Introduction to Diffraction

Maria Beatriz Gay Ducati

GFPAE – IF-UFRGS

beatriz.gay@ufrgs.br

CTEQ :: 2012 Summer School
July 30-August 09, 2012 – Lima, Peru
Outline

➢ Review of diffraction
  ✓ Mandelstam variables
  ✓ Regge Theory
  ✓ Pomeron

➢ Diffraction at HERA
  ✓ Deep Inelastic Scattering
  ✓ Diffractive DIS
  ✓ Diffractive Structure Functions

➢ Diffraction at Tevatron
  ✓ Diffraction at Tevatron
  ✓ Hadronic case
  ✓ Diffractive Structure Functions

➢ Diffraction at LHC

✓ Partonic Structure of the Pomeron
✓ Results
✓ W / Z production
✓ Higgs Production
Processes in channels s and t

Two body scattering can be calculated in terms of two independent invariants, s and t, Mandelstam variables.

\[
\begin{align*}
    s &= (A + B)^2 = (C + D)^2 \\
    t &= (A - C)^2 = (B - D)^2
\end{align*}
\]

where \(s \sim 2\pi^2\), \(t \sim 2\pi^2\), \(\sqrt{s} \sim m\), \(\sqrt{t} \sim m\) and \(t > 0\) in s-channel diagram.

\[A_{AB\rightarrow CD}(s,t) = A_{AC\rightarrow BD}(t,s)\]

by crossing symmetry

\[A(s,t) \approx \frac{g^2}{m^2 - t}\]

pion exchange

\[g\quad \text{coupling constant}\]

Singularity (pole) in non-physical region \(t > 0\) in s-channel diagram \(t = m^2\).
What is Diffraction?

- Diffraction is characterized as a colour singlet exchange process in $pp$ physics

- Described in terms of t channel exchanges
What is exchanged in the $t$ channel?

- Elastic
- SD
- DPE
Regge Theory

✓ Ressonances as observables in $t$ channel

✓ $t$ channel trajectory

$$\alpha(t) = \alpha(0) + \alpha' t$$

slope

✓ Amplitudes through partial waves decomposition

$$A(s, t) \approx \sum_{t=0}^{\infty} (2l + 1) A_l(t) P_l(\cos \theta)$$

$$A_l(t) \quad \rightarrow \quad \text{sum on poles (Reggeons)}$$

$$\frac{d\sigma}{dt} \approx \frac{1}{s^2} |A(s, t)|^2 = g(t) \left( \frac{s}{s_0} \right)^{2\alpha(t)-2}$$

Good for hadron interactions with low momentum transfer $\pi^- p \rightarrow \pi^0 n$
Regge Theory

• At fixed $t$, with $s \gg t$

• Amplitude for a process governed by the exchange of a trajectory $\alpha(t)$ is

$$A(s,t) \sim \left(\frac{s}{s_0}\right)^{\alpha(t)}$$

• No prediction for $t$ dependence

• Elastic cross section

$$\frac{d\sigma_{el}}{dt} \sim s^{2\alpha(t)-2}$$

• Total cross section considering the optical theorem
Diffractive scattering

Consider elastic \( A \ B \rightarrow A \ B \)

\[
\frac{d\sigma_{el}}{dt} \approx \frac{1}{s^2} \sum_X \]

\[
\sigma_{tot} = \frac{1}{2s} \sum_X \]

by Regge

\[
\alpha(0) \approx 1 + \varepsilon, \quad \alpha(0) \leq 0.5
\]

optical theorem

\[
\sigma_{tot}^{AB} \approx \frac{1}{s} \text{Im}(A_{el}^{AB})_{t=0} \approx s^{\alpha(0)-1}
\]

vacuum trajectory

Pomeron \( \alpha_{IP}(t) \)

vacuum quantum numbers

\[
\sim \frac{1}{s}
\]
Regge theory

\[ \sigma_T \propto \text{IP exchange} \]

\[ \sigma_{\text{dif}} \propto \text{vacuum quantum numbers} \]

Hadrons scattering

- Elastic
- Single
- Double
- Double Pomeron Exchange
- Totally Inelastic

Following particle distribution in rapidity
Rapidity

\[ y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z} \approx -\ln \tan \frac{\theta}{2} = \eta \]

\[ \eta \] pseudorapidity for a particle with \((E, \bar{p}_\perp, p_z)\) and polar angle \(\theta\)

Diffraction defined by
- leading proton
- large rapidity gap
Diffractive processes

- Elastic scattering
- Single diffraction
- Double diffraction
- Double Pomeron (Photon) Exchange
- Multi Pomeron Exchange

**Tevatron/LHC**
- Higgs: photo-, NLO
- W, Z: NLO
- QQ: NRQCD, NLO

**Tevatron/LHC**
- Higgs: photo-, NLO
- QQ: NLO
Regge phenomenology in QCD

- Elastic amplitude mediated by the Pomeron exchange

\[ A_{el}(t) \propto \left[ i - \cotg \frac{\pi \alpha_P(t)}{2} \right] \left( \frac{s}{s_0} \right)^{\alpha_P(t) t} \]

\[ \alpha_P(t) = \alpha_P^0 + \alpha'_P t \]

What is the Pomeron?

- A Regge pole: not exactly, since \( \alpha_{IP}(t) \) varies with \( Q^2 \) in DIS
- DGLAP Pomeron \( \Rightarrow \) specific ordering for radiated gluon
  \[ k_{i+1}^2 < k_i^2 \leq Q^2 \quad \text{and} \quad x \leq x_{i+1} \leq x_i \]
- BFKL Pomeron \( \Rightarrow \) no ordering \( \Rightarrow \) no evolution in \( Q^2 \)
- Other ideas?
The Pomeron

- Regge trajectory has intercept which does not exceed 0.5
- Reggeon exchange leads to total cross sections decreasing with energy
- Experimentally, hadronic total cross sections as a function of $s$ are rather flat around

$$\sqrt{s} \sim (10 - 20) \text{ GeV}^2$$

INCREASE AT HIGH ENERGIES

- Chew and Frautschi (1961) and Gribov (1961) introduced a Regge trajectory with intercept 1 to account for asymptotic total cross sections
- This reggeon was named Pomeron (IP)
The Pomeron

- From fitting elastic scattering data
  
  \[ \alpha_{IP} \approx 0.25 \text{ GeV}^{-2} \]

- For the intercept total cross sections implies
  
  \[ \alpha_{IP}(0) \approx 1 \]

- Pomeron dominant trajectory in the elastic and diffractive processes

- Known to proceed via the exchange of vacuum quantum numbers in the \( t \)-channel

\[ \text{IP: } P = +1; \quad C = +1; \quad I = 0; \]
**Pomeron trajectory**

Regge-type

\[
\frac{d\sigma}{dt}(W) = \exp(b_0 t) W^{2[2\alpha_{IP}(t)+2]}
\]

- First measurements in h-h scattering

\[
\alpha(t) = \alpha(0) + \alpha' t
\]

- \(\alpha(0)\) and \(\alpha'\) are fundamental parameters to represent the basic features of strong interactions

- \(\alpha(0)\) energy dependence of the diffractive cross section

\[
\frac{d\sigma}{dt}(W) = W^{4\alpha(0)-4} \exp(bt)
\]

- \(\alpha'\) slope

**Soft Pomeron values**

\[
\alpha(0) \sim 1.09 \\
\alpha' \sim 0.25
\]
Diffractive scattering

\[
\alpha_{IP}(t) = 1.085 + 0.25t \quad (p \, p, \, p \, \bar{p})
\]

The interactions described by the exchange of a IP are called diffractive so

\[
\frac{d\sigma_{tot}^{AB}}{dt} \approx \frac{\beta_{AIP}^2(t) \beta_{BIP}^2(t)}{16\pi} s^{2\alpha_{IP}-2}
\]

\[\beta_{iIP} \quad \text{Pomeron coupling} \quad \text{with external particles}\]

Valid for \[s \rightarrow \infty, \quad \frac{t}{s} \rightarrow 0\]

High s \[\sigma_{tot}^{AB} \approx \beta_{AIP}(0) \beta_{BIP}(0) s^{\alpha_{IP}-1}\]
Studies of diffraction

- In the beginning hadron-hadron interactions

- Exclusive diffractive production: $\rho$, $\varphi$, $J/\psi$, $Y$, $\gamma$

Gluon exchange

SOFT
low momentum transfer

HARD
high momentum transfer
Studies of diffraction

- Cross section
  \[ \sigma(W) \propto W^\delta \]

- \( \delta \) expected to increase from soft (\( \sim 0.2 \) is a “soft” Pomeron) to hard (\( \sim 0.8 \) is a “hard” Pomeron)

- Differential cross section
  \[ \frac{d\sigma}{dt} \propto e^{-b|t|} \]

- \( b \) expected to decrease from soft (\( \sim 10 \text{ GeV}^{-2} \)) to hard (\( \sim 4 - 5 \text{ GeV}^{-2} \))
Froissart limit

- No diffraction within a black disc

- It occurs only at periphery, \( b \sim R \) in the Froissart regime, \( R \propto \ln(s) \)

- Unitarity demands

  \[
  \begin{align*}
  \sigma_{\text{tot}} &\propto \sigma_{\text{el}} \propto \ln^2(s) \\
  \sigma_{\text{sd}} &\propto \ln(s)
  \end{align*}
  \]

  i.e. \( \frac{\sigma_{\text{sd}}}{\sigma_{\text{tot}}} \propto \frac{1}{\ln(s)} \)

- Donnachie-Landshoff approach may not be distinguishable from logarithmic growth

  Any \( s^\lambda \) power behaviour would violate unitarity

  At some point should be modified by unitarity corrections

  - Rate of growth \( \sim s^{0.08} \) would violate unitarity only at large energies
Some results

✓ Many measurements in pp

✓ Pomeron exchange trajectory

\[ \alpha(t) \sim 1.10 + 0.25 t \]

Pomeron universal and factorizable applied to total, elastic, diffractive dissociation cross sections in \( ep \) collisions
DIFFRACTION AT HERA
HERA

ep collisions at sqrt(s) ~ 300 GeV (1992 - 2007)

P (920 GeV)

e (27.5 GeV)

HERA


PETRA
HERA experiments and diffraction

**HERA:** ~10% of low-x DIS events are diffractive

→ study QCD structure of high energy diffraction with virtual photon
DEEP INELASTIC SCATTERING

- Scattering of a charged (neutral) lepton off a hadron at high momentum transfer

\[
x = \frac{Q^2}{2P \cdot q} = \frac{Q^2}{Q^2 + W^2 - m_N^2}
\]

Bjorken’s \( x \)

- Measurement of the energy and scattering angle of the outgoing lepton

- Inclusive DIS: Probes partonic structure of the proton \( (F_2) \)

- Electron-proton centre of mass energy

\[
s = (k + P)^2 \approx 4E_e E_p
\]

- Photon virtuality

\[
Q^2 = -q^2 = -(k - k')^2 \approx 4E_e E_e' \sin^2 \frac{\theta}{2}
\]

- Photon-proton centre of mass energy

\[
W^2 = (q + P)^2
\]

- Square 4-momentum at the \( p \) vertex

\[
t = (P' - P)^2
\]
DEEP INELASTIC SCATTERING

• Introducing the hadronic tensor $W_{\mu\nu}$

$$W_{\mu\nu} = \frac{1}{2\pi} \int d^4z e^{iq \cdot z} < N | J^\mu(z) J^\nu(0) | N >$$

