) \frac{1}{m_h^2} W_2(x, Q^2)$$
$$-i\epsilon_{\mu\nu\lambda\sigma} p^\lambda q^\sigma \; \frac{1}{m_h^2} W_3^{(Vh)}(x, Q^2)$$

- **with dimensionless structure functions:**

$$F_1 = W_1\,, \qquad F_2 = \frac{p \cdot q}{m_h^2} W_2\,, \qquad F_3 = \frac{p \cdot q}{m_h^2} W_3$$

- **And with spin (back to the photon). Note equivalent expression for $W^{\mu\nu}$.**

$$W^{\mu\nu} = \frac{1}{4\pi} \int d^4z \, e^{iq\cdot z} \; \langle h(P,S) \,|\, J^\mu(z) J^\nu(0) \,|\, h(P,S) \rangle$$

$$= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) F_1(x,Q^2)$$

$$+ \left(P^\mu - q^\mu \frac{P\cdot q}{q^2} \right) \left(P^\nu - q^\nu \frac{P\cdot q}{q^2} \right) F_2(x,Q^2)$$

$$+ \, i m_h \, \epsilon^{\mu\nu\rho\sigma} q_\rho \left[\frac{S_\sigma}{P\cdot q} g_1(x,Q^2) + \frac{S_\sigma (P\cdot q) - P_\sigma (S\cdot q)}{(P\cdot q)^2} g_2(x,Q^2) \right]$$

- **Parton model structure functions**

$$F_2^{(eh)}(x) = \sum_f e_f^2 \, x \, \phi_{f/h}(x)$$

$$g_1^{(eh)}(x) = \frac{1}{2} \sum_f e_f^2 \left(\Delta\phi_{f/n}(x) + \Delta\bar{\phi}_{f/h}(x) \right)$$

- **Notation:** $\Delta\phi_{f/h} = \phi^{+}_{f/h} \; - \; \phi^{-}_{f/h}$ **with** $\phi^{\pm}_{f/h}(x)$ **probability for struck quark** f **to have momentum fraction** x **and helicity with (+) or against (−)** h**'s helicity.**

3. Getting at the Quark Distributions

- **Relating the parton distributions to experiment**

- **Simplifying assumptions (adequate to early experiments; generally no longer adequate) that illustrate the general approach.**

$$
\begin{array}{ll}
\phi_{u/p} = \phi_{d/n} \qquad \phi_{d/p} = \phi_{u/n} & \text{isospin} \\
\phi_{\bar{u}p} = \phi_{\bar{u}/n} = \phi_{\bar{d}/p} = \phi_{\bar{d}/n} & \text{symmetric sea} \\
\phi_{c/p} = \phi_{b/N} = \phi_{t/N} = 0 & \text{no heavy quarks}
\end{array}
$$

$$F_2^{(eN)}(x) = 2xF_1^{(eN)}(x) = \sum_{f=u,d,s} e_F^2 x\phi_{f/N}(x)$$

$$F_2^{(W^+N)} = 2x\left(\sum_{D=d,s,b} \phi_{D/N}(x) + \sum_{U=u,c,t} \phi_{\bar{U}/N}(x)\right)$$

$$F_2^{(W^-N)} = 2x\left(\sum_{D} \phi_{\bar{D}/N}(x) + \sum_{U} \phi_{U/N}(x)\right)$$

$$F_3^{(W^+N)} = 2x\left(\sum_{D} \phi_{D/N}(x) - \sum_{U} \phi_{\bar{U}/N}(x)\right)$$

$$F_3^{(W^-N)} = 2x\left(\sum_{D} \phi_{\bar{D}/N}(x) - \sum_{U} \phi_{U/N}(x)\right)$$

- **Overdetermined with the assumptions: checks consistency.**
- **Further consistency checks: Sum Rules, e.g.:**

$$N_{u/p} = \int_0^1 dx\ \left[\,\phi_{u/p}(x) - \phi_{\bar{u}/p}(x)\,\right] = 2$$

etc. for $N_{d/p} = 1$**.**

The most interesting ones make predictions on measurable structure functions . . .

