

UNDERSTANDING PERTURBATIVE PHYSICS THROUGH SINGLE-TOP-QUARK PRODUCTION



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t Outline

- 1. Understanding electroweak (EW) physics
 - What is single-top-quark production?
 - Why do we study it?
- 2. Understanding perturbative QCD
 - The new Drell-Yan (DY) and Deep Inelastic Scattering (DIS) (or, dealing with the lump we swept under the rug)
 - What we've had to learn about the cross section
- 3. Applied understanding
 - A new paradigm for interpreting higher-order calculations
 - Examining the connection between theory and experiment
 - The impact of angular correlations



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What is single-top-quark production?

Why do we study it?

What is single-top-quark production?

Single-top-quark production is an electroweak (EW) process.



In the Standard model (SM) this involves the exchange of a W boson, whose LO (or NLO) virtuality labels the process.

ProcessW virtualityt-channel $q^2 < 0$ ($q^2 \sim -m_W^2/4$)s-channel $q^2 > m_t^2$ Wt-associated $q^2 = m_W^2$ (on-shell)

Production modes distinguished by the number of tagged b jets.



The Tevatron has produced ~ 5000 single-top-quark events (2 fb⁻¹)

A candidate event from $D\emptyset$

+



This flagship measurement of the Fermilab Tevatron has been sighted!



3 measurements by $D\emptyset$ find single-top at $3 + \sigma$. 2 measurements by CDF "exclude" single-top at 95% C.L. 1 measurement by CDF is consistent w/ single-top. Net result DØ observes: $s + t = 4.9 \pm 1.4$ pb (Theory expects 2.9 ± 0.3 pb) It works!

Twice as much data is currently being analyzed for late summer.



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What is single-top-quark production?

Why do we study it?

Why we look at single-top-quark production



Weak interaction structure

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Goal: Determine the structure of the W-t-q vertex.

- Measure CKM couplings "direct measurement of V_{tb} "
- Measure Lorentz structure "spin correlations"



The partial width $\Gamma(t \to bW) = |V_{tb}|^2 \frac{G_F m_t^3}{8\pi\sqrt{2}} (1 - \frac{m_W^2}{m_t^2}) (1 + \frac{2m_W^2}{m_t^2}) (1 - \mathcal{O}(\alpha_s))$ $\approx |V_{tb}|^2 \times 1.42 \text{ GeV.}$

cf. K.G. Chetyrkin *et al.*, PRD 60, 114015 (1999); A. Denner and T. Sack, NPB 358, 46 (1991); R. Migneron and A. Soni, PRL 66, 3105 (1991).

Next most likely Standard Model decays are:

BR ($t \rightarrow sW$)	\approx	$1.6 imes 10^{-3}$ assuming $ V_{ts} = 0.04$
BR ($t ightarrow dW$)	\approx	1×10^{-4} assuming $ V_{td} = 0.01$
BR ($t ightarrow bWZ$)	\approx	$10^{-6} - 10^{-7}$

BR ($t \rightarrow X$) < 10^{-11} , X from a FCNC

Can we see these at the LHC? How would we know? Any decay other than $t \rightarrow bW$ is a good sign of new physics.



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Both CDF and DØ have measured branching fractions in $t\bar{t}$.

$$\frac{\mathrm{BR}(t \to Wb)}{\mathrm{BR}(t \to Wq)} = \frac{|V_{tb}|^2}{|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2} = 1.03 \pm 0.2$$

DØ, PLB 639, 616 (06); CDF, PRL 95, 102002 (05)

Assuming exactly 3 generations and no new physics, unitarity implies: $|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 = 1$

 \Rightarrow $|V_{tb}| > 0.78$ at 95% C.L. (cf. Particle Data Book)

But if we assume 3 generations, unitarity tells us V_{tb} to 4 decimals:

 $\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.9739 - 0.9751 & 0.221 & -0.227 & 0.0029 - 0.0045 \\ 0.221 & -0.227 & 0.9730 - 0.9744 & 0.039 & -0.044 \\ 0.0048 - 0.014 & 0.037 & -0.043 & 0.9990 - 0.9992 \end{pmatrix}$

PDG, PLB 592, 1 (2004)

We have shown consistency, but not much more. ALL we really know is $|V_{tb}| \gg |V_{td}|, |V_{ts}|$.

Is this a problem? How DO we get V_{tb} ?

• Relaxing the assumption of 3 generations, V_{tb} is barely constrained.

$$\Rightarrow \left(\begin{array}{cccccccccc} 0.9730 - 0.9746 & 0.2174 - 0.2241 & 0.0030 - 0.0044 \dots \\ 0.213 & -0.226 & 0.968 & -0.975 & 0.039 & -0.044 \dots \\ 0 & -0.08 & 0 & -0.11 & 0.07 & -0.9993 \dots \\ \vdots & \vdots & \vdots & \vdots & \end{array}\right)$$

• New physics could add to the branching fraction in the denominator, or lead to a fake signal. (e.g. $\tilde{q} \to q' \tilde{\chi}_1^+ \to q' W^+ \tilde{\chi}_1^0$)

There is no way to measure V_{tb} in top-quark decays without measuring the full and all partial widths (say, at a linear collider).

Single-top-quark production cross section is proportional to $|V_{tb}|^2$. q— Measure BR $(t \rightarrow Wb)$ in $t\bar{t}$, extract $|V_{tb}|$ from σ_t with an error $\sim \delta \sigma_t/2$.





 ΔV_{tb} falls along the black line.

DØ

- Extracted: $V_{tb} = 1.3 \pm 0.2$ (s + t)
- s only: $V_{tb} \approx 1.0$; t only: $V_{tb} \approx 1.5$

<u>CDF</u>

- s + t (ME): $V_{tb} \approx 1.0$
- $s \text{ or } t \text{ (ML): } V_{tb} \approx 0.3$
- s only (NN): $V_{tb} \approx 0.9$; t only (NN): $V_{tb} \approx 0.3$

The additional $1fb^{-1}$ of data on tape will clarify this.