• Spin average absorbed in the nucleon state $|N>$$

• The leptonic tensor $L_{\mu\nu}$ defined as (lepton masses neglected)

$$L_{\mu\nu} = 2(l_\mu l'_\nu + l_\nu l'_\mu - g_{\mu\nu} l \cdot l')$$

• The differential cross section for DIS takes the form

$$\frac{d\sigma}{dE' d\Omega} = \frac{\alpha_{em}^2}{2m_N Q^4} \frac{E'}{E} L_{\mu\nu} W_{\mu\nu}$$

$\Omega \equiv (\theta, \varphi)$

• It can be expressed in terms of two structure functions $W_1$ and $W_2$

$$\frac{d\sigma}{dE' d\Omega} = \frac{4\alpha_{em}^2 E'^2}{Q^4} \left[ 2W_1 \sin^2 \frac{\theta}{2} + W_2 \cos^2 \frac{\theta}{2} \right]$$

Solid angle identifying the direction of the outgoing lepton
DEEP INELASTIC SCATTERING

• Introducing the dimensionless structure functions

\[ F_1(x, Q^2) \equiv m_N W_1(\nu, Q^2) \]

\[ F_2(x, Q^2) \equiv \nu W_1(\nu, Q^2) \]

• The hadronic tensor in terms of \( F_1 \) and \( F_2 \) reads

\[ W_{\mu\nu} = 2 \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) F_1(x, Q^2) + \frac{2}{(P \cdot q)} \left[ \left( P_\mu - \frac{P \cdot q}{q^2} q_\mu \right) \left( P_\nu - \frac{P \cdot q}{q^2} q_\nu \right) \right] F_2(x, Q^2) \]

• The differential cross section for DIS takes the form

\[
\frac{d\sigma}{dx dy} = \frac{4\pi\alpha_{em}^2 s}{Q^4} \left\{ \frac{xy^2 F_1(x, Q^2) + \left( 1 - y - \frac{xy m_N^2}{s} \right) F_2(x, Q^2)}{s} \right\}
\]

\[ F_T = 2xF_1 \]

\[ F_L = F_2 - 2xF_1 \]

\[
\sigma^{\gamma^*N}(x, Q^2) = \frac{4\pi^2\alpha_{em}}{Q^2} F_2(x, Q^2)
\]
DIFFRACTIVE DIS

- Proton escapes in the beam pipe
- No quantum numbers exchanged between $\gamma^*$ and $p$

**NO COLOUR FLUX**

**LARGE RAPIDITY GAP**

- pQCD motivated description of strong interactions
Kinematics of DDIS

- Described by 5 kinematical variables
- Two are the same appearing in DIS:
  - Bjorken’s $x$
  - Squared momentum transfer at the lepton vertex

\[ x = \frac{Q^2}{2P.q} = \frac{Q^2}{W^2 + Q^2 - m_N^2} \approx \frac{Q^2}{W^2 + Q^2} \]

\[ Q^2 = -q^2 = -(k - k')^2 \]

or

\[ y = \frac{P.q}{P.k} \approx \frac{Q^2}{xs} \]
**Kinematics of DDIS**

- New kinematic variables are dependent of the three-momentum $P'$ of the outgoing proton.

- Invariant quantities
  
  $t = -(P' - P)^2 \approx -\frac{P'^2}{x_F}$

- $x_{IP} = \frac{(P - P') \cdot q}{P \cdot q} = \frac{M^2 + Q^2 - t}{W^2 + Q^2 - m_N^2} \approx \frac{M^2 + Q^2}{W^2 + Q^2} = 1 - x_F$

- $M^2$ is the invariant mass of the $X$ system.

- $x_F$ is the Feynman variable
  
  $x_F \equiv \frac{|p_z'|}{p_z}$

- $\beta$ is the momentum fraction of the parton inside the Pomeron
  
  $\beta = \frac{Q^2}{2q \cdot (P - P')} = \frac{Q^2}{M^2 + Q^2 - t} \approx \frac{Q^2}{M^2 + Q^2}$


**Diffractive Structure Functions**

- DDIS differential cross section can be written in terms of two structure functions

\[ F_1^{D(4)} \quad \text{and} \quad F_2^{D(4)} \]

- Dependence of variables: \( x, Q^2, x_{IP}, t \)

- Introducing the longitudinal and transverse diffractive structure functions

\[
F_L^{D(4)} = F_2^{D(4)} - 2x F_1^{D(4)} \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \}
Diffractive Structure Functions

✓ Data are taken predominantly at small $y$

✓ Cross section little sensitivity to $R_{D}^{(4)}$

✓ $F_{L}^{D(4)} \ll F_{T}^{D(4)}$ for $\beta < 0.8 - 0.9$ neglect $R_{D}^{(4)}$ at this range

\[
\frac{d\sigma_{\gamma^{*}p}^{D}}{dx dQ^2 dx_{IP} dt} = \frac{4\pi\alpha_{em}^2}{xQ^4} \left(1 - y + \frac{y^2}{2}\right) F_{2}^{D(4)}(x, Q^2, x_{IP}, t)
\]

✓ $F_{2}^{D(4)}$ proportional to the cross section for diffractive $\gamma^{*}p$ scattering

\[
F_{2}^{D(4)}(x, Q^2, x_{IP}, t) = \frac{Q^2}{4\pi\alpha_{em}^2} \frac{d\sigma_{\gamma^{*}p}^{D}}{dx_{IP} dt}
\]

✓ $F_{2}^{D(4)}$ dimensional quantity

$F_{2}^{D(4)} \equiv \frac{dF_{2}^{D}(x, Q^2, x_{IP}, t)}{dx_{IP} dt}$

$F_{2}^{D}$ is dimensionless
Diffractive Structure Functions

✓ When the outgoing proton is not detected
  
  no measurement of \( t \)

✓ Only the cross section integrated over \( t \) is obtained

\[
\frac{d\sigma_{\gamma^* p}^D}{dx dq^2 dx_{IP}} = \frac{4\pi\alpha^2_{em}}{xQ^4} \left( 1 - y + \frac{y^2}{2} \right) F_2^{D(3)}(x, Q^2, x_{IP})
\]

✓ The structure function \( F_2^{D(4)} \) is defined as

\[
F_2^{D(3)}(x, Q^2, x_{IP}) = \int_0^\infty d|t| F_2^{D(4)}(x, Q^2, x_{IP}, t)
\]
Diffractive Parton Distributions

✓ Factorization theorem holds for diffractive structure functions

✓ These can be written in terms of the diffractive partons distributions

✓ It represents the probability to find a parton in a hadron $h$, under the condition the $h$ undergoes a diffractive scattering

✓ QCD factorization formula for $F_2^D$ is

$$
\frac{dF_2^D(x, Q^2, x_{IP}, t)}{dx_{IP} dt} = \sum_i \int_x^{x_{IP}} d\xi \frac{df_i (\xi, \mu^2, x_{IP}, t)}{dx_{IP} dt} F_2^i \left( \frac{x}{\xi}, Q^2, \mu^2 \right)
$$

✓ $df_i (\xi, \mu^2, x_{IP}, t) / dx_{IP} dt$ is the diffractive distribution of parton $i$

✓ Probability to find in a proton a parton of type $i$ carrying momentum fraction $\xi$

✓ Under the requirement that the proton remains intact except for a momentum transfer quantified by $x_{IP}$ and $t$
Diffractive Parton Distributions

✓ Perturbatively calculable coefficients

\[
F_2 \left( \frac{x}{\xi}, Q^2, \mu^2 \right)
\]

✓ Factorization scale \( \mu^2 = M^2 \)

✓ Diffractive parton distributions satisfy DGLAP equations

✓ Thus

\[
\frac{\partial}{\partial \ln \mu^2} \frac{df_i (\xi, \mu^2, x_{IP}, t)}{dx_{IP} dt} = \sum_j \int_{\xi}^1 \frac{d\xi}{\xi} P_{ij} \left( \frac{\xi}{\zeta}, \alpha_s (\mu) \right) \frac{df_j (\xi, \mu^2, x_{IP}, t)}{dx_{IP} dt}
\]

✓ “Fracture function” is a diffractive parton distribution integrated over \( t \)

\[
\frac{df_i (\xi, \mu^2, x_{IP})}{dx_{IP}} = \int_{x_{IP}^2 m_N^2}^{\infty} d | t | \frac{df_i (\xi, \mu^2, x_{IP}, t)}{dx_{IP} dt}
\]
Partonic Structure of the Pomeron

✓ It is quite usual to introduce a partonic structure for $F_{2}^{IP}$

✓ At Leading Order ➔ Pomeron Structure Function written as a superposition of quark and antiquark distributions in the Pomeron

$$F_{2}^{IP} (\beta, Q^2) = \sum_{q,q'} e_q^2 \beta q^{IP} (\beta, Q^2)$$

✓ $\beta = \frac{x}{x_{IP}}$ ➔ interpreted as the fraction of the Pomeron momentum carried by its partonic constituents

✓ $q^{IP} (\beta, Q^2)$ ➔ probability to find a quark $q$ with momentum fraction $\beta$

inside the Pomeron

✓ This interpretation makes sense only if we can specify unambiguously the probability of finding a Pomeron in the proton and assume the Pomeron to be a real particle (INGELMAN-SCHLEIN / 1985)
Partonic Structure of the Pomeron

✓ Diffractive quark distributions and quark distributions of the Pomeron are related

\[
\frac{df_q (\beta, Q^2, x_{IP}, t)}{dx_{IP} dt} = \frac{1}{16\pi^2} \left| g_{IP} (t) \right|^2 x_{IP}^{-2\alpha_{IP}(t)} q^{IP} (\beta, Q^2)
\]

• Introducing gluon distribution in the Pomeron

\[ g^{IP} (\beta, Q^2) \]

• Related to \( df_g / dx_{IP} dt \) by

\[
\frac{df_g (\beta, Q^2, x_{IP}, t)}{dx_{IP} dt} = \frac{1}{16\pi^2} \left| g_{IP} (t) \right|^2 x_{IP}^{-2\alpha_{IP}(t)} g^{IP} (\beta, Q^2)
\]

• At Next-to-Leading order, Pomeron Structure Function acquires a term containing \( g^{IP} (\beta, Q^2) \)
QCD factorization

PDFs from inclusive diffraction predict cross sections for exclusive diffraction

Inclusive

Exclusive (dijet)

Hard scattering QCD matrix element perturbatively calculated

Diffractive parton densities are the same for all processes

\[ \sigma^D(\gamma^* p \rightarrow Xp) = \sum_{\text{parton}_i} f^D_i(x, Q^2, x_{IP}, t) \cdot \sigma^{\gamma^*i}(x, Q^2) \]

\( \sigma^{\gamma^*i} \) universal hard scattering cross section (same as in inclusive DIS)

\( f^D_i \) diffractive parton distribution functions → obey DGLAP universal for diffractive ep DIS (inclusive, di-jets, charm)
Analysis of $F_2^D (\beta, Q^2)$

- Hard partons in IP
  weak $\beta$ dependence

- QCD evolution
  weak log $Q^2$ dependence

- Scattering on point-like charges
  approximate scaling

$$F_2^{IP} (\beta) = F_2^{D(4)} / f_{IP/p} \approx \frac{x}{\beta} F_2^{D(4)} (x)$$
Results from inclusive diffraction (2002)

\[ f_i^D(x, Q^2, x_{IP}, t) = f_{IP/p}(x_{IP}, t) \cdot f_i^{IP}(\beta = x / x_{IP}, Q^2) \]

\[ \sigma_{diff} = flux(x_P) \cdot \mathcal{O}(\beta, Q^2) \]

\[ f_{IP/p}(x_{IP}, t) = A_{IP} \frac{e^{B_{IP}t}}{x_{IP}^{2\alpha_{IP}(t)-1}} \]