- **The Adler Sum Rule:**

$$
\begin{aligned}
1 = N_{u/p} - N_{d/p} &= \int_0^1 dx\, \left[\phi_{d/n}(x) - \phi_{d/p}(x)\right] \\
&= \int_0^1 dx\, \left[\sum_D \phi_{D/n}(x) + \sum_U \phi_{\bar{U}/n}(x)\right] \\
&\qquad - \int_0^1 dx\, \left[\sum_D \phi_{D/p}(x) + \sum_U \phi_{\bar{U}/p}(x)\right] \\
&= \int_0^1 dx\, \frac{1}{2x}\left[F_2^{(\nu n)} - F_2^{(\nu p)}\right]
\end{aligned}
$$

In the second equality, use isospin invar., in the third, all the extra terms cancel.

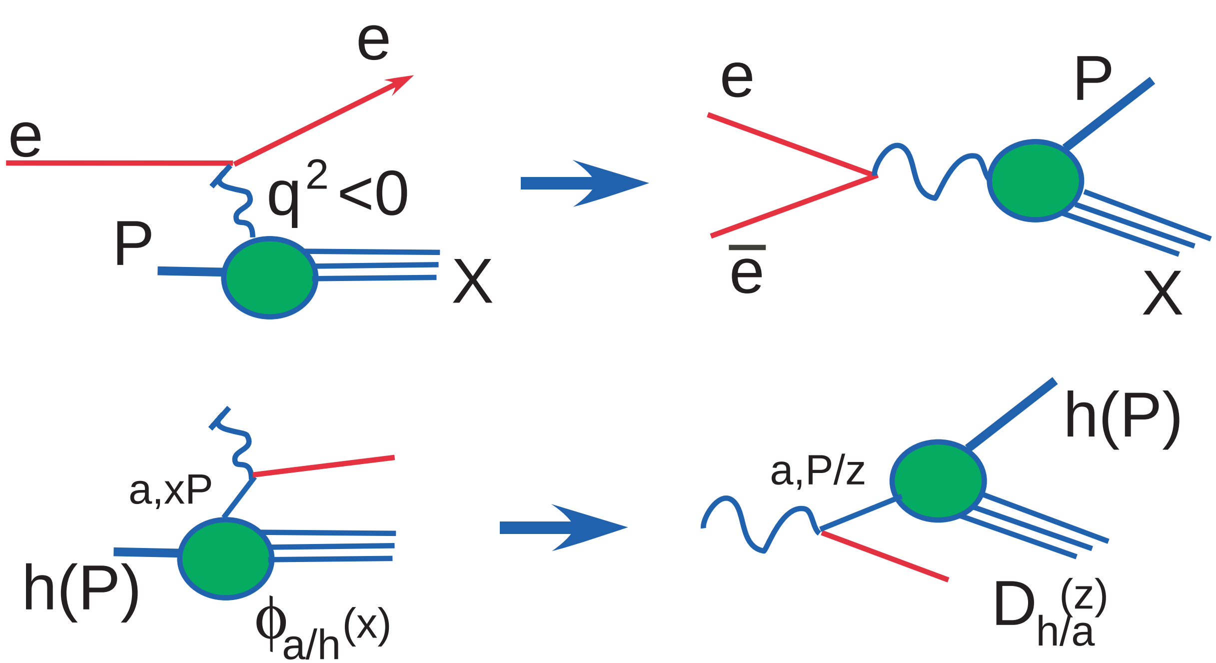
- **And similarly, the Gross-Llewellyn-Smith Sum Rule:**

$$3 = N_{u/p} + N_{d/p} = \int_0^1 dx\, \frac{1}{2x} \left[xF_3^{(\nu n)} + xF_3^{(\nu p)} \right]$$

4. Extensions

- **Fragmentation functions**

  "Crossing" applied to DIS: "Single-particle inclusive" (1PI)
  From scattering to pair annihilation.
  Parton distributions become "fragmentation functions".