Observing Lorentz structure in single-top

$$\mathcal{L} = -\frac{g}{\sqrt{2}} \sum_{q=dsb} \overline{t} \gamma^{\mu} \frac{1}{2} (1-\gamma_5) V_{tq} q W_{\mu}^+$$

- The V A structure of the Lagrangian produces a 100% correlation between the direction of the d quark and the spin s_t of the top quark. M. Jeżabek, NPBPS 37B, 197 (1994)
- The large width of the top quark (~ 1.5 GeV) allows it to decay before it depolarizes (~ $\lambda_{QCD}^2/m_t = 1$ MeV), or hadronizes (~ $\lambda_{QCD} = 300$ MeV). A. Falk, M. Peskin, PRD 49, 3320 (1994)

• $\frac{1}{\Gamma_{(t \to b l \nu)}} \frac{d\Gamma_{(t \to b l \nu)}}{d \cos \theta} = \frac{1}{2} (1 + \frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}} \cos \theta)$ θ is the angle, in the top-quark rest frame, between the direction of the charged lepton and the spin of the top quark. Does this hold at NLO? after cuts?

We'll come back to this...



T. Stelzer, Z.S., S. Willenbrock PRD 58, 094021 (98)

Why we look at single-top-quark production



Weak interaction structure



Goal: Determine the structure of the W-t-q vertex.

- Measure CKM couplings "direct measurement of V_{tb} "
- Measure Lorentz structure "spin correlations"

Direct or indirect new physics New *t*-*q* couplings mostly affect *t*-channel measurement (*Wbj*).

- Larger V_{ts} or V_{td} give PDF enhancement to σ_t .
- FCNC production modes from, e.g. Z-t-c, increase σ_t .

s-channel looks like *t*-channel, since distinguished by number of *b*-tags.

New physics in s-channel vs. t-channel





Using single-top to search for arbitrary charged-vector currents (W')

Working definition: A W' boson is any particle that mediates a flavor-changing charged vector or axial-vector current.

Some model classes

- Left-right symmetric models: Broken $SU(2)_L \times SU(2)_R$
 - · Generic mixing of W_L – W_R
 - R. N. Mohapatra, J. Pati, A. Salam, G. Senjanovic, ...
 - Orbifold-breaking suppressed mixing, enhanced couplings Y. Mimura, S. Nandi, ...
 - \cdot Supersymmetric *L*-*R* models
 - M. Cvetic, J. Pati, ...

Models with additional left-handed W'

- Little Higgs: SU(5)/SO(5), SU(6)/SP(6), SU(N)/SU(N-1), ...
 - T. Gregoire, N. Arkani-Hamed, S. Chang, H. C. Cheng, A. Cohen, I. Low,
 - D. E. Kaplan, E. Katz, O. C. Kong, A. Nelson, M. Schmaltz, W. Skiba,
 - D. Smith, J. Terning, J. Wacker, ...
- · Topcolor topflavor, leptophobic topflavor seesaw, generic mixing
 - H. Georgi, H. J. He, E. Jenkins, X. Li, E. Ma, E. Malkawi, D. Muller, S. Nandi, E. Simmons, T. Tait, C. P. Yuan, ...
- \cdot Extra dimensions: Kaluza-Klein modes of the W
- A. Datta, P. O'Donnell, T. Huang, Z. Lin, X. Zhang, ...
- Non-commuting extended technicolor
 - R. Chivukula, E. Simmons, J. Terning, ...

Fully differential NLO W' cross section and widths w/ arbitrary Lagrangian:

$$\mathcal{L} = \frac{1}{\sqrt{2}} \overline{f}_i \gamma_\mu \left(g_R e^{i\omega} \cos \zeta \, V_{f_i f_j}^R P_R + g_L \sin \zeta \, V_{f_i f_j}^L P_L \right) W' f_j + \text{H.c.}$$

Complete factorization of couplings proven for ALL models. Z.S., PRD 66, 075011 (2002) (hep-ph/0207290).

The differential cross section looks like:

$$\sigma_0 = \int q_1 \otimes \bar{q}_2 \otimes \frac{1}{2\hat{s}} \frac{2N_c}{3} \frac{R_t t(t-m^2) + R_u u(u-m^2)}{(\hat{s}-M_{W'}^2)^2 + M_{W'}^2 \Gamma_{W'}^2} d\mathbf{P}S_2$$

If $W' \equiv W'_L$, W'_R , or small mixing: |V| = |A| $\Rightarrow R_u = 0$; $R_t = \frac{g^4 |V'_i|^2 |V'_f|^2}{8}$ $\Rightarrow \sigma_{NLO} = \sigma^{SM}_{NLO}$ up to overall factor: $(g'/g_{SM})^4$ This holds for any final state, but *s*-channel single-top is special...

The final state is fully reconstructable!



Model-independent searches at the Tevatron



The invariant mass of the *Wbb* provides a nice sharp peak with little to no background.

- Run I: CDF set bound (SM-like) $M_{W'} > 536(566)$ GeV. PRL 90, 081802 (03)
- Run II: $M_{W'} > 630(670) \text{ GeV. D}\emptyset, \text{PLB } 641, 423 \text{ (06)} \\ M_{W'} > 760(790) \text{ GeV. CDF, Note } 8747$
- Run II reach $\sim 900 \text{ GeV}$ (w/ 2 fb⁻¹).
- Use spin correlations to tell if W' has left- or right-handed interactions.



Robust model-independent searches at LHC



Single-top-final state has: $4 \times$ the signal as $W' \rightarrow l\nu$, a mass peak, a signal whether W' is left- or right-handed, better controlled backgrounds, ...



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t Complete coverage of Littlest Higgs early at LHC

Not just Little Higgs, but all models that mix with the W to produce ratios of couplings satisfy

$$\frac{1}{g_1^2} + \frac{1}{g_2^2} + \dots + \frac{1}{g_n^2} = \frac{1}{g_{SM}^2} \approx \frac{1}{0.427}$$

Thus, $1.02g_{_{SM}} < g_{1,2,...} < \sqrt{4\pi}$ (upper limit of effective theory). \Rightarrow For all Littlest Higgs models there will be at least 1 W' with $0.187 < g'/g_{_{SM}} < 5.34$, and preferentially $g'/g_{_{SM}} \sim 1$.



You either discover or rule out the entire parameter space in 1 year!