Reduced cross section from inclusive diffractive data

\[ \sigma_r^{D(3)}(\beta, Q^2, x_{IP}) \approx F_2^{D(3)} \]

- get diffractive PDFs from a NLO (LO) DGLAP QCD Fit to inclusive data from 6.5 GeV^2 to 120 GeV^2

extrapolation of the Fit to lower Q2 to higher Q2
Results from inclusive diffraction (2008)

$3.5 \leq Q^2 \leq 1600 \text{ GeV}^2$

Gives a reasonably good description of inclusive data from $3.5 \text{ GeV}^2 - 1600 \text{ GeV}^2$

Data on low $\beta$ for high $Q^2$
Diffractive Parton Densities (H1-02)

- Determined from NLO QCD analysis of diffractive structure function
- More sensitive to quarks
- Gluons from scaling violation, poorer constraint
- Gluon carries about 75% of pomeron momentum
- Large uncertainty at large $z_P$

If factorisation holds, jet and HQ cross sections give better constraint on the gluon density
Diffractive Parton Densities (H1-06)

- Total quark singlet and gluon distributions obtained from NLO QCD H1. DPDF Fit A,

- Range $0.0043 < z < 0.8$, corresponding to experiment

- Central lines surrounded by inner errors bands
  - experimental uncertainties

- Outer error bands
  - experimental and theoretical uncertainties

$z$ is the momentum fraction of the parton inner the Pomeron
Diffractive Parton Densities (ZEUS-06)

- Recent Zeus fits to higher statistical large rapidity gaps
- Improved heavy flavour treatment
- DPDFs dominated by gluon density
- It extends to large z
DIFFRACTION AT TEVATRON
TEVATRON

ep collisions at \( \sqrt{s} \sim 300 \text{ GeV} \) (1992–2007)
Diffraction at Tevatron

- Proton-antiproton scattering at highest energy
- Soft & Hard Diffraction

\[ \xi < 0.1 \Rightarrow \mathcal{O}(1) \text{ TeV } \text{“Pomeron beams”} \]

Structure function of the Pomeron
\[ F(\beta, Q^2) \]

Diffraction dynamics?
Exclusive final states?

- Gap dynamics in pp presently not fully understood!

\[ \xi = \frac{M_X^2}{s} \]
Diffraction at Tevatron

• IS paper (1985) first discussion of high-$p_T$ jets produced via Pomeron exchange

• Events containing two jets of high transverse energy and a leading proton were observed in proton-antiproton scattering at $\sqrt{s} = 630$ GeV by the CERN UA8 experiment (Bonino et al. 1988)

• Rate of jet production in this scattering 1 – 2%

• It was in agreement with the predicted order of magnitude made by IS

• Since then hard diffraction in proton-proton scattering was pursued by the CDF and D0 Collaborations at the Tevatron

• UA8 group reported some evidence for a hard Pomeron substructure $\beta (1- \beta)$ (Brand et al. 1992)
Diffraction at Tevatron

- Kinematical range for physical process at Tevatron broader
- Experiments have been investigating diffractive reactions
- First results of diffractive events were reported in 1994-1995 (Abachi et al. 1994; Abe et al. 1995)
- Then, three different classes of processes investigated at the Tevatron
  - Double diffraction
  - Single diffraction
  - Double Pomeron Exchange
- Both CDF and D0 detectors covered the pseudorapidity range $|\eta| \leq 4 - 5$
Diffractive Physics at 1.96 TeV

- Physics observed at the Tevatron described by colour exchange perturbative QCD
- There is also electroweak physics on a somewhat smaller scale
- There is a significative amount of data that is not described by colour exchange pertubative interaction
Hard Single diffraction

- Large rapidity gap
- Intact hadrons detected

- Diffractive production of some objects can be studied

**Jets, W, J/ψ, b ...**

- Measurement of the ratio of diffractive to non-diffractive production

<table>
<thead>
<tr>
<th>Hard component</th>
<th>Fraction (R) %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dijet</td>
<td>0.75 ± 0.10</td>
</tr>
<tr>
<td>W</td>
<td>1.15 ± 0.55</td>
</tr>
<tr>
<td>b</td>
<td>0.62 ± 0.25</td>
</tr>
<tr>
<td>J/ψ</td>
<td>1.45 ± 0.25</td>
</tr>
</tbody>
</table>

All fractions ~ 1%
Hadronic case

To calculate diffractive hard processes at the Tevatron

- Using diffractive parton densities from HERA
- Obtain cross sections one order of magnitude higher

D0 data on diffractive dijets

IP approach

- IP parton densities
- IP flux

Universal?

IP an effect from QCD dynamics?
Hadronic case

Factorization breakdown between HERA and Tevatron

IS doesn’t describe DATA diffractive cross section

Momentum fraction of parton in the Pomeron
Pomerion as composite

• Considering Regge factorization we have

\[ F_{2}^{D(4)}(x, Q^2, x_{IP}, t) = f_{IP/p}(x_{IP}, t) F_{2}^{IP}(\beta, Q^2) \]

Data \rightarrow Good fit with added Reggeon for HERA

Pomerion as gluons

• Elastic amplitude \rightarrow neutral exchange in t-channel

• Smallness of the real part of the diffractive amplitude \rightarrow nonabeliance

Born graphs in the abelian and nonabelian (QCD) cases look like
Diffractive dijet cross section

\[ \sigma(\bar{p}p \rightarrow \bar{p}X) \approx F_{jj} \otimes F_{jj}^{D} \otimes \hat{\sigma}(ab \rightarrow jj) \]

- Study of the diffractive structure function

\[ F_{jj}^{D} = F_{jj}^{D}(x, Q^2, t, \xi) \]

- Experimentally determine diffractive structure function

\[ R_{SD}^{ND}(x, \xi) = \frac{\sigma(SD_{jj})}{\sigma(ND_{jj})} = \frac{F_{jj}^{D}(x, Q^2, \xi)}{F_{jj}(x, Q^2)} \]

Will factorization hold at Tevatron?
Gap Survival Probability (GSP)

\[
\langle S \rangle^2 = \frac{\int d^2 b |A(s,b)|^2 P^S(s,b)}{\int d^2 b |A(s,b)|^2}
\]

- **\( A(s,b) \)** amplitude of the particular process (parameter space \( b \)) of interest at center-of-mass energy \( \sqrt{s} \)
- **\( P^S(s,b) \)** probability that no inelastic interaction occurs between scattered hadrons

GAP
region of angular phase space devoid of particles
Survival probability
fulfilling of the gap by hadrons produced in interactions of remanescent particles

**GAP**
Large Rapidity Gap

\( f_0 \)
\( f \)
\( \text{IP} \)
KMR – Gap Survival Probability


- Survival probability of the rapidity gaps
- Associated with the Pomeron (double vertical line)
  - Calculated
    - * single diffraction (SD)
    - * central diffraction (CD)
    - * double diffraction (DD)
- FPS (cal) forward photon spectrometer (calorimeter),
- Detection of isolated protons (events where leading baryon is either a proton or a N*)
KMR model

• **t dependence** of elastic pp differential cross section in the form \( \exp(Bt) \)
• **Pion-loop insertions** in the Pomeron trajectory
• **Non-exponential form** of the proton-Pomeron vertex \( \beta(t) \)
• **Absorptive corrections**, associated with eikonalization

(a) Pomeron exchange contribution;
(b-e) Unitarity corrections to the pp elastic amplitude.
(f) Two pion-loop insertion in the Pomeron trajectory
KMR model

- GSP KMR values

<table>
<thead>
<tr>
<th>$\sqrt{s}$ (TeV)</th>
<th>2b (GeV$^{-2}$)</th>
<th>Survival probability $S^2$ for:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>SD (FPS)</td>
</tr>
<tr>
<td>0.54</td>
<td>4.0</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>5.5</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>7.58</td>
<td>0.27</td>
</tr>
<tr>
<td>1.8</td>
<td>4.0</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>5.5</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>8.47</td>
<td>0.24</td>
</tr>
<tr>
<td>14</td>
<td>4.0</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>5.5</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>10.07</td>
<td>0.21</td>
</tr>
</tbody>
</table>

- GSP considering multiple channels
• Survival probability as a function of $\Omega (s,b = 0)$

• $\Omega$ opacity (optic density) of interaction of incident hadrons

• Ratio of the radius in soft and hard interactions $a = R_s / R_h$

• Suppression due to secondary interactions by additional spectators hadrons
GLM model

- Eikonal model originally explain the exceptionally mild energy dependence of soft diffractive cross sections

- s-channel unitarization enforced by the eikonal model

- Operates on a diffractive amplitude in different way than elastic amplitude

- Soft input obtained directly from the measured values of $\sigma_{\text{tot}}$, $\sigma_{\text{el}}$ and hard radius $R_H$

- $F1C$ and $D1C$ different methods from GLM model

<table>
<thead>
<tr>
<th>$\sqrt{s}$ (GeV)</th>
<th>$S_{CD}^2(F1C')$</th>
<th>$S_{CD}^2(D1C')$</th>
<th>$S_{SD\text{incl}}^2(F1C')$</th>
<th>$S_{SD\text{incl}}^2(D1C')$</th>
<th>$S_{DD}^2(F1C')$</th>
<th>$S_{DD}^2(D1C')$</th>
</tr>
</thead>
<tbody>
<tr>
<td>540</td>
<td>14.4%</td>
<td>13.1%</td>
<td>18.5%</td>
<td>17.5%</td>
<td>22.6%</td>
<td>22.0%</td>
</tr>
<tr>
<td>1800</td>
<td>10.9%</td>
<td>8.9%</td>
<td>14.5%</td>
<td>12.6%</td>
<td>18.2%</td>
<td>16.6%</td>
</tr>
<tr>
<td>14000</td>
<td>6.0%</td>
<td>5.2%</td>
<td>8.6%</td>
<td>8.1%</td>
<td>11.5%</td>
<td>11.2%</td>
</tr>
</tbody>
</table>
Applications

- Electroweak Vector boson processes
  - $W^+$ and $Z^0$ production

- Quarkonium hadroproduction at NLO
  - Application to Heavy-Ion Collisions

- Quarkonium production in NRQCD factorization
  - $J/\psi$ + gamma
  - Upsilon + gamma
  - Nuclear production

- Higgs boson production
  - Diffractive factorization
  - Ultraperipheral Collisions
Electroweak vector boson production

W/Z Production

Leading order \( \Rightarrow \) W and Z produced by a quark in the Pomeron

- General cross section for W and Z

\[
\frac{d\sigma}{d\mu^2} = \sum_{a,b} \int dx_a f_{a/p}(x_a, \mu^2) f_{b/p}(x_b, \mu^2) \frac{d\hat{\sigma}(pp \rightarrow [W/Z]X)}{d\hat{t}}
\]

- \( W^+ (W^-) \) inclusive cross section

\[
\frac{d\sigma}{d\eta_{e^-(e^+)}} = \sum_{a,b} \int dE_T f_{a/p}(x_a) f_{b/p}(x_b) \left[ \frac{V_{ab}^2 G_F^2}{6\Gamma_W M_W} \right] \hat{t}^2(\hat{u}^2) \frac{1}{\sqrt{A^2 - 1}}
\]

\[
\mu^2 = M_W^2 \quad \hat{t} = -E_T M_W \left[ A + \sqrt{(A^2 - 1)} \right]
\]

- Total decay width \( \Gamma_W = 2.06 \text{ GeV} \)