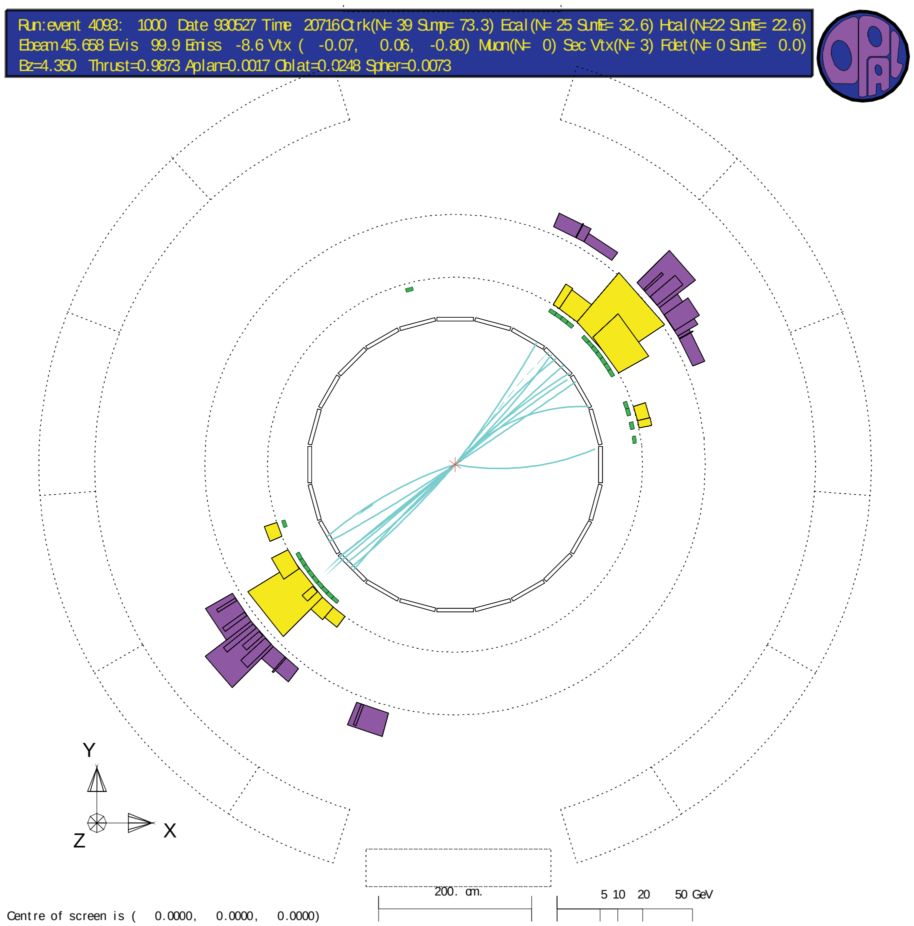
- **Parton model relation for 1PI cross sections**