Understanding perturbative QCD through single-top-quark production

• DIS

The traditional testbed of perturbative QCD have been restricted to Drell-Yan production, e^+e^- to jets, or deep inelastic scattering (DIS).



A key property that all three processes share is a complete factorization of QCD radiation between different parts of the diagrams.

- Drell-Yan \rightarrow Initial-state (IS) QCD radiation only.
- $e^+e^- \rightarrow \text{jets} \rightarrow \text{Final-state}$ (FS) QCD radiation only.
 - \rightarrow Proton structure and fragmentation functions probed. Simple color flow.

s-/t-channel single-top-quark production (A generalized Drell-Yan and DIS)

A perfect factorization through next-to-leading order (NLO) makes single-top-quark production mathematically *identical*[†] to DY and DIS!



Color conservation forbids the exchange of 1 gluon between the fermion lines. Generalized Drell-Yan.

IS/FS radiation are independent.



Double-DIS (DDIS) w/ 2 scales: $\mu_l = Q^2$, $\mu_h = Q^2 + m_t^2$

[†] Massive forms: m_t , m_b , and m_t/m_b are relevant.

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$$\sigma_{\text{obs.}} = \int f_1(x_1, \mu_1) f_2(x_2, \mu_2) \otimes \overline{|M|^2} \otimes d\text{P.S.} \otimes D_i(p_i) \dots D_n(p_n)$$

Theorists factorize (break) the cross section into:

- Initial-state IR singularities swept into parton distribution "functions". These are not physical, but include scheme dependent finite terms: $\overline{\mathrm{MS}}$ — the current standard DIS — ill-defined in all modern PDF sets, could be fixed, but useless.
- A squared matrix element, which represents the bulk of the perturbative calculation effort.
- Phase space which you may not want to completely integrate out.
 ⇒ Exclusive cross sections (jet counting), angular correlations
- Fragmentation functions or jet definitions. These provide the coarse graining to hide final-state IR singularities.



Rethinking the initial state: W-gluon fusion $\rightarrow t$ -channel single-top

W-gluon fusion (circa 1996)



Look at the internal *b*.
The propagator is

$$\frac{1}{(P_g - P_{\bar{b}})^2 - m_b^2} = \frac{1}{-2P_g \cdot P_{\bar{b}}}$$

$$P_g = E_g(1, 0, 0, 1), P_{\bar{b}} = (E_b, \vec{p}_T, p_z)$$

$$P_g \cdot P_{\bar{b}} = E_g(p_z \sqrt{1 + \frac{p_T^2 + m_b^2}{p_z^2}} - p_z)$$

$$\approx E_g p_z(\frac{p_T^2 + m_b^2}{2p_z^2}) \sim (p_T^2 + m_b^2)$$

$$\int_{p_T \text{ cut}} \frac{dp_T^2}{p_T^2 + m_b^2} \to \ln\left(\frac{1}{p_T^2 \text{ cut} + m_b^2}\right)$$

 $m_t \approx 35 m_b! \ \alpha_s \ln \sim .7$ -.8

The same procedure for the W leads to the massive formula for DIS.



Each additional order adds another $\frac{1}{n!} \left[\alpha_s \ln \left(\frac{Q^2 + m_t^2}{m_b^2} \right) \right]^n$

Looks bad for perturbative expansion...

Resummation of large logs and b PDF

Use Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation to sum large logs due to (almost) collinear singularities in gluon splitting.

$$\frac{dQ(\mu^2)}{d\ln(\mu^2)} \approx \frac{\alpha_s}{2\pi} P_{Qg} \otimes g + \frac{\alpha_s}{2\pi} P_{QQ} \otimes Q; Q \ll g$$

$$P_{Qg}(z) = \frac{1}{2}[z^2 + (1-z)^2].$$

$$Q(x,\mu^2) = \frac{\alpha_s(\mu^2)}{2\pi} \ln\left(\frac{\mu^2}{m_Q^2}\right) \int_x^1 \frac{dz}{z} P_{Qg}(z) g\left(\frac{x}{z},\mu^2\right)$$



Olness, Tung, NPB 308, 813 (88) Stelzer, ZS, Willenbrock, Aivazis, Collins, Olness, Tung, PRD 50, 3102 (94) PRD 56, 5919 (1997)

Aside: In the $\overline{\text{MS}}$ scheme, $b(\mu \le m_b) \equiv 0$. DIS scheme is not uniquely defined for heavy quarks. Do you choose $F_2 \equiv 0$ (traditional) or define w.r.t. MS? The first attempt to calculate single-top failed because the DIS scheme was used. Bordes, van Eijk, NPB435, 23 (95)

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Improved perturbative calculation



This is not just a mathematical trick.

The *b* (and *c*) quarks are full-fledged members of the proton structure. Leads to: $b\bar{b} \rightarrow h$, the largest SUSY Higgs cross section at large $\tan \beta$. Zb/Zc, Zbj/Zcj, Wbj, ...

- h

t-channel single-top is Double-DIS

 \mathcal{F}_i^q

 H_i

As mentioned before, color conservation forbids the exhange of a gluon between the light-quark lines and heavy-quark lines.

The inclusive cross section is reduced to calculating structure functions.

$$\begin{split} d\sigma &= \frac{1}{2S} 4 \left(\frac{g^2}{8} \right)^2 \frac{1}{(Q^2 + M_W^2)^2} (2\pi)^2 \frac{1}{4S} dQ^2 dW_1^2 dW_2^2 \left[3F_1(x_1, Q^2) F_1(x_2, Q^2) - \frac{1}{Q^2 + M_W^2} \right] \\ &- \frac{1}{2} F_1(x_1, Q^2) F_2(x_2, Q^2) \frac{W_2^2 + Q^2}{Q^2} - \frac{1}{2} F_2(x_1, Q^2) F_1(x_2, Q^2) \frac{W_1^2 + Q^2}{Q^2} \\ &+ F_2(x_1, Q^2) F_2(x_2, Q^2) \frac{1}{(W_1^2 + Q^2)(W_2^2 + Q^2)} \left(S - \frac{(W_1^2 + Q^2)(W_2^2 + Q^2)}{2Q^2} \right)^2 \\ &+ F_3(x_1, Q^2) F_3(x_2, Q^2) \left(\frac{SQ^2}{(W_1^2 + Q^2)(W_2^2 + Q^2)} - \frac{1}{2} \right) \right]_{P_1}, \end{split}$$