- \( V_{ab} \) is the CKM Matrix element

- \( W^+ (W^-) \) dependence in \( t \) (u) channel

\( G_F = 1.166 \times 10^{-5} \text{ GeV}^2 \)
W (Z) Diffractive cross sections

- **W^+(-)** diffractive cross section

\[
\frac{d\sigma}{d\eta_{e^-(e^+)}} = \sum_{a,b} \int dx_{IP} g(x_{IP}) \int dE_T f_{a/IP}(x_a) f_{b/p}(x_b) \left[ \frac{V_{ab}^2 G_F^2}{6s\Gamma_W M_W} \right] \frac{\hat{t}^2 (\hat{u}^2)}{\sqrt{A^2 - 1}}
\]

- **Z^0** diffractive cross section

\[
\sigma = \sum_{a,b} \int \frac{dx_{IP}}{x_{IP}} \int \frac{dx_b}{x_b} \int \frac{dx_a}{x_a} f(x_{IP}) f_{a/IP}(x_a, \mu^2) f_{b/p}(x_b, \mu^2) \left[ \frac{2\pi C_{ab}^Z G_F M_Z^2}{3\sqrt{2} s} \right] \frac{d\hat{\sigma}(ab \to ZX)}{d\hat{t}}
\]

- **f_{a/IP}** is the quark distribution in the IP parametrization of the IP structure function (H1)

- **g (x_{IP})** is the IP flux integrated over t

\[
g(x_{IP}) = \int_{-\infty}^0 f_{IP/p}(x_{IP}, t) dt
\]

- **C_{qq}^Z = 1/2 - 2 |e_q|^2 \sin^2 \theta_W + 4 |e_q|^2 \sin^4 \theta_W

- **\theta_W** is the Weinberg or weak-mixing angle
Energies and Mandelstan Variables

- Total Energy

\[ E_e = \frac{\sqrt{s}}{4} [x_a (1 + \cos \theta) + x_b (1 - \cos \theta)] \]

- Longitudinal Energy

\[ E_L = \frac{\sqrt{s}}{4} [x_a (1 + \cos \theta) - x_b (1 - \cos \theta)] \]

- Transversal Energy

\[ E_T = \frac{M_w}{2} \sin \theta \]

- Mandelstan variables of the process

\[ \hat{t} = (p_c - p_a)^2 = -\frac{s}{2} (1 - \cos \theta) \]

\[ \hat{u} = (p_c - p_b)^2 = -\frac{s}{2} (1 + \cos \theta) \]

\[ \hat{s} = (p_a + p_b)^2 = M_w^2 \]

\[ \cos \theta = \pm \frac{\sqrt{A^2 - 1}}{A} \]

\[ A = \frac{M_w}{2 E_T} \]
W$^+$ and W$^-$ Cross Sections

Tevatron [ sqrt(s) = 1.8 TeV ]

IS + GSP models

<table>
<thead>
<tr>
<th>Pseudo-rapidity</th>
<th>Data (%)</th>
<th>R(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$</td>
<td>\eta_e</td>
</tr>
<tr>
<td></td>
<td>$1.5 &lt;</td>
<td>\eta_e</td>
</tr>
<tr>
<td></td>
<td>Total $W \rightarrow e\nu$</td>
<td>$0.64 \pm 0.24$</td>
</tr>
<tr>
<td></td>
<td>Total $Z \rightarrow e^+e^-$</td>
<td>$0.89 \pm 0.25$</td>
</tr>
<tr>
<td></td>
<td>Total $Z \rightarrow e^+e^-$</td>
<td>$1.44 \pm 0.80$</td>
</tr>
</tbody>
</table>

GSP is an average of KMR ($S_2 = 0.09$) and GLM ($S_2 = 0.086$) estimations

• Tevatron, without GSP – 7.2 %

$R = \frac{\int_{-\eta}^{\eta} \sigma_{W^+}^{inc} + \sigma_{W^-}^{inc}}{\int_{-\eta}^{\eta} \sigma_{W^+}^{inc} + \sigma_{W^-}^{inc}}$

• Ranges

$|\eta_e| < 1.1$

$1.5 < |\eta_e| < 2.5$

$|\eta| < 1.1$
Quarkonium production in NRQCD

Focus on the following single diffractive processes:

\[ pp \rightarrow p + \left( J/\psi + \gamma \right) + X \quad\text{and}\quad pp \rightarrow p + \left( Y + \gamma \right) + X \]

Diffractive ratios as a function of transverse momentum \( p_T \) of quarkonium state:

- Quarkonia produced with large \( p_T \) are easy to detect.

**Singlet contribution**

\[ g + g \rightarrow \gamma + (c\bar{c})\left(3S_1^{(1)}\right) \rightarrow J/\psi \]

**Octet contributions**

\[ g + g \rightarrow \gamma + (c\bar{c})\left(1S_0^{(8)}\right) \rightarrow J/\psi, \]
\[ g + g \rightarrow \gamma + (c\bar{c})\left(3P_J^{(8)}\right) \rightarrow J/\psi, \]

Higher contribution on high \( p_T \).
NRQCD factorization

$x_1 (x_2)$ is the momentum fraction of the proton carried by the gluon

$\frac{M^2}{s} \leq x_1 < 1$

$x_2 = \frac{x_1 \bar{x}_T e^{-y} - 2\tau}{2x_1 - \bar{x}_T e^y}$

$\tau = \frac{m_{\psi}^2}{s}$

Invariant mass of $J/\psi + \gamma$ system

Cross section written as

$\sigma(H) = \sum_n c_n 0 | O^n_H | 0 \rangle$

Coefficients are computable in perturbation theory

Matrix elements of NRQCD operators

Matrix elements

\[
\langle 0 | O_n^H | 0 \rangle = \sum_X \sum_\lambda \langle 0 | \kappa_{n}^\dagger | H(\lambda) + X \rangle \langle H(\lambda) + X | \kappa_n | 0 \rangle
\]

Bilinear in heavy quarks fields which create as a pair $Q\bar{Q}$ Quarkonium state

\[
\frac{d\sigma}{dt}(g + g \to J/\psi + \gamma) = \frac{\pi^2 e_c^2 \alpha_s \alpha_s^2 m_c}{s^2} \left[ \frac{10}{9} \left( \frac{s^2 s_1^2 + t^2 t_1^2 + u^2 u_1^2}{s_1^2 t_1^2 u_1^2} \right) \langle O_8^{J/\psi}(3S_1) \rangle + \frac{3}{2} \frac{tu}{s_1^2 m_c^2} \langle O_1^{J/\psi}(3S_1) \rangle + \frac{16}{27} \left( \frac{s^2 s_1^2 + t^2 t_1^2 + u^2 u_1^2}{s_1^2 t_1^2 u_1^2} \right) \langle O_8^{J/\psi}(3S_1) \rangle + \frac{3}{2} \frac{tu}{s_1^2 m_c^2} \langle O_1^{J/\psi}(3S_1) \rangle \right]
\]

\[
s_1 = s - 4m_c^2, \quad t_1 = t - 4m_c^2, \quad u_1 = u - 4m_c^2
\]

\[
e_c = \frac{2}{3}
\]

$\alpha_s$ running
Diffractive cross section

\[
\frac{d^2 \sigma_{SD}}{dy dp_T} = \int_{x_{\mathbf{p}}^{\min}}^{x_{\mathbf{p}}^{\max}} dx_{\mathbf{p}} \int_1 \frac{M^2}{s x_{\mathbf{p}}} dx_1 \int_{-1}^0 dt f_{\mathbf{p}/p}(x_{\mathbf{p}}, t) \times g_{\mathbf{IP}}(x_{\mathbf{IP}}, \mu_F^2) g_{\mathbf{p}}(x_2, \mu_F^2) \frac{4 x_1 x_{\mathbf{IP}} x_2 p_T}{2 x_1 x_{\mathbf{IP}} - \bar{x}_T e^y} \frac{d\hat{\sigma}}{d\hat{t}}
\]

Momentum fraction carried by the Pomeron

Squared of the proton's four-momentum transfer

Pomeron flux factor

\[
f_{\mathbf{IP}/p}(x_{\mathbf{IP}}, t) \propto x_{\mathbf{IP}}^{1-2\alpha(t)} F^2(t)
\]

Pomeron trajectory

\[
\alpha(t) = \alpha_{\mathbf{IP}}(0) + \alpha'_\mathbf{IP} t
\]
Results for $J/\psi + \gamma$

- Predictions for inclusive and diffractive cross sections
- LHC, Tevatron and RHIC
- Diffractive cross sections considering GSP ($<|S|^2>$)
- $B = 0.0594$ is the branching ratio into electrons

$1 \leq p_T \leq 20$ at LHC
Results for $\Upsilon + \gamma$

- Predictions of inclusive cross section
- LHC, Tevatron and RHIC
  
  $-1 < |y| < 1$

- $B = 0.0238$ is the branching ratio into electrons
Results for $J/\psi + \gamma$ at LHC

- $B = 0.0594$
- Absolute value cross section strongly dependent

- Diffractive cross sections (DCS) without GSP ($\langle |S|^2 \rangle$)
- Comparison between two different sets of diffractive gluon distribution (H1)

Quark mass
NRQCD matrix elements
Factorization scale
Results for $\Upsilon + \gamma$ at LHC

- $B = 0.0238$
- Absolute value cross section strongly dependent
- Diffractive cross sections (DCS) without GSP ($<|S|^2>$)
- Comparison between two different sets of diffractive gluon distribution (H1)

Quark mass
NRQCD matrix elements
Factorization scale
Diffractive ratio at LHC

<table>
<thead>
<tr>
<th>$p_T$ [GeV]</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{d\sigma_{\text{inc}}}{dp_T}$ ($J/\Psi$)</td>
<td>97.04</td>
<td>14.54</td>
<td>2.82</td>
<td>0.68</td>
</tr>
<tr>
<td>$\frac{d\sigma_{\text{SD}}}{dp_T}$ ($J/\Psi$)</td>
<td>0.78</td>
<td>0.10</td>
<td>0.017</td>
<td>0.0036</td>
</tr>
<tr>
<td>$R_{\text{SD}}$ [%] ($J/\Psi$)</td>
<td>0.8</td>
<td>0.69</td>
<td>0.6</td>
<td>0.53</td>
</tr>
<tr>
<td>$\frac{d\sigma_{\text{inc}}}{dp_T}$ ($\Upsilon$)</td>
<td>5.91</td>
<td>2.49</td>
<td>1.00</td>
<td>0.41</td>
</tr>
<tr>
<td>$\frac{d\sigma_{\text{SD}}}{dp_T}$ ($\Upsilon$)</td>
<td>0.036</td>
<td>0.013</td>
<td>0.0054</td>
<td>0.0018</td>
</tr>
<tr>
<td>$R_{\text{SD}}$ [%] ($\Upsilon$)</td>
<td>0.6</td>
<td>0.53</td>
<td>0.54</td>
<td>0.44</td>
</tr>
</tbody>
</table>

- Slightly large diffractive ratio in comparison to **
- Could explain the $p_T$ dependence in our results

$[\sigma] = \text{pb}$ considering FIT A

** C. S. Kim, J. Lee and H. S. Song, Phys Rev D59 (1999) 014028

$J/\psi + \gamma$

<table>
<thead>
<tr>
<th>$R_T$ (GeV)</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_T$ ($p_T$)(%)</td>
<td>0.52</td>
<td>0.52</td>
<td>0.50</td>
<td>0.48</td>
<td>0.47</td>
<td>0.46</td>
<td>0.44</td>
</tr>
</tbody>
</table>

This work

$$\mu_F = \sqrt{\frac{p_T^2 + m_{\psi}^2}{4}}$$

Ref **

$$\mu_F = E_T$$

Renormalized Pomeron flux

$<|S|^2> = 0.06$

Q$^2$ evolution in the gluon density

No Q$^2$ evolution in the gluon density
Higgs production

Higgs production

✓ Standard Model (SM) of Particle Physics has unified the Electromagnetic interaction and the weak interaction;