$$\sigma_h(P,q) = \sum_a \int_0^1 dz \;\hat{\sigma}_a(P/z,q)\; D_{h/a}(z)$$

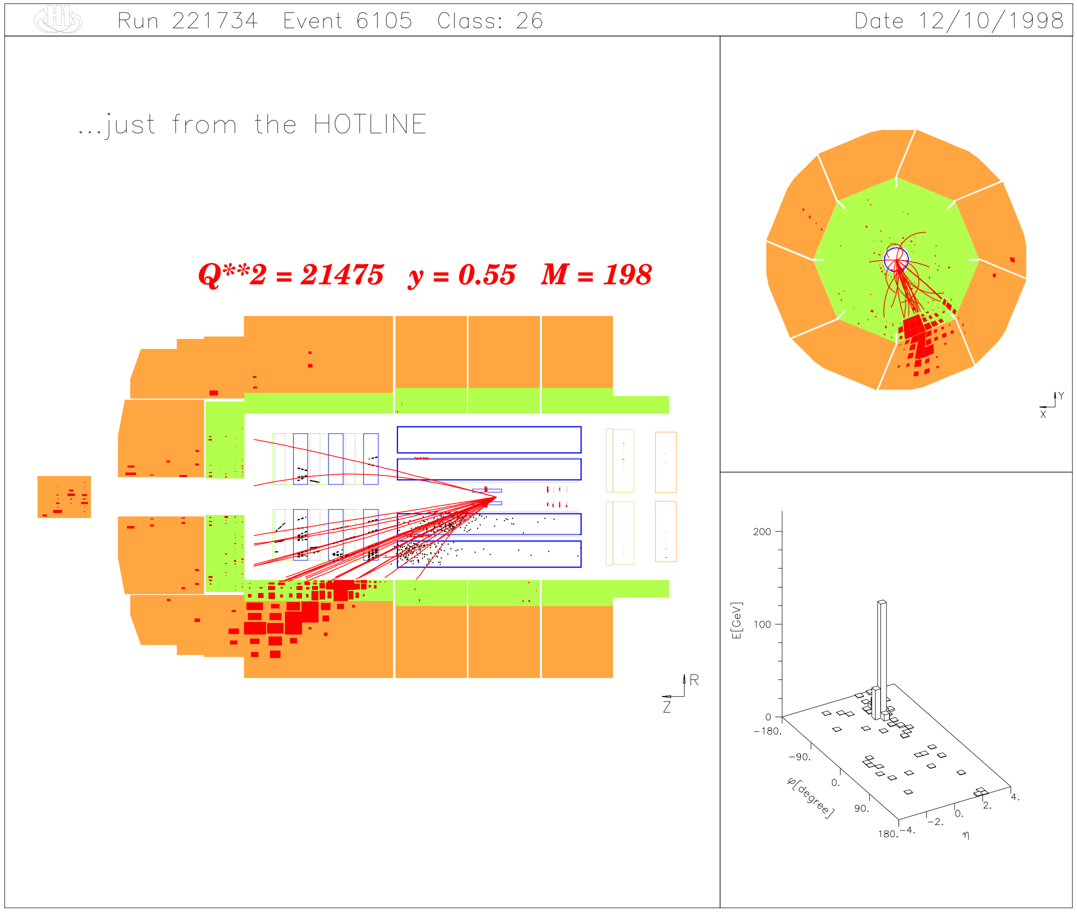
Heuristic justification: Formation of hadron C from parton a takes a time τ_0 in the rest frame of a, but much longer in the CM frame – this "fragmentation" thus decouples from $\hat{\sigma}_a$, and is independent of q (scaling).

- **Fragmentation picture suggests that hadrons are aligned along parton direction $\Rightarrow$ jets. And this is what happens.**

- **For** e^+e^-**:**


Run:event 4093: 1000 Date 930527 Time 20716 Ctrk(N= 39 Sump= 73.3) Ecal(N= 25 SumE= 32.6) Hcal(N=22 SumE= 22.6)
Ebeam 45.658 Evis 99.9 Emiss -8.6 Vtx (-0.07, 0.06, -0.80) Muon(N= 0) Sec Vtx(N= 3) Fdet(N= 0 SumE= 0.0)
Bz=4.350 Thrust=0.9873 Aplan=0.0017 Oblat=0.0248 Spher=0.0073
Y
X
Z
200. cm.
5 10 20 50 GeV
Centre of screen is (0.0000, 0.0000, 0.0000)