$$x_1 &= \frac{Q^2}{W_1^2 + Q^2}, x_2 = \frac{Q^2 + m_t^2}{W_2^2 + Q^2}, F_1 \equiv \mathcal{F}_1, F_2 \equiv 2x\mathcal{F}_2, F_3 \equiv 2\mathcal{F}_3 \qquad \texttt{W} \neq \texttt{Q} \\ \mathcal{F}_i^q(x, Q^2) &= q(x, \mu^2) \\ &+ \frac{\alpha_s(\mu^2)}{2\pi} \int_x^1 \frac{dz}{z} \left[H_i^q(z, Q^2, \mu^2, \lambda) q\left(\frac{x}{z}, \mu^2\right) + H_i^g(z, Q^2, \mu^2, \lambda) g\left(\frac{x}{z}, \mu^2\right) \right] \\ H_i \sim \text{splitting functions plus corrections} \end{split}$$



Uncertainty in the *b* PDF will soon dominate the *t*-channel single-top uncertainty

$$b_{LO}(x,\mu^2) = \frac{\alpha_s(\mu^2)}{2\pi} \ln\left(\frac{\mu^2}{m_b^2}\right) \int_x^1 \frac{dz}{z} P_{bg}\left(\frac{x}{z}\right) g\left(z,\mu^2\right)$$

Find the uncertainty using the 41 PDFs of CTEQ6. Since the minimum (z_i^0) of the PDF fit is not the minimum of the observable O, we define the Modified Tolerance Method as an improved measure of the uncertainty:

$$\delta O_{+} = \sqrt{\sum_{i=1}^{20} \left(\max[O(z_{i}^{0}+t) - O(z_{i}^{0}), O(z_{i}^{0}-t) - O(z_{i}^{0}), 0] \right)^{2}}$$

 $\delta O_{-} = \sqrt{\sum_{i=1}^{20} \left(\max[O(z_{i}^{0}) - O(z_{i}^{0} + t), O(z_{i}^{0}) - O(z_{i}^{0} - t), 0] \right)^{2}}$

Z.S., PRD 66, 075011 (2002); Z.S., P. Nadolsky, eConf C010630, P511 (2002); eConf C010630, P510 (2002)

Single-top motivated:

- The first heavy-quark PDF uncertainties.
- Asymmetric PDF error equations for observables.





Using scale variation to estimate higher-order uncertainty

Standard lore says that the choice of scale in a perturbative calculation is arbitrary... Standard lore is not quite correct.

If single-top-quark production were exactly Drell-Yan or DIS, then there are unique scale choices.



The PDFs were extracted assuming these scales. Therefore, it is mathematically incorrect to choose any other scale for DY or DIS.

This means there is a subtle (small?) systematic error in all calculations that had not been previously recognized.

With the DDIS choice of scales, the NLO correction to the inclusive cross section is zero within errors.

This will be true for some particle distributions as well!

Scale (μ) dependence of the t-channel jets and top

- The shapes of the p_T and η distributions do not change if you vary the scales. Only the normalization changes.
- If you vary the 4 independent scales[†] at the same time you underestimate the uncertainty.

$\mu/2 - 2\mu$	LO_t (m_t)	NLO_t (m_t)	LO_t (DDIS)	NLO_t (DDIS)
fixed	0.95 pb	1.03 pb	1.07 pb	1.06 pb
μ_l & μ_h	$\pm 1\%$	$\pm 2.5\%$	$^{+0.1}_{-2}\%$	$\pm 3.5\%$
μ_h	$^{-7.5}_{+5.5}\%$	$^{-3.5}_{+4}\%$	$^{-7.2}_{+5.2}\%$	$^{-3}_{+4}\%$
μ_l	$^{+6.7}_{-5.8}\%$	$\pm 1\%$	$^{+8}_{-6.8}\%$	$\pm 0.6\%$

- Summing the independent variations in quadrature predicts $\sim\pm11\%$ uncertainty at LO (consistent with the results).
- At NLO we get $\sim \pm 4\%$ uncertainty due to scale variation.
- [†] (2 factorization, 2 renormalization)



Rethinking the matrix element: A practical problem for experiments

The same large logs that lead to a reordered perturbation for *t*-channel single-top, implied a potentially large uncertainty in measurable cross sections when cuts were applied.

Recall: *t*-channel and *s*-channel are distinguished by the number of *b*-jets. A problem: About 20% of the time, the extra \overline{b} -jet from the *t*-channel process is hard and central.

Real problem: Is the b contamination 20%, 30%, 10%?



Another problem: To distinguish from $t\bar{t}$, the cross section in the W + 2 jet bin has to be known. Counting jets is IDENTICAL to performing a jet veto.

Again: Counting jets is IDENTICAL to performing a jet veto. Inclusive cross sections are not enough.

Fully Differential NLO Techniques

- In 2001, there were few matrix-element techniques or calculations that could deal IR singularities in processes with massive particles.
- Experiments were mostly stuck using LO matrix elements to predict semi-inclusive or exclusive final states.
- We needed methods to provide the 4-vectors, spins, and corresponding weights of exclusives final-state configurations. These needs led to work on 3 techniques:
 - Phase space slicing method with 2 cutoffs.

L.J. Bergmann, Ph.D. Thesis, FSU (89) cf. H. Baer, J. Ohnemus, J.F. Owens, PRD 40, 2844 (89) B.W. Harris, J.F. Owens, PRD 65, 094032 (02)

• Phase space slicing method with 1 cutoff.