✓ Particles acquire mass through their interaction with the Higgs Field;

✓ Existence of a new particle: the Higgs boson

✓ The theory does not predict the mass of H;

✓ Predicts its production rate and decay modes for each possible mass;

➤ Exclusive diffractive Higgs production $pp \rightarrow p\,H\,p : 3-10\,\text{fb}$

➤ Inclusive diffractive Higgs production $pp \rightarrow p + X + H + Y + p : 50-200\,\text{fb}$

Albert de Roeck X BARIONS (2004)
Focus on the gluon fusion

\[ pp \rightarrow gg \rightarrow H \]

Main production mechanism of Higgs boson in high-energy pp collisions

Gluon coupling to the Higgs boson in SM

triangular loops of top quarks

Lowest order to \( gg \) contribution
Gluon fusion

Lowest order partonic cross section expressed by the gluonic width of the Higgs boson

\[ \hat{\sigma}_{LO}(gg \rightarrow H) = \frac{\sigma_0}{m_H^2} \delta(\hat{s} - m_H^2) \]

\[ \sigma_0 = \frac{8\pi^2}{m_H^3} \Gamma_{LO}(H \rightarrow gg) \]

\[ \Gamma_{LO}(H \rightarrow gg) = \frac{G_F \alpha_s^2}{36\sqrt{2}\pi^3} m_H^3 \left| \frac{3}{4} \sum_Q A_Q(\tau_Q) \right|^2 \]

\[ A_Q(\tau_Q) = 2[\tau + (\tau - 1)f(\tau)]/\tau^2 \]

\[ f(\tau) = \arcsin^2 \sqrt{\tau} \]

\[ \hat{s} \quad gg \text{ invariant energy squared} \]

\[ \tau_Q = m_H^2/4m_Q^2 \]

Quark Top

dependence
LO hadroproduction

- Lowest order two-gluon decay width of the Higgs boson

\[ \sigma_0 = \frac{G_F \alpha_s^2(\mu^2)}{288 \sqrt{2\pi}} \left| \frac{3}{4} \sum_q A_Q(\tau_Q) \right|^2 \]

- Gluon luminosity

\[ \frac{d\mathcal{L}^{gg}}{d\tau} = \int_{\tau}^1 \frac{dx}{x} g(x, M^2) g(\tau/x, M^2) \]

- Lowest order proton-proton cross section

\[ \sigma_{LO}(pp \rightarrow H) = \sigma_0 \tau_H \frac{d\mathcal{L}^{gg}}{d\tau_H} \]

- Renormalization scale

\[ \mu_Q \quad \mathcal{T} = \tau_H \quad \tau_H = \frac{m_H^2}{s} \]

- Invariant pp collider energy squared
Virtual diagrams

- Coefficient \( C(\tau_Q) \) contributions from the virtual two-loop corrections

- Regularized by the infrared singular part of the cross section for real gluon emission

\[
C'(\tau_Q) = \pi^2 + c(\tau_Q) + \frac{33 - 2N_F}{6} \ln \frac{\mu^2}{m_H^2}
\]

- Infrared part
- Finite \( \tau_Q \) dependent piece
- Logarithmic term depending on the renormalization scale \( \mu \)
Delta functions

- Contributions from gluon radiation in $gg$, $gq$ and $qq$ scattering

- Dependence of the parton densities

\[ \Delta \sigma_{gg} = \int_{\tau_H}^1 d\tau \frac{dL_{gg}}{d\tau} \times \frac{\alpha_s}{\pi} \sigma_0 \left\{ -\hat{\tau} P_{gg}(\hat{\tau}) \log \frac{M^2}{\hat{s}} + d_{gg}(\hat{\tau}, \tau_Q) \right. \\
+ 12 \left[ \left( \frac{\log (1 - \hat{\tau})}{1 - \hat{\tau}} \right)_+ - \hat{\tau} [2 - \hat{\tau} (1 - \hat{\tau})] \log (1 - \hat{\tau}) \right] \left\} \right. \\
\Delta \sigma_{gq} = \int_{\tau_H}^1 d\tau \sum_{q, \bar{q}} \frac{dL_{gq}}{d\tau} \times \frac{\alpha_s}{\pi} \sigma_0 \left\{ \hat{\tau} P_{gq}(\hat{\tau}) \left[ -\frac{1}{2} \log \frac{M^2}{\hat{s}} + \log (1 - \hat{\tau}) \right] + d_{gq}(\hat{\tau}, \tau_Q) \right\} \\
\Delta \sigma_{qq} = \int_{\tau_H}^1 d\tau \sum_q \frac{dL_{qq}}{d\tau} \times \frac{\alpha_s}{\pi} \sigma_0 d_{qq}(\hat{\tau}, \tau_Q) \]

\( \hat{\tau} = \tau_H / \tau \)

Renormalization scale

QCD coupling $\alpha_s(\mu^2)$ in the radiative corrections and LO cross sections
NLO Cross Section

- Gluon radiation two parton final states
  \[ gg \rightarrow H \]

- Invariant energy \( \hat{s} \geq m_H^2 \) in the \( gg, gq \) and \( q\bar{q} \) channels

- New scaling variable \( \hat{\tau} = \frac{m_H^2}{\hat{s}} \) supplementing \( \tau_H \) and \( \tau_Q \)

- The final result for the pp cross section at NLO
  \[
  \sigma(pp \rightarrow H + X) = \sigma_0 \left[ 1 + C \frac{\alpha_s}{\pi} \right] \tau_H \frac{d\mathcal{L}^{gg}}{d\tau_H} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}}
  \]

- Renormalization scale in \( \alpha_s \) and the factorization scale of the parton densities to be fixed properly
Diffractive processes

Single diffractive

Double Pomeron Exchange
Diffractive cross sections

Single diffractive

\[ \sigma_{IPp \to H}(M_H, M_X) = C_g \int_0^1 \int_0^1 F_{g/p}(\xi_p) \cdot F_{g/IP}(\beta) \cdot \sigma_{gg \to H}(M_H, \hat{s}) \ d\beta \ d\xi_p \]

Double Pomeron Exchange

\[ \sigma_{IPp \to H}(M_H, M_X) = C_g \int_0^1 \int_0^1 F_{g/IP_A}(\beta) \cdot F_{g/p}(\xi_p) \cdot \sigma_{gg \to H}(M_H, \hat{s}) \ d\beta \ d\xi_p \]

Normalization

\[ C_g \]

Momentum fractions: pomeron and quarks

\[ \xi = 1 - x_p \]

\[ \beta = \frac{x}{x_{IP}} \]

Gluon distributions in the proton

MSTW (2008)

Pomeron flux

\[ f_{IP/h}\left(\frac{x_{IP}}{x}\right) \]

H1 parametrization (2006)

Gluon distributions \((i)\) in the Pomeron \(IP\)
FIT Comparison :: SD vs. DPE

- For $E_{CM} = 7$ TeV and $M_H = \mu = M$,
  - Inclusive curve is shown.
  - Single Diffractive is represented by a dashed line.
  - GLM and KKMR are indicated with specific markers.

- For $E_{CM} = 14$ TeV,
  - CED is shown with a brown line.
  - GLM is depicted with a red dotted line.
  - KKMR is illustrated with a blue solid line.

The graphs show the dependence of $\sigma_{NLO}$ on the Higgs mass for different collider energies.
**SD production as $M_H$ function (NLO)**

<table>
<thead>
<tr>
<th>Mass (GeV)</th>
<th>$\sqrt{s}$ (TeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.96</td>
</tr>
<tr>
<td>120</td>
<td>5.36(4.23)</td>
</tr>
<tr>
<td>140</td>
<td>2.57(2.02)</td>
</tr>
<tr>
<td>160</td>
<td>1.24(0.98)</td>
</tr>
<tr>
<td>180</td>
<td>0.60(0.47)</td>
</tr>
<tr>
<td>200</td>
<td>0.31(0.24)</td>
</tr>
</tbody>
</table>

**GLM**  
**KKMR**
Exclusive Higgs boson production

Diffractive Higgs Production

- The reaction: $pp \rightarrow p + H + p$

- Protons lose a small fraction of their energy :: **scattering in small angles**

- Nevertheless enough to produce the Higgs Boson

$$\frac{d\sigma}{dy} = \frac{|M|^2}{16^2 \pi^3 b^2}$$

$G_F$ is the Fermi constant and $Q_T^2 \equiv -Q_T^2$
The probability for a quark to emit 2 gluon in the t-channel is given by the integrated gluon distribution

\[ f(x, Q) \equiv K \frac{\partial G(x, Q)}{\partial \ln Q^2} \]

The factor \( K \) is related to the non-diagonality of the distribution:

\[ K \approx e^{-bk^2/2} \frac{2^{2\lambda+3} \Gamma(\lambda + 5/2)}{\sqrt{\pi} \Gamma(\lambda + 4)} \]

\[ \frac{d\sigma}{dy} \approx \frac{\alpha_s G_F \sqrt{2}}{9b^2} \left[ \int \frac{d^2 Q_T}{Q_T^4} f(x_1, Q_T) f(x_2, Q_T) \right]^2 \]
Sudakov form factors

- The former cross section is **infrared divergent**!
- The regulation of the amplitude can be done by suppression of gluon emissions from the production vertex;
- The Sudakov form factors accounts for the probability of emission of one gluon

\[
\frac{C_A \alpha_s}{\pi} \int_{Q_T^2}^{m_H^2/4} \frac{dp_T^2}{p_T^2} \int_{p_T}^{m_H/2} \frac{dE}{E} \sim \frac{C_A \alpha_s}{4\pi} \ln^2 \left( \frac{m_H^2}{Q_T^2} \right)
\]

- The **suppression** of several gluon emissions exponentiates

\[
e^{-S} = \exp \left( -\int_{Q_T^2}^{m_H^2/4} \frac{dp_T^2}{p_T^2} \alpha_s(p_T^2) \right) \int_0^{1-\Delta} dz \left[ z P_{gg}(z) + \sum_q P_{qg}(z) \right]
\]

- Then, the gluon distributions are modified in order to include S
Photoproduction mechanism

• The Durham group’s approach is applied to the photon-proton process;

• This is a subprocess of Ultraperipheral Collisions;

• Hard process: photon splitting into a color dipole, which interacts with the proton;

\[ \mathcal{A}_T = \frac{s M_H^2 \alpha_s^3 \alpha}{\pi \nu} \sum_q e_q^2 \left( \frac{2C_F}{N_c} \right) \int \frac{dk^2}{k^6} \int_0^1 \frac{[\tau^2 + (1 - \tau)^2][\alpha_{\ell}^2 + (1 - \alpha_{\ell})^2]k^2}{k^2 \tau (1 - \tau) + Q^2 \alpha_{\ell}(1 - \alpha_{\ell})} \, d\alpha_{\ell} \, d\tau. \]
\[ \frac{d\sigma}{dy_H \, dt} \bigg|_{y_H, t=0} = \frac{S_{gap}^2}{2\pi B} \left( \frac{\alpha_s^2 M_H^2}{3 N_c \pi v} \right)^2 \left( \sum_q e_q^2 \right)^2 \left[ \int_{k_0^2}^{\infty} \frac{dk^2}{k^6} e^{-S(k^2, M_H^2)} f_g(x, k^2) \chi(k^2, Q^2) \right]^2 \]

- The cross section is calculated for central rapidity \((y_H = 0)\)

- Proton content\(^1\): \(\alpha_s C_F / \pi \rightarrow f_g(x, k^2) = C_\ell \delta(\ell \cdot k^2) x G(x, k^2)\)

- Gap Survival Probability\(^2\): \(S_{gap}^2 \rightarrow 3\% \ (5\%) \) for LHC (Tevatron)

- Gluon radiation suppression\(^3\): Sudakov factor \(S(k^2, M_H^2) \sim \ell n^2 (M_H^2 / 4k^2)\)

- Cutoff \(k_0^2\): Necessary to avoid infrared divergencies :: \(k_0^2 = 1 \text{ GeV}^2\).