- **And for DIS:**



Run 221734 Event 6105 Class: 26
Date 12/10/1998
...just from the HOTLINE
Q**2 = 21475 y = 0.55 M = 198
E[GeV]
200
100
0
−180.
−90.
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−4.
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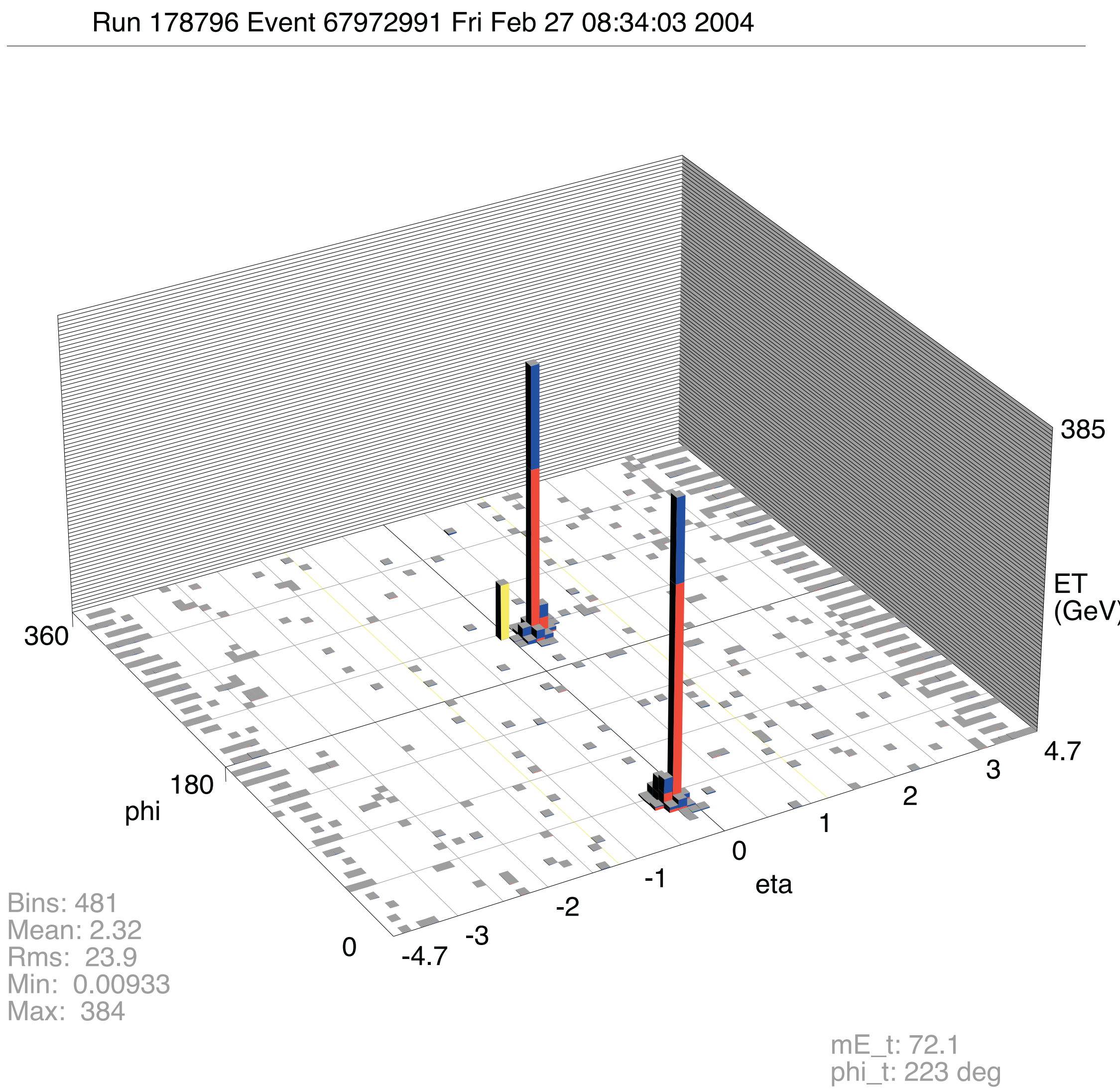
- **And in nucleon-nucleon collisions:**



- **Finally: the Drell-Yan process**

  Parton Model (1970).
  Drell and Yan: look for the annihilation of quark pairs into virtual photons of mass Q . . . any electroweak boson in NN scattering.