W.T. Giele, E.W.N. Glover, PRD 46, 1980 (92) cf. W.T. Giele, E.W.N. Glover, D.A. Kosower, NPB 403, 633 (93) E. Laenen, S. Keller, PRD 59, 114004 (99)

 Massive dipole formalism (a subtraction method) coupled with a helicity-spinor calculation. Invented to solve single-top production. cf. L. Phaf, S. Weinzierl, JHEP 0104, 006 (01)

S. Catani, S. Dittmaier, M. Seymour, Z. Trocsanyi, NPB 627,189 (02)

Massive Dipole Formalism (subtraction)

$$\sigma_{NLO} = \int_{n+1} d\sigma^{Real} + \int_n d\sigma^{Virtual}$$
$$= \int_{n+1} \left(d\sigma^R - d\sigma^A \right) + \int_n \left(d\sigma^V + \int_1 d\sigma^A \right)$$

- $d\sigma^A$ is a sum of color-ordered dipole terms.
 - $d\sigma^A$ must have the same point-wise singular behavior in *D* dimensions as $d\sigma^R$.
 - $\Rightarrow d\sigma^A$ is a local counterterm for $d\sigma^R$.
 - $\int_1 d\sigma^A$ is analytic in *D* dimensions, and reproduces the soft and collinear divergences of $d\sigma^R$.
- Some advantages over Phase Space Slicing are:
 - You can easily project out spin eigenstates.
 ⇒ Explicitly test different spin bases at NLO after cuts.
 - Event generators use color-ordered matrix elements.
- Both methods have some contribution to n-body final states from n+1 phase-space. Hence, you must do 2 separate integrations.

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Phase Space Slicing Method (2 cutoffs)

B.W. Harris, J.F. Owens, PRD 65, 094032 (02)



Phase space is divided into 3 types of regions using two parameters: δ_s , δ_c .

- collinear: for any invariant $\hat{s}_{34}, \hat{t}_{13}, \hat{s}_{35}, \ldots < \delta_c \hat{s};$
- soft: $E_g \leq \delta_s \sqrt{\hat{s}}/2$

both are integrated out analytically.

• hard non-collinear: (finite, all particles well separated and non-soft) is integrated numerically.

After adding virtual and mass factorization terms, all poles cancel.

The triangles marked m give vanishing contribution for $\delta_c \ll \delta_s$.

Logarithmic dependence on the cutoffs cancels in any IR-safe observable at the histogramming stage.

Subtraction vs. phase space slicing

In practical terms, the difference in methods is in how to integrate in the presence of infrared singularities.

$$I = \lim_{\epsilon \to 0^+} \left\{ \int_0^1 \frac{dx}{x} x^{\epsilon} F(x) - \frac{1}{\epsilon} F(0) \right\}$$

Subtraction: Add and subtract F(0) under the integral

$$I = \lim_{\epsilon \to 0^+} \left\{ \int_0^1 \frac{dx}{x} x^{\epsilon} \left[F(x) - F(0) + F(0) \right] - \frac{1}{\epsilon} F(0) \right\}$$
$$= \int_0^1 \frac{dx}{x} \left[F(x) - F(0) \right], \text{ finite up to machine precision}$$

PSS: Integration region divided into two parts $0 < x < \delta$ and $\delta < x < 1$, with $\delta \ll 1$. A Maclaurin expansion of F(x) yields

$$I = \lim_{\epsilon \to 0^+} \left\{ \int_0^{\delta} \frac{dx}{x} x^{\epsilon} F(x) + \int_{\delta}^{1} \frac{dx}{x} x^{\epsilon} F(x) - \frac{1}{\epsilon} F(0) \right\}$$
$$= \int_{\delta}^{1} \frac{dx}{x} F(x) + F(0) \ln \delta + \mathcal{O}(\delta), \text{ take } \lim_{\delta \to 0} \text{ numerically}$$

Remaining $\ln \delta$ singularities removed by summing all integrals I_i .

Explicit t-channel calculation (soft)

Soft region: $0 \le E_g \le \delta_s \frac{\sqrt{s}}{2}$

$$\begin{split} d\sigma_t^{(S)} &= d\sigma_t^{(0)} \left[\frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu^2}{s} \right)^{\epsilon} \right] \left(\frac{A_2^t}{\epsilon^2} + \frac{A_1^t}{\epsilon} + A_0^t \right) \\ A_2^t &= 3C_F \\ A_1^t &= C_F \left[1 - 6\ln\delta_s - 2\ln\left(\frac{-t}{s\beta}\right) - \ln\left(\frac{(m^2-t)^2}{m^2s}\right) \right] \\ A_0^t &= C_F \left[6\ln^2\delta_s - 2\ln\delta_s + 4\ln\delta_s\ln\left(\frac{-t}{s\beta}\right) \\ &+ 2\ln\delta_s\ln\left(\frac{(m^2-t)^2}{m^2s}\right) + \frac{s+m^2}{s-m^2}\ln\left(\frac{s}{m^2}\right) + \ln^2\left(\frac{-t}{s\beta}\right) + 2\text{Li}_2\left(1 + \frac{t}{s\beta}\right) \\ &- \frac{1}{2}\ln^2\left(\frac{s}{m^2}\right) + \ln^2\left(\frac{m^2}{m^2-t}\right) + 2\text{Li}_2\left(\frac{t}{m^2}\right) - 2\text{Li}_2\left(\frac{u}{s+u}\right) \right], \end{split}$$

where the top-quark mass is denoted as m, and $\beta = 1 - m^2/s$.

t-channel (collinear)

Hard region divided into hard collinear (HC) and hard-noncollinear (H \overline{C})

- $-H\overline{C}$ computed numerically in 4 dimensions.
- HC where invariants $s_{ij} = (p_i + p_j)^2$ or $t_{ij} = (p_i p_j)^2$ in the denominator become smaller in magnitude than $\delta_c s$.