- Electroweak vacuum expectation value: \(v = 246 \text{ GeV}\)

- Gluon-proton form factor: \(B = 5.5 \text{ GeV}^{-2}\)

---

\(^1\) Khoze, Martin, Ryskin, EJPC 14 (2000) 525

\(^2\) Khoze, Martin, Ryskin, EJPC 18 (2000) 167

\(^3\) Forshaw, hep-ph/0508274
Ultraperipheral Collisions

- Photon emission from the proton

\[
\sigma(p p(A) \rightarrow p + H + p(A)) = 2 \int_{\omega_0}^{\sqrt{s}/2} d\omega \frac{dn_i}{d\omega} \sigma_{\gamma p}(\omega, M_H),
\]

with photon fluxes

\[
\frac{dn_p}{d\omega} = \frac{\alpha_{em}}{2\pi\omega} \left[ 1 + \left( 1 - \frac{2\omega}{\sqrt{s}} \right)^2 \right] \left( \ell nA - \frac{11}{6} + \frac{3}{A} - \frac{3}{2A^2} + \frac{1}{3A^2} \right).
\]

\[
\frac{dn_A}{d\omega} = \frac{2Z^2\alpha_{em}}{\pi\omega} \left[ \mu K_0(\mu)K_1(\mu) - \frac{\mu^2}{2} [K_1^2(\mu) - K_0^2(\mu)] \right].
\]

- The photon virtuality obey the Coherent condition for its emission from a hadron under collision

\[
Q^2 \lesssim 1/R^2
\]
Photoproduction cross section

$M_H = 120$ GeV
Cross section = 1.77-6 fb

Estimations for the GSP in the LHC energy
pA collisions

<table>
<thead>
<tr>
<th>Process</th>
<th>$\sigma$ (fb)</th>
<th>BR $\times\sigma$ (fb)</th>
<th>$\mathcal{L}$ (fb$^{-1}$)</th>
<th>Events/yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>$pp$</td>
<td>1.77</td>
<td>1.27</td>
<td>1.30</td>
<td>1 (30)</td>
</tr>
<tr>
<td>$pp$</td>
<td>5.92</td>
<td>4.26</td>
<td>1.30</td>
<td>6 (180)</td>
</tr>
<tr>
<td>$pPb$</td>
<td>617.</td>
<td>444.</td>
<td>0.035</td>
<td>21</td>
</tr>
<tr>
<td>$pPb$</td>
<td>2056.</td>
<td>1480.</td>
<td>0.035</td>
<td>72</td>
</tr>
</tbody>
</table>

$\text{BR}(H\to bb-bar) = 72\%$
Summary results

✓ Diffraction with absorptive corrections (gap survival probability)

- describe Tevatron data for $W^+$ and $Z^0$ production
- rate production for quarkonium + photon at LHC energies
  \[ R_{SD}^{(J/\psi)} = 0.8 - 0.5 \% \quad R_{SD}^{(\Upsilon)} = 0.6 - 0.4 \% \text{(first in literature)} \]

- predictions for heavy quark production (SD and DPE) at LHC energies possible to be verified in AA collision
  (diffractive cross section in pp, pA and AA collisions)
  \[ \overline{C}\overline{C} \quad \overline{B}\overline{B} \]
  A = Lead and Calcium

Higgs predictions in agreement with Hard Pomeron Exchange

Cross sections of Higgs production 1 fb (DPE); 60-80 fb (SD)
Summary results

- Exclusive photoproduction is promising for the LHC
  - strong suppression of backgrounds
  - cross section prediction: $2-6 \text{ fb}$
  - expecting between 1 and 6 events per year
  - additional signature with the H associated production
  - High event rates for pA collisions
    - $\sigma = 1 \text{ pb}$ for pPb collisions
Where we are

IP approach $\rightarrow$ successes and failures

Perturbative + non-perturbative QCD $\rightarrow$ How exactly contribute?

✓ Diffraction at HERA (Soft diffraction) described by factorization model (IS)

✓ Same model doesn’t describe Tevatron data (Hard diffraction)

✓ Solution? $\rightarrow$ Factorization + Gap Survival Probability is a possibility,

**BUT NOT THE ONLY ONE**

✓ Breaking of factorization?

NEXT $\rightarrow$ Overall theoretical understanding

$\rightarrow$ LHC $\rightarrow$ Diffractive Higgs production?

$\rightarrow$ Diffraction at nuclei collisions?

$\rightarrow$ Diffractive production of $X_c$, $X_b$, ... ?
Next

DIFFRACTION IN NUCLEAR COLLISIONS

✓ Gap survival probability for nuclear collisions

✓ Dijets in hadronic and nuclear collisions

✓ ...
BACKUP
Predictions (LHC – 14 TeV)

High diffractive ratio

\[ R_{KMR} = \frac{1}{\int_{-1}^{1} \sigma_{\text{inclusive}} - \int_{-1}^{1} \sigma_{\text{dиффрактивное}}} = 0.311 \]

Large range of pseudorapidity

\[-6 \leq \eta \leq 6\]
Bialas-Landshoff approach

Double Pomeron Exchange

\[ p + p \rightarrow p + Q\bar{Q} + p \]

\[
\sigma_{\Pi\Pi}(BL) = \frac{1}{2s (2\pi)^8} \int |M_{fi}|^2 \left[ F(t_1) F(t_2) \right]^2 dPH.
\]

\[ F(t) \quad \text{nucleon form-factor} \]

\[ F(t) = \exp(bt) \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad b = 2 \text{ GeV}^{-2} \]

Differential phase-space factor

\[
dPH = d^4k_1 \delta(k_1^2) d^4k_2 \delta(k_2^2) d^4r_1 \delta(r_1^2 - m_Q^2) \times d^4r_2 \delta(r_2^2 - m_Q^2) \Theta(k_1^0) \Theta(k_2^0) \Theta(r_1^0) \Theta(r_2^0) \times \delta^{(4)}(p_1 + p_2 - k_1 - k_2 - r_1 - r_2),
\]

\[ m_Q \quad \text{mass of produced quarks} \]
Bialas-Landshoff approach

Sudakov parametrization for momenta

\[
Q = \frac{x}{s} p_1 + \frac{y}{s} p_2 + v, \quad k_1 = x_1 p_1 + \frac{y_1}{s} p_2 + v_1,
\]

\[
k_2 = \frac{x_2}{s} p_1 + y_2 p_2 + v_2, \quad r_2 = x_Q p_1 + y_Q p_2 + v_Q,
\]

- \(v, v_1, v_2, v_Q\): two-dimensional four-vectors describing the transverse component of the momenta
- \(p_1, p_2, (k_1, k_2)\): momenta for the incoming (outgoing) protons
- \(r_2, (r_1)\): momentum for the produced quark (antiquark)
- \(Q\): momentum for one of exchanged gluons
Bialas-Landshoff approach

Square of the invariant matrix element averaged over initial spins and summed over final spins

\[
|M_{fi}|^2 = \frac{x_1 y_2 H}{(sx_Q y_Q)^2 (\delta_1 \delta_2)^{1+2\epsilon} \delta_1^{2\alpha' t_1} \delta_2^{2\alpha' t_2}} \left(1 - \frac{4 m_Q^2}{s\delta_1 \delta_2}\right) \exp \left[2\beta (t_1 + t_2)\right]
\]

\[
\delta_1 = 1 - x_1, \quad \delta_2 = 1 - y_2, \quad t_1 = -\vec{v}_1^2, \quad t_2 = -\vec{v}_2^2
\]

\[
\beta = 1 \text{GeV}^{-2}
\]

\[
\exp \left[2\beta (t_1 + t_2)\right]
\]

effect of the momentum transfer dependence of the non-perturbative gluon propagator

\[
H = S_{\text{gap}}^2 \times 2s \left[\frac{4\pi m_Q (G^2 D_0)^3 \mu^4}{9 (2\pi)^2}\right]^2 \left(\frac{\alpha_s}{\alpha_0}\right)^2
\]

\[
\epsilon = 0.08, \quad \alpha' = 0.25 \text{ GeV}^{-2}, \quad \mu = 1.1 \text{ GeV}
\]

\[
G^2 D_0 = 30 \text{ GeV}^{-1} \mu^{-1}
\]
Partonic Structure of the Pomeron

✓ Diffractive quark distributions and quark distributions of the Pomeron are related

\[
\frac{df_q(\beta, Q^2, x_{IP}, t)}{dx_{IP} dt} = \frac{1}{16\pi^2} \left| g_{IP}(t) \right|^2 x_{IP}^{-2\alpha_{IP}(t)} q^{IP}(\beta, Q^2)
\]

• Introducing gluon distribution in the Pomeron

\[
g^{IP}(\beta, Q^2)
\]

• Related to \( df_g / dx_{IP} dt \) by

\[
\frac{df_g(\beta, Q^2, x_{IP}, t)}{dx_{IP} dt} = \frac{1}{16\pi^2} \left| g_{IP}(t) \right|^2 x_{IP}^{-2\alpha_{IP}(t)} g^{IP}(\beta, Q^2)
\]

• At Next-to-Leading order, Pomeron Structure Function acquires a term containing \( g^{IP}(\beta, Q^2) \)

Representation of D* diffractive production in the infinite-momentum frame description of DDIS
Diffractive processes

- Hadronic processes can be characterized by an energy scale.

  - Soft processes – energy scale of the order of the hadron size (~ 1 fm)
    - pQCD is inadequate to describe these processes
    \[ \alpha_{soft}(t) = 1.08 + 0.25t \]

  - Hard processes – “hard” energy scale ( > 1 GeV²)
    - can use pQCD
    - “factorization theorems”
    - Separation of the perturbative part from non-perturbative
    \[ \alpha_{hard}(t) = 1.30 + 0.02t \]

- Most of diffractive processes at HERA “soft processes”
Pomeron as composite

• Considering Regge factorization we have

\[ F_2^{D(4)}(x, Q^2, x_{IP}, t) = f_{IP/p}(x_{IP}, t)F_2^{IP} (\beta, Q^2) \]

Data  Good fit with added Reggeon for HERA

Pomeron as gluons

• Elastic amplitude  neutral exchange in t-channel
• Smallness of the real part of the diffractive amplitude  nonabeliance

Born graphs in the abelian and nonabelian (QCD) cases look like

see MBGD & M. V. T. Machado 2001
The Pomeron

- From fitting elastic scattering data, the IP trajectory is much flatter than others.
- For the intercept, $\alpha_{IP} \approx 0.25 \text{ GeV}^{-2}$ total cross sections implies $\alpha_{IP}(0) \approx 1$.
- The Pomeron is the dominant trajectory in the elastic and diffractive processes.
- Known to proceed via the exchange of vacuum quantum numbers in the $t$-channel.