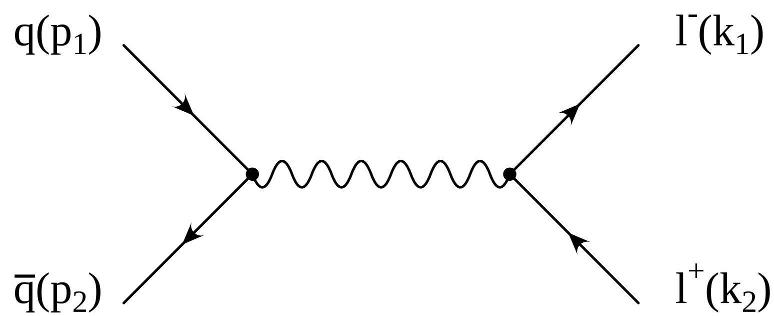
$$\frac{d\sigma_{NN\to\mu\bar{\mu}+X}(Q,p_1,p_2)}{dQ^2 d\ldots} \sim \int d\xi_1 d\xi_2 \sum_{a=\mathrm{q}\bar{\mathrm{q}}} \frac{d\sigma^{\mathrm{EW,\,Born}}_{\mathrm{a}\bar{\mathrm{a}}\to\mu\bar{\mu}}(Q,\xi_1 p_1,\xi_2 p_2)}{dQ^2 d\ldots}$$
$$\times(\text{probability to find parton a}(\xi_1)\text{ in }N)$$
$$\times(\text{probability to find parton }\bar{\mathrm{a}}(\xi_2)\text{ in }N)$$

The probabilities are $\phi_{q/N}(xi_i)$'s from DIS!

How it works (with colored quarks) . . .

- **The Born cross section**

$\sigma^{\mathrm{EW,Born}}$ **is all from this diagram (ξ's set to unity):**



With this matrix element

$$M = e_q \frac{e^2}{Q^2} \overline{u}(k_1)\gamma_\mu v(k_2)\overline{v}(p_2)\gamma^\mu u(p_1)$$

- **First square and sum/average M. Then evaluate phase space.**

- **Total cross section:**

$$\sigma^{\mathrm{EW,\,Born}}_{\mathrm{q\bar{q}}\to\mu\bar{\mu}}(x_1p_1, x_2p_2) = \frac{1}{2\hat{s}}\int \frac{d\Omega}{32\pi^2}\frac{e_q^2 e^4}{3}(1+\cos^2\theta)$$

$$= \frac{4\pi\alpha^2}{9Q^2}\sum_q e_q^2 \equiv \sigma_0(M)$$

With Q the pair mass and 3 for color average

Now we're ready for the parton model <u>differential</u> cross section for NN scattering:

Pair mass (Q) and rapidity

$$\eta \equiv (1/2)\ln(Q^+/Q^-) = (1/2)\ln[(Q^0+Q^3)/(Q^0-Q^3)]$$

overdetermined $\to$ delta functions in the Born cross section

$$\frac{d\sigma^{(PM)}_{NN\to\mu\bar{\mu}+X}(Q,p_1,p_2)}{dQ^2 d\eta} = \int_{\xi_1,\xi_2} \sum_{a=q\bar{q}} \sigma^{\mathrm{EW,\,Born}}_{a\bar{a}\to\mu\bar{\mu}}(\xi_1 p_1, \xi_2 p_2)$$

$$\times\delta\left(Q^2 - \xi_1\xi_2 S\right)\,\delta\left(\eta - \frac{1}{2}\ln\left(\frac{\xi_1}{\xi_2}\right)\right)$$

$$\times\phi_{a/N}(\xi_1)\,\phi_{\bar{a}/N}(\xi_2)$$

and integrating over rapidity,

$$\frac{d\sigma}{dQ^2} = \left(\frac{4\pi\alpha^2_{\mathrm{EM}}}{9Q^4}\right)\int_0^1 d\xi_1\,d\xi_2\,\delta\left(\xi_1\xi_2 - \tau\right)\sum_a \lambda_a^2\,\phi_{a/N}(\xi_1)\,\phi_{\bar{a}/N}(\xi_s)$$

Drell and Yan, 1970 (aside from 1/3 for color)

Analog of DIS: scaling in $\tau = Q^2/S$