Singular regions from FS radiation give:

$$d\sigma_p^{(HC,FS)} = d\sigma_p^{(0)} \left[\frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu^2}{s} \right)^{\epsilon} \right] \left(\frac{A_1}{\epsilon} + A_0 \right)$$

$$A_1 = C_F \left(2\ln\delta_s + \frac{3}{2} - 2\ln\beta \right)$$

$$A_0 = C_F \left[\frac{7}{2} - \frac{\pi^2}{3} - \ln^2\delta_s - \ln^2\beta + 2\ln\delta_s\ln\beta - \ln\delta_c \left(2\ln\delta_s + \frac{3}{2} - 2\ln\beta \right) \right]$$

Singular regions from IS radiation give:

$$d\sigma_{p,C}^{ij} = d\sigma_p^{(0)} \left[\frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu^2}{s} \right)^{\epsilon} \right] \left[\tilde{f}_j^H(z,\mu_F) + \left(\frac{A_1^{sc}}{\epsilon} + A_0^{sc} \right) f_j^H(z,\mu_F) \right]$$
$$A_1^{sc} = C_F \left(2\ln\delta_s + \frac{3}{2} \right) \quad A_0^{sc} = C_F \left(2\ln\delta_s + \frac{3}{2} \right) \ln\left(\frac{s}{\mu_F^2} \right)$$
$$\tilde{f}_i^H(z,\mu_F) \text{ is a universal modified PDF.}$$

t-channel (virtual)

Virtual contribution has two pieces. One \propto Born, one not:

$$\begin{split} d\sigma_{p}^{(V)} &= d\sigma_{p}^{(0)} \left[\frac{\alpha_{s}}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu^{2}}{s} \right)^{\epsilon} \right] \left(\frac{A_{2}^{V}}{\epsilon^{2}} + \frac{A_{1}^{V}}{\epsilon} + A_{0}^{V} \right) + \left(\frac{\alpha_{s}}{2\pi} \right) d\tilde{\sigma}_{p}^{(V)} \\ A_{2}^{V} &= C_{F} \left\{ \left[-2 \right] - [1] \right\} \qquad \text{Note: } \lambda = t/(t-m^{2}) \\ A_{1}^{V} &= C_{F} \left\{ \left[-3 - 2\ln\left(\frac{s}{-q^{2}}\right) \right] + \left[-\frac{5}{2} - 2\ln\left(1-\lambda\right) - \ln\left(\frac{s}{m^{2}}\right) \right] \right\} \\ A_{0}^{V} &= C_{F} \left\{ \left[-\ln^{2}\left(\frac{s}{-q^{2}}\right) - 3\ln\left(\frac{s}{-q^{2}}\right) - 8 - \frac{\pi^{2}}{3} \right] \\ &+ \left[-\frac{1}{2}\ln^{2}\left(\frac{s}{m^{2}}\right) - \frac{5}{2}\ln\left(\frac{s}{m^{2}}\right) - 2\ln\left(1-\lambda\right)\ln\left(\frac{s}{m^{2}}\right) - 6 \\ &- \frac{1}{\lambda}\ln\left(1-\lambda\right) - \ln^{2}\left(1-\lambda\right) - 2\ln\left(1-\lambda\right) + 2\text{Li}_{2}\left(\lambda\right) - \frac{\pi^{2}}{3} \right] \right\} \\ d\tilde{\sigma}_{t}^{(V)} &= \frac{1}{2s}\frac{1}{4}g^{4}|V_{ud}|^{2}|V_{tb}|^{2}C_{F}\frac{m^{2}su}{t}\ln\left(\frac{m^{2}}{m^{2}-t}\right)\left(\frac{1}{t-M_{W}^{2}}\right)^{2}d\Gamma_{2} \,, \end{split}$$

We can keep track of light- and heavy-quark contributions separately.

t-channel (summing it up)

We now see cancellation of singularities: $\frac{1}{c^2}$: $A_2^t + A_2^V = 3C_F + (-3C_F) = 0$ $\frac{1}{\epsilon} : A_1^t + A_1^V + A_1 + 2A_1^{sc} = 0 \quad \text{(e.g., } C_F[1 + 3/2 + (-3 - 5/2) + 2(3/2)] = 0\text{)}$ Final 2-to-2 result $\sigma^{(2)} = \left(\frac{\alpha_s}{2\pi}\right) \sum_{i} \int dx_1 dx_2 \left\{ f_a^{H_1}(x_1, \mu_F) f_b^{H_2}(x_2, \mu_F) \times \right\}$ $\left| d\sigma_{p}^{(0)} \left(A_{0}^{p} + A_{0}^{V} + A_{0} + 2A_{0}^{sc} \right) + d\tilde{\sigma}_{p}^{(V)} \right|$ $+ d\sigma_{p}^{(0)} \left[f_{a}^{H_{1}}(x_{1},\mu_{F}) \tilde{f}_{b}^{H_{2}}(x_{2},\mu_{F}) + \tilde{f}_{a}^{H_{1}}(x_{1},\mu_{F}) f_{b}^{H_{2}}(x_{2},\mu_{F}) \right] + (x_{1} \leftrightarrow x_{2}) \right\}$ Final 2-to-3 result

$$\sigma^{(3)} = \sum_{a,b} \int dx_1 dx_2 \frac{1}{2s} \int_{H\overline{C}} g^4 |V_{ud}|^2 |V_{tb}|^2 \overline{\Psi}(p_i) d\Gamma_3$$

 $\alpha_{s\,l(h)}$ and the luminosity functions $L_{l(h)} = f_a^{H_1}(x_1, \mu_{F\,l(h)}) f_b^{H_2}(x_2, \mu_{F\,l(h)})$ are evaluated using the scales at the light(heavy)-quark lines, respectively. $\sigma^{\text{final}} = \sigma^{(2)} + \sigma^{(3)}$ is cutoff independent

Cut-off dependence of NLO correction

Here $\delta_c = \delta_s/300$, using CTEQ5M1 PDFs.



The 2-to-2 and 2-to-3 components of the correction each depend logarithmically on the cutoffs, but the sum depends only linearly on δ_c and δ_s . So take δ_c and δ_s to 0.