Regge-type:
\[
\alpha(t) = \alpha(0) + \alpha' t
\]

First measurements in h-h scattering:
\[
W^2 = (q+p)^2
\]

- $\alpha(0)$ and $\alpha'$ are fundamental parameters to represent the basic features of strong interactions.

\[
\frac{d\sigma}{dt}(W) = \exp(b_0 t)W^{2[2\alpha_{IP}(t)+2]}
\]

- $b = b_0 + 4\alpha' \ln(W)$

- Energy dependence of the transverse system.
Pomeran structure function

- Pomeran structure function has been modeled in terms of a light flavor singlet distribution $\Sigma(z)$

- Consists of u, d and s quarks and antiquarks and a gluon distribution $g(z)$

- $z$ is the longitudinal momentum fraction of the parton entering the hard subprocess with respect of the diffractive exchange

- $(z = \beta)$ for the lowest order quark-parton model process and $0 < \beta < z$ for higher order processes

- Quark singlet and gluon distributions are parametrized at $Q^2_0$

$$zf_{i/IP}(z, Q^2_0) = A_i z^{B_i} (1 - z)^{C_i} \exp \left[ - \frac{0.01}{(1 - z)} \right]$$
### Pomerion structure function

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha'_IP$</td>
<td>$0.06^{+0.19}_{-0.06} GeV^{-2}$</td>
</tr>
<tr>
<td>$B_{IP}$</td>
<td>$5.5^{+2.0}_{-0.7} GeV^{-2}$</td>
</tr>
<tr>
<td>$\alpha_{IR}(0)$</td>
<td>$0.50 \pm 0.10$</td>
</tr>
<tr>
<td>$\alpha'_IR$</td>
<td>$0.3^{+0.6}_{-0.3} GeV^{-2}$</td>
</tr>
<tr>
<td>$B_{IR}$</td>
<td>$1.6^{+1.6}_{-0.4} GeV^{-2}$</td>
</tr>
<tr>
<td>$m_c$</td>
<td>$1.4 \pm 0.2 GeV$</td>
</tr>
<tr>
<td>$m_b$</td>
<td>$4.5 \pm 0.5 GeV$</td>
</tr>
<tr>
<td>$\alpha_8^{(5)} (M_Z^2)$</td>
<td>$0.118 \pm 0.002$</td>
</tr>
</tbody>
</table>

- Values of fixed parameters (masses) and their uncertainties, as used in the QCD fits.

- $\alpha'_IP$ and $B_{IP}$ (strongly anti-correlated) are varied simultaneously to obtain the theoretical errors on the fits (as well as $\alpha'_IR$ and $B_{IR}$).

- Remaining parameters are varied independently.

- Theoretical uncertainties on the free parameters of the fit are sensitive to the variation of the parametrization scale $Q^2_0$.
## The HERA Collider

Publications on diffraction made by H1 Collaboration *

<table>
<thead>
<tr>
<th>Event</th>
<th>Number of papers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diffractive Cross Sections (SD, DD)</td>
<td>11</td>
</tr>
<tr>
<td>Diffractive Final States</td>
<td>14</td>
</tr>
<tr>
<td>Quasi-elastic Cross Sections</td>
<td>20</td>
</tr>
<tr>
<td>Total cross sections / decomposition</td>
<td>2</td>
</tr>
</tbody>
</table>

Similar numbers to ZEUS Collaboration

* Fazio, Summerschool Acquafredda 2010
The Tevatron Collider

Publications on diffraction made by CDF Collaboration

**Soft Diffraction**

Double Diffraction – PRL 87, 141802 (2001)

**Hard Diffraction**

Dijets – PRL 85, 4217 (2000); PRD 77, 052004 (2008)
Charmonium – PRL 102, 242001 (2009)
W – PRL 78, 2698 (1997)

J/ψ – PRL 87, 241802 (2001)
Roman Pot Tag Dijets – PRL 84, 5043 (2000)

Mesropian, Summerschool Acquafredda (2010)
Diffractive processes

- Hadronic processes can be characterized by an energy scale

  ▶ Soft processes – energy scale of the order of the hadron size (≈ 1 fm)
  
  pQCD is inadequate to describe these processes

  \[ \alpha_{soft}(t) = 1.08 + 0.25t \]

  ▶ Hard processes – “hard” energy scale (> 1 GeV^2)

  can use pQCD

  “factorization theorems”

  Separation of the perturbative part from non-perturbative

  \[ \alpha_{hard}(t) = 1.30 + 0.02t \]

- Most of diffractive processes at HERA

  “soft processes”
Pomeron flux factor

• $x_{IP}$ dependence is parametrized using a flux factor

$$f_{IP/p}(x_{IP}, t) = A_{IP} \frac{e^{B_{IP}t}}{x_{IP}^{2\alpha_{IP}(t) - 1}}$$

• IP trajectory is assumed to be linear

$$\alpha_{IP}(t) = \alpha_{IP}(0) + \alpha'_{IP} t$$

• $B_{IP}$, $\alpha'_{IP}$

their uncertainties

obtained from the fits to H1 forward proton spectometer (FPS) data

Normalization parameter $x_{IP}$ is chosen such that

$$x_{IP} \int_{t_{cut}}^{t_{min}} f_{IP/p} dt = 1 \text{ at } x_{IP} = 0.003$$

• $|t_{min}| \approx m_p^2 x_{IP} / (1 - x_{IP})$ is the proton mass

• $|t_{cut}| = 1.0 \text{ GeV}^2$ is the limit of the measurement
J/ψ+γ production

✓ Considering the Non-relativistic Quantum Chromodynamics (NRQCD)

✓ Gluons fusion dominates over quarks annihilation

✓ Leading Order cross section convolution of the partonic cross section with the PDF

✓ MRST 2001 LO no relevant difference using MRST 2002 LO and MRST 2003 LO

✓ Non-perturbative aspects of quarkonium production

  • Expansion in powers of ν

    □ ν is the relative velocity of the quarks in the quarkonia

NLO expansions in αs one virtual correction and three real corrections
NRQCD Factorization

\[ g + g \rightarrow \gamma + (c\bar{c}) \left[ ^3 S_1^1, ^3 S_1^8 \right] \]
\[ g + g \rightarrow \gamma + (c\bar{c}) \left[ ^1 S_0^8, ^3 P_J^8 \right] \]

- Negligible contribution of quarks annihilation at high energies

\[ \frac{d^2\sigma_{\text{inc}}}{d\eta dp_T} = \int dx_1 g_{p}(x_1, \mu_F^2) g_{p}(x_2, \mu_F^2) \frac{4x_1 x_2 p_T}{2x_1 - x_T \eta} \frac{d\hat{\sigma}}{d\hat{t}} \]

\[ \bar{x}_T = \frac{2m_T}{\sqrt{s}} \]

\[ m_T = \sqrt{p_T^2 + m_\psi^2} \]

\[ \sqrt{s} \] is the center mass energy (LHC = 14 TeV)

\[ J/\psi \text{ rapidity} \quad 9.2 \text{ GeV}^2 \]
# Matrix elements (GeV$^3$)

<table>
<thead>
<tr>
<th>$\langle O_1^{J/\psi} (^3 S_1) \rangle$</th>
<th>1.16</th>
<th>$\langle O_1^{\gamma} (^3 S_1) \rangle$</th>
<th>10.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle O_8^{J/\psi} (^3 S_1) \rangle$</td>
<td>1.19 x 10^{-2}</td>
<td>$\langle O_8^{\gamma} (^3 S_1) \rangle$</td>
<td>0.02</td>
</tr>
<tr>
<td>$\langle O_8^{J/\psi} (^1 S_0) \rangle$</td>
<td>0.01</td>
<td>$\langle O_8^{\gamma} (^1 S_0) \rangle$</td>
<td>0.136</td>
</tr>
<tr>
<td>$\langle O_8^{J/\psi} (^1 P_0) \rangle$</td>
<td>0.01 x $m_c^2$</td>
<td>$\langle O_8^{\gamma} (^1 P_0) \rangle$</td>
<td>0</td>
</tr>
</tbody>
</table>

$e_b = -\frac{1}{3}$

$m_b = 4.5$ GeV

$m_{\Upsilon} = 9.46$ GeV/c$^2$
Variables to DDIS

Cuts for the integration over $x_{IP}$

$$x_{IP}^{min} \leq x_{IP} \leq 0.05$$

$$x_{IP}^{min} = \frac{\bar{x} T e^{-y} - 2\tau}{\bar{x} T e^{-y} - 2}$$

Scales

$$Q_0^2 = 2.5 \text{ GeV}^2$$

$$\Lambda_{QCD} = 0.2$$

$$\mu_F^2 = \frac{(p_T^2 + m_{\psi}^2)}{4}$$

$$x_2 = \frac{x_1 x_{IP} \bar{x} T e^{-y} - 2\tau}{2x_1 x_{IP} - \bar{x} T e^y},$$

$$\hat{s} = x_1 x_2 x_{IP} s,$$

$$\hat{t} = m_{\psi}^2 - x_2 \sqrt{s} m_T e^y$$

$$\hat{u} = m_{\psi}^2 - x_1 x_{IP} \sqrt{s} m_T e^{-y}.$$
Heavy quark production


Heavy quark hadroproduction

- Focus on the following single diffractive processes

\[ pp \rightarrow p + (c\bar{c}) + X \quad \text{and} \quad pp \rightarrow p + (b\bar{b}) + X \]

- Diffractive ratios as a function of energy center-mass \( E_{CM} \)

\[ g + g \rightarrow Q + \bar{Q} \]

- Diagrams contributing to the lowest order cross section

NLO functions

\[ f_{gg}^{(1)} = \frac{7}{1536\pi} \left[ 12\beta \ln^2(8\beta^2) - \frac{366}{7}\beta \ln(8\beta^2) + \frac{11}{42}\pi^2 \right] + \beta \left[ a_0 + \beta^2(a_1 \ln(8\beta^2) + \right.
\]
\[ + a_3\beta^4 \ln(8\beta^2) + \rho^2(a_4 \ln \rho + a_5 \ln^2 \rho) + \rho(a_6 \ln \rho + a_7 \ln^2 \rho) \right] 
\]
\[ + (n_{1f} - 4)\frac{\rho^2}{1024\pi} \left[ \ln \left( \frac{1 + \beta}{1 - \beta} \right) - 2\beta \right] \]

<table>
<thead>
<tr>
<th>( a_0 )</th>
<th>0.108068</th>
<th>( a_4 )</th>
<th>0.0438768</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 )</td>
<td>-0.114997</td>
<td>( a_5 )</td>
<td>-0.0760996</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>0.0428630</td>
<td>( a_6 )</td>
<td>-0.165878</td>
</tr>
<tr>
<td>( a_3 )</td>
<td>0.131429</td>
<td>( a_7 )</td>
<td>-0.158246</td>
</tr>
</tbody>
</table>

\[ \beta = \sqrt{1 - \rho} \]

\[ \tilde{f}_{gg}^{(1)}(\rho) = \frac{1}{8\pi^2} \left[ \left\{ 2\rho(59\rho^2 + 198\rho - 288) \ln \left( \frac{1 + \beta}{1 - \beta} \right) + 12\rho(\rho^2 + 16\rho + 16)h_2(\beta) \right. \right. 
\]
\[ - 6\rho(\rho^2 - 16\rho + 32)h_1(\beta) - \frac{4}{15}\beta(7449\rho^2 - 3328\rho + 724) \right\} + 2f_{gg}^{(0)}(\rho) \ln \left( \frac{\rho}{4\beta^2} \right) \]
Total cross section LO

\[ \sigma_{h_1 h_2}(s, m^2_Q) = \sum_{i,j} \int^1_0 dx_1 \int^1_0 dx_2 f_i^{h_1}(x_1, \mu^2_F) f_j^{h_2}(x_2, \mu^2_F) \hat{\sigma}_{ij}(\hat{s}, m^2_Q, \mu_F, \mu_R) \]

\[ \rho = \frac{4m^2}{\hat{s}} \]

\[ \hat{s} = x_1 x_2 s \]

\[ f_i^{h_1}(x_1, \mu^2_F) f_j^{h_2}(x_2, \mu^2_F) \]

are the parton distributions inner the hadron i=1 and j=2

\[ x_{1,2} \] are the momentum fraction

Partonical cross section

\[ \hat{\sigma}_{ij}(\hat{s}, m^2, \mu^2) = \frac{\alpha_s^2(\mu^2)}{m^2} f_{ij}(\rho, \frac{\mu^2}{m^2}) \]

\[ \hat{\sigma}(gg \to Q\bar{Q}) = \sigma_0 \left( \frac{1}{NV} \right) \left[ 3L(\beta)\xi_0 + 2(V-2)(1+\rho) + \rho(6\rho - N^2) \right] \]

\[ \sigma_0 = \frac{\alpha_s^2}{m^2} \frac{\pi\beta}{24\rho} \]

\[ L(\beta) = \frac{1}{\beta} \log \left( \frac{1+\beta}{1-\beta} \right) - 2 \]

\[ \beta = \sqrt{1-\rho} \]

factorization (renormalization) scale

\[ \alpha_s = \frac{g^2}{4\pi} \]
NLO Production

\[ g + g \rightarrow Q + \bar{Q} + g \]

\[ \hat{\sigma}_{ij}(\hat{s}, m_Q^2, \mu^2, \mu_R^2) = \frac{\alpha_s^2(\mu_R)}{m_Q^2} \sum_{k=0}^{\infty} [4\pi \alpha_s(\mu_R)]^k \sum_{l=0}^{k} f_{ij}^{(k,l)}(\rho) \ln \left( \frac{\mu_F^2}{m_Q^2} \right) \]

\[ f_{gg}(\rho, \mu^2/m^2) = f_{gg}^{(0)}(\rho) + g^2(\mu^2) \left[ f_{gg}^{(1)}(\rho) + \tilde{f}_{gg}^{(1)}(\rho) \ln(\mu^2/m^2) \right] + O(g^4) \]

Running of the coupling constant

\[ \frac{d\alpha_S(\mu^2)}{d\ln(\mu^2)} = -b_0 \alpha_S^2 - b_1 \alpha_S^3 + O(\alpha_S^4) \]

\[ b_0 = \frac{33 - 2n_{1f}}{12\pi}, \quad b_1 = \frac{153 - 19n_{1f}}{24\pi^2} \]

\[ n_{1f} = 3 \ (4) \text{ charm (bottom)} \]

\[ f_{gg}^{(0)}(\rho) = \frac{\pi/\beta}{192} \left[ \frac{1}{\beta}(\rho^2 + 16\rho + 16) \ln \left( \frac{1 + \beta}{1 - \beta} \right) - 28 - 31\rho \right] \]
Diffractive cross section

$$\sigma_{h_1 h_2}^\text{SD}(s, m_Q^2) = \sum_{i,j=q\bar{q},g} \int_0^1 dx_1 \int_{x_1}^1 dx_2 \int_{x_1}^{x_{\text{IP}}^\text{max}} dx_{1(1)}^\text{IP} \left( \frac{x_{1(1)}}{x_{\text{IP}}} \right) \bar{f}_{\text{IP}/h_1}^1 \left( \frac{x_{1(1)}}{x_{\text{IP}}} \right) f_{i/\text{IP}} \left( \frac{x_{1(1)}}{x_{\text{IP}}} , \mu^2 \right) f_{j/h_2}(x_2, \mu^2) \hat{\sigma}_{ij}(\hat{s}, m_Q^2, \mu^2) + (1 \Rightarrow 2)$$

Pomeron flux factor

$$f_{\text{IP}/h_1}^1 \left( \frac{x_{1(1)}}{x_{\text{IP}}} \right)$$

Pomeron Structure Function (H1)

$$\beta = \frac{x}{x_{\text{IP}}}$$

KKMR model

$$<|S|^2> = 0.06$$ at LHC single diffractive events

Parametrization of the pomeron flux factor and structure function

H1 Collaboration
Heavy quarks production at the LHC

<table>
<thead>
<tr>
<th>Heavy Quark</th>
<th>$\sigma_{\text{inc}}(\sqrt{s} = 14 \text{ TeV})$</th>
<th>$\sigma_{\text{diff}}(\sqrt{s} = 14 \text{ TeV})$</th>
<th>$R_{\text{diff}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c\bar{c}$</td>
<td>7811 $[\mu b]$</td>
<td>178 $[\mu b]$</td>
<td>2.3 %</td>
</tr>
<tr>
<td>$b\bar{b}$</td>
<td>393 $[\mu b]$</td>
<td>7 $[\mu b]$</td>
<td>1.7 %</td>
</tr>
</tbody>
</table>

Heavy quarks cross sections in NLO to pp collisions
GSP value decreases the diffractive ratio ($<|S|^2> = 0.06$)

Inclusive nuclear cross section at NLO

$$\sigma_A = A^2 \sigma_N$$

$A_{\text{PbPb}} = 208\ (5.5 \text{ TeV});\ 40\ (6.3 \text{ TeV})$

$$\sigma^{\text{SD}}_{pPb} = 0.76\ (0.018)\ \text{mb}$$

charm (bottom)

$$\sigma^{\text{DPE}}_{PbPb} = 32.5\ (0.32)\ \mu b$$
Diffractive cross sections @ LHC

Inclusive cross section

Nucleus-Nucleus collision

<table>
<thead>
<tr>
<th></th>
<th>Pb-Pb ((C\bar{C}))</th>
<th>Pb-Pb ((B\bar{B}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_A[\text{mb}])</td>
<td>188165, 16</td>
<td>7340, 23</td>
</tr>
</tbody>
</table>

**Diffractive cross sections**

- **Coherent**
  - Pomeran emmitted by the nucleus
  - \(A + A \rightarrow X + A + [\text{LRG}] + A\)
  - \(F(t) \approx \exp(R_A^2 t/6)\)
  - \(R_A = r_0 A^{1/3}\)
  - \(r_0 = 1.2 \text{ fm}\)

- **Predictions to cross sections possible to be verified at the LHC**

- **Very small diffractive ratio**

\[A_{Pb} = 240\]
pA cross sections @ LHC

\[ \sigma_{PPb}^{SD} = 0.76 (0.018) \text{ mb} \]

\[ A_{eff} = 4.39 \]

\[ S_{GAP}^2 = 0.0287 \]


\[ p + p \rightarrow \bar{Q}QX + p \]

Suppression factor

\[ K = \left\{ 1 - \frac{1}{\pi} \frac{\sigma_{tot}^{pp}(s)}{B_{sd}(s) + 2B_{el}^{pp}(s)} \right\} + \frac{1}{(4\pi)^2} \frac{[\sigma_{tot}^{pp}(s)]^2}{B_{el}^{pp}(s) [B_{sd}(s) + B_{el}^{pp}(s)]} \]

than is suggested by Eq. (70). Therefore, the predicted energy dependence of the survival probability Eq. (70) might be quite wrong and the diffractive cross section at the LHC energy may be overestimated.

\[ \sigma_{pA} \sim 0.8 \text{ mb (charm)} \]

\[ A_{eff} \approx 10. \]
Diffractive cross sections @ LHC

<table>
<thead>
<tr>
<th>Incoherent</th>
<th>PbPb ($c\bar{c}$)</th>
<th>PbPb ($b\bar{b}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{inc}/A^2$</td>
<td>1.68 mb</td>
<td>0.03 mb</td>
</tr>
<tr>
<td>$\sigma_{inc}^{abs}$</td>
<td>4356 – 0.07 mb</td>
<td>85 – 0.001 mb</td>
</tr>
<tr>
<td>$R_{inc} [%]$</td>
<td>38</td>
<td>19</td>
</tr>
<tr>
<td>$R_{inc}^{abs} [%]$</td>
<td>$2.28 - 3.8 \times 10^{-5}$</td>
<td>$1.14 - 1.9 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

- No values to $<|S|^2>$ for single diffractive events in AA collisions
- Estimations to central Higgs production $<|S|^2> \sim 8 \times 10^{-7}$
- Values of diffractive cross sections possible to be verified experimentally

$A_{Pb} = 240$
# DPE results at LHC

<table>
<thead>
<tr>
<th>$Q\bar{Q}$</th>
<th>$\sigma_{\text{inc}}$ $[\mu b]$</th>
<th>$\sigma_{\text{DPE}}$ $[\mu b]$</th>
<th>$R_{\text{DPE}}$ $[%]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c\bar{c}$</td>
<td>7811</td>
<td>13.6–0.53</td>
<td>0.17–7\times10^{-3}</td>
</tr>
<tr>
<td>$b\bar{b}$</td>
<td>393</td>
<td>0.053–0.027</td>
<td>0.01–0.007</td>
</tr>
</tbody>
</table>

$S^2_{\text{gap}} = 0.026$

## pp collisions LHC (14 TeV)

### Ingelman-Schlein

### Bialas-Landshoff

<table>
<thead>
<tr>
<th></th>
<th>CaCa [$c\bar{c}$]</th>
<th>PbPb [$c\bar{c}$]</th>
<th>CaCa [$b\bar{b}$]</th>
<th>PbPb [$b\bar{b}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\text{DPE}}^{\text{AA}}$ $[\mu b]$</td>
<td>22.8–2.8</td>
<td>31.1–4.2</td>
<td>0.25–0.14</td>
<td>0.32–0.2</td>
</tr>
<tr>
<td>$R_{\text{coh}}^{\text{DPE}}$ $[%]$</td>
<td>$3 - 0.4 \times 10^{-4}$</td>
<td>$2 - 0.2 \times 10^{-4}$</td>
<td>$8 - 4 \times 10^{-5}$</td>
<td>$4 - 3 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

$S^2_{\text{gap}} = 0.032 (0.031)$, $A^2_{\text{eff}} = 9.52 (6.21)$

$S^2_{\text{gap}} = a/[b + \ln(\sqrt{s/s_0})]$

$a = 0.126$, $b = -4.688$, $s_0 = 1 \text{ GeV}^2$

## AA collisions LHC

- CaCa (6.3 TeV)
- PbPb (5.5 TeV)
\( d \) functions

\[
P_{gg}(\hat{\tau}) = 6 \left\{ \left( \frac{1}{1 - \hat{\tau}} \right)_+ + \frac{1}{\hat{\tau}} - 2 + \hat{\tau}(1 - \hat{\tau}) \right\} + \frac{33 - 2N_F}{6} \delta(1 - \hat{\tau})
\]

\[
P_{gq}(\hat{\tau}) = \frac{4}{3} \frac{1 + (1 - \hat{\tau})^2}{\hat{\tau}}
\]

\( F_+ : \) usual + distribution

\[
F(\hat{\tau})_+ = F(\hat{\tau}) - \delta(1 - \hat{\tau}) \int_0^1 d\hat{\tau}' F(\hat{\tau}')
\]

\[
\tau_Q = \frac{m_H^2}{4m_Q^2} \ll 1
\]

Considering only the heavy-quark limit

Region allowed by Tevatron combination

\[
c(\tau_Q) \rightarrow \frac{11}{2}
\]

\[
d_{gg}(\hat{\tau}, \tau_Q) \rightarrow -\frac{11}{2} (1 - \hat{\tau})^3
\]

\[
d_{gq}(\hat{\tau}, \tau_Q) \rightarrow -1 + 2\hat{\tau} - \frac{\hat{\tau}^2}{3}
\]

\[
d_{q\bar{q}}(\hat{\tau}, \tau_Q) \rightarrow \frac{32}{27} (1 - \hat{\tau})^3
\]
CDF Detector
D0 Detector