Rethinking jet definitions and phase space: Experiments need exclusive t + 1 jet at NLO

ZTOP, Z.S., PRD 70, 114012 (2004) (hep-ph/0408049)

	# b-jets	tj~(Wbj)	$tjj \; (Wbjj)$
s-channel	= 2	0.620 pb $^{+13}_{-11}\%$	0.168 pb $^{+24}_{-19}\%$
	= 1	0.022 pb $^{+24}_{-19}\%$	(NNLO)
t-channel	= 1	0.950 pb $^{+16}_{-15}\%$	0.152 pb $^{+17}_{-14}\%$
	= 2	0.146 pb $^{+21}_{-16}\%$	0.278 pb $^{+21}_{-16}\%$

Cuts: $p_{Tj} > 15$ GeV, $|\eta_j| < 2.5$, no cuts on t Jet definition: $\Delta R_{k_T} < 1.0 ~(\approx \Delta R_{\text{cone}} < 0.74)$

Breakdown of shape-independent uncertainties

Process	$\times \delta m_t$ (GeV)	$\mu/2$ – 2μ	PDF	b mass	$lpha_s(\delta_{ m NLO})$
s -channel $par{p}$	$+2.33 \\ +2.71 $	$^{+5.7}_{-5.0}\%$	$^{+4.7}_{-3.9}\%$	< 0.5%	$\pm 1.4\%$
pp	$^{-1.97}_{+2.26}\%$	$\pm 2\%$	$^{+3.3}_{-3.9}\%$	< 0.4%	$\pm 1.2\%$
t-channel $par{p}$	$^{-1.6}_{+1.75}\%$	$\pm 4\%$	$^{+11.3}_{-8.1}\%$	< 1%	$\pm 0.01\%$
pp	$^{-0.73}_{+0.78}\%$	$\pm 3\%$	$^{+1.3}_{-2.2}\%$	< 1%	$\pm 0.1\%$

Every number on this page, even the concept of *t*-channel single-top, required a new or revised understanding of QCD.

- b PDFs \rightarrow t-channel
- PDF uncertainties
- multiple scales
- 2 expansions: α_s , $1/\ln$
- Fully differential NLO jet calculations



Applied Understanding

Jet calculations Theory vs. experiment Angular correlations

Zack Sullivan, Southern Methodist University - p.44/64

Physically you can think of phase space slicing as forming a "pre-jet" that is much smaller than your final jet of radius R. ($\delta_c \ll \delta_R$)



The essential challenge of NLO differential calculations is dealing with final-state soft or collinear IR divergences.

Unlike inclusive NLO calculations, exclusive NLO calculations are only well-defined in the presence of a jet definition or hadronization function.



How do we interpret fully differential NLO calculations? Paradigm of "jet calculations"

- We are calculating jets not "better partons."
 - NLO calculations are not well defined w/o a jet definition.



• "Bad things" happen if you treat jets as partons...

Transverse momenta distributions at NLO

At LO, a *d*-quark recoils against the top quark in *t*-channel.



NLO "*d*-jet" (no cuts)



- Perturbation theory is not terribly stable at low p_{Td} (or even high p_{Td}).
- This is not what we want.
 Be careful what you ask for!

We measure the highest E_T jet



The highest E_T jet recoils against the top. The measurable change in shape is comparable to the scale uncertainty.

Pseudorapidity of highest- E_T jet j_1 .

One of the most distinct features of the *t*-channel exchange is that the final-state light-jet tends to be very forward.



The highest- E_T jet (j_1) is slightly more central at NLO than at LO. This is expected since j_1 takes most of the recoil of the top quark. Note the double-DIS character of isolation between heavy-quark radiation and light-quark radiation is maintained. Having an additional \bar{b} does not change η_{j_1} . You can study the effect of the cone size used in the k_T algorithm on the reconstructed p_T and η of the jet.



For "reasonable" values of R the variation is < 10%, but must be checked in any given analysis.

Upshot: NLO exclusive calculations give jets not partons. Without some thought, mismatches between theory and experiment can be larger than the theory error alone would indicate. What do theory and experiment have to do with each other?



Experiment_{theory} = Hadrons from Monte Carlo tuned to data. Theory_{experiment} = Monte Carlo event records of π , e^+ , γ , etc.

Event generators vs. NLO t-channel $t\overline{b}$ $(Wb\overline{b})$

Z.S., PRD 70, 114012 (04)

Initial-state radiation (ISR) is generated by backward evolution of angular-ordered showers.

 \Rightarrow The jet containing the extra \overline{b} comes from soft ISR.





- PYTHIA/HERWIG completely underestimate the $Wb\bar{b}$ final state.
- The background to $WH \rightarrow Wb\bar{b}$ is much larger than we thought!
- Lesson: *n*-jets+showers $\neq n + 1$ jets. \Rightarrow Need NLO matching.

Zack Sullivan, Southern Methodist University – p.51/64

Matching showering event generators to NLO

A simple prescription for success

- 1. Generate all phase-space configurations using MadEvent/CompHEP.
- 2. Feed the events into PYTHIA/HERWIG and shower them.
- 3. Create 4 samples with a given jet definition, e.g. cones with $\Delta R = 0.7$, or k_T with $\Delta R = 1$, and minimal cuts:
 - t + j, t + b, t + j + j, t + j + b
 - Remember that comparisons are made at generator level, i.e. apply jet reconstruction to $\pi^{\pm}s$, γs , e^{\pm} or μ^{\pm} inside jets, etc. in the event record.
- 4. Normalize each sample to the NLO prediction after cuts, and with the same jet definition.
 - NLO jets must be E_T ordered.
- 5. Finally, feed into the detector simulations.

CDF and DØ have signals, and yet...



-1.0 -0.5 0.0 cost

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1.0



Angular correlations: the current frontier

Both *s*- and *t*-channel single-top are matrix elements go like:

$$[p_d \cdot (p_t - m_t s_t)][p_e \cdot (p_t - m_t s_t)]$$

In top rest frame, $p_t = m_t(1, 0, 0, 0)$, and $s_t = (0, \hat{s})$. Choose top spin projection $\hat{s} = \hat{d}$. $\Rightarrow \sigma \propto (1 + \cos \theta_{e^+ d}^t)$

- s-channel 98% of \bar{d} from \bar{p} $\Rightarrow \sigma \propto (1 + \cos \theta_{e^+ \bar{p}}^t)$
- *t*-channel *d* in highest- E_t non-*b*-tagged jet j_1 3/4 of the time. $\Rightarrow \sigma \propto (1 + \cos \theta_{e^+ j_1}^t)$ For rest, $\Rightarrow \sigma \propto (1 + \cos \theta_{d j_1}^t \cos \theta_{e^+ j_1}^t)$ dilution $\cos \theta_{d j_1}^t = 1 - Q^2 / (E_d^t E_{j_1}^t) \sim 0.86$

We are saved by kinematically-induced correlations. i.e., *t*-channel pole pushes jet forward.





Angular correlations in single-top-quark and Wjj production at NLO

Z.S., PRD 72, 094034 (2005) (hep-ph/0510224)



Original comparison of t-channel single-top and Wjj background done at LO.

Used in neural-nets by CDF and DØ.



- 1. Do spin-induced angular correlations survive higher-order radiation?
- 2. Is the background really insensitive to the angular distributions that typify the signal? If so, does this survive complex cuts on the data?
- 3. The angular distributions are properly defined in the top quark rest frame. How much of these correlations is an artifact of that frame?
- 4. Does this lead to better discriminates between S, B? e.g., ways to avoid b-tagging? Are there other useful particle correlations?

t LO vs. NLO

<u>t-channel</u>

- Insensitive to top reconstruction (similar in LAB frame) top is non-relativistic, so little boost.
- Additional ISR b-jets confuse which jet has the d.

s-channel

- NLO = LO \times K-factor
- Issue: Dominated by top reconstruction.
 - W fit to $e + \not\!\!E_T$.
 - I naively assigned a random b jet to top decay.

Wjj (+ $Wb\bar{b}$, $Wc\bar{c}$)

• NLO = LO \times K-factor

Spin-dependent ME fed into PYTHIA/HERWIG get all correlations (not all shown), as long as NLO-matched ME are used for *t*-channel.



†

Can you avoid *b*-tagging? No, but it raises a subtlety...

In the top rest frame, the *b* recoils against the W (and the *e*), while j_1 wants to be close to *e*.

- Proposal: Define "b" to be the e^+ jet with the largest angle w.r.t. e^+ in the top rest frame. Correct b > 80% for s-/t-chan.
 - Equiv. cut: $\cos heta_{e``b''}^t < \cos heta_{ej_1}^t$

Angular cuts generically induce correlations. This is why we need reliable predictions. Warning: Two experimental biases select the largest angle jet (this cut):

- 1. *b*-tagging $\propto E_{Tb}$, picks jet recoiling vs. *W*.
- 2. Top-mass cut, also picks jet recoiling vs. W.





NLO $\cos \theta_{ej_1}^t$ VS. $\cos \theta_{eb}^t$ VS. $\cos \theta_{bj_1}^t$

t-channel NLO-LO< 3%

all Wjj



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I propose these acceptance cuts as a starting point: 1. $\cos \theta_{eb}^t < \cos \theta_{ej_1}^t$.

- $2. \cos \theta_{bj_1}^t < \cos \theta_{ej_1}^t.$
- 3. $\cos \theta_{bj_1}^t < 0.6-0.8$.
- 4. $\cos \theta_{ej_1}^t > 0-0.4 \text{ or } \cos \theta_{eb}^t > -0.8.$
- 5. $M_{bj_1} > 80-120 \text{ GeV}$
- Result: $S/\sqrt{B} \approx 1.25 \times S_0/\sqrt{B_0}$, $S/B \approx 3 \times S_0/B_0$ Overall $S \sim 0.4 \times S_0$, but $B \sim B_0/7!$



- These correlations are not completely utilized in the Tevatron analyses.

— Strong angular cuts are typical in difficult analyses: SUSY, $H \rightarrow WW$, ... We MUST check angular correlations for the LHC analyses. $\cos \theta_{ej_1}^t$ VS. $\cos \theta_{eb}^t$ VS. $\cos \theta_{bj_1}^t$ at LHC

t-channel Similar to

 $Wb\overline{b}$





The main background at LHC is from $t\bar{t}$, but there are large handles here.

NOTE: \overline{t} production is just like s-channel, i.e., if you boost the system to average $\eta = 0$, $\cos \theta_{ep}^{t}$ is the relevant angle, where p is on the same side as the electron.

Single-top-quark production is the new Drell-Yan and DIS.

Single-top-quark production forces us to reconsider our intuitions and develop new technologies that push the frontiers of perturbative physics:

- 1. Understanding electroweak physics
 - We have a first measurement of weak interaction structure.

$$t \longrightarrow \int_{V_W}^{q} -i \frac{g}{\sqrt{2}} V_{tq} \gamma^{\mu} \frac{1}{2} (1 - \gamma_5) \qquad V_{tb} = 1.3 \pm 0.2 \text{ (D}\text{)}$$

Angular correlations will play an important role in improving S/B

- Anything that modifies the effective coupling of t to anything effects single-top
- Any new charged current (W') is observable up to 5.5 TeV

Single-top-quark production is the new Drell-Yan and DIS.

- 2. Single-top has changed how we think about the cross section. $\sigma_{obs.} = \int f_1(x_1, \mu_1) f_2(x_2, \mu_2) \otimes \overline{|M|^2} \otimes dP.S. \otimes D_i(p_i) \dots D_n(p_n)$
 - The *b* (and *c*) are fundamental parts of the proton. Including their contribution in improved pQCD calculations is essential in obtaining the correct cross sections and kinematics.
 - Studying the effect of scales is subtle. s-channel = Drell Yan, $\mu = M_{tb}^2$ t-channel = DDIS, $\mu_l = Q^2$, $\mu_h = Q^2 + m_t^2$
 - There are 3 methods to calculate NLO ME: PSSM, PSSM2, MDF Differential production included in ZTOP.
 - Exclusive jet cross sections including spin effects and top decay:

MCFM 5.1

Matrix-element calculation of *t*-,*s*-channel, *Wt* Gives distributions

MC@NLO 3.3

Showering MC (w/ HERWIG) of *t-,s*-channel

Gives events

Need to verify angular correlations Zack Sullivan, Southern Methodist University – p.63/64

Single-top-quark production is the new Drell-Yan and DIS.

- 3. Study of single-top has forced us to be more exact
 - The "paradigm of jet calculations" Differential NLO calculations describe jets, not partons
 - NLO matching is essential to model many processes There are now many schemes: MLM, CKKW, MC@NLO — significantly better understanding is needed
 - Angular correlations are relatively untested at NLO.
 Most searches for new physics require tight cuts on phase space.
 This induces large sensitivity to correlations.

It will be vital to the success of the LHC and beyond to develop close interactions between theory and experiment of the type single-top-quark production has enjoyed.

The study of single-top production has led to a decade of discoveries You will lead the next decade of discovery