Infrared Renormalons and Power Corrections

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1. IR renormalons + Power corrections
2. Inclusive Vs Exclusive measurements
3. Event shapes: $1/Q$ Corrections and "Universality"
4. Higher twist in DIS
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IR Renormalons and Power Corrections

Precision Data on hard processes in QCD ask for a theoretical description to power-like accuracy (GLS, event shapes, ..)

Can we improve perturbation theory by including power like terms?

Renormalons allow us to include these systematically if perturbation theory is known to sufficiently high order. The Coefficient of the power corrections cannot be uniquely determined.

To see how this can happen consider $\langle 1-T \rangle$. "Improved" leading order perturbative prediction:

$$\langle 1-T \rangle \sim \int_0^Q dk_\perp \, \alpha_s(k_\perp) \, F(k_\perp, Q)$$

↑ improved!

$k_\perp$ of emitted gluon.

$$F \sim \frac{4C_F}{\pi Q} \quad \text{for } k_\perp < Q.$$

$$\alpha_s(k_\perp) = \alpha_s(Q) + 2\beta_1 \ln\frac{Q}{k_\perp}\alpha_s(Q^2) + \cdots$$

$$\langle 1-T\rangle \sim \frac{4C_F}{\pi}\alpha_s(Q)\sum_n n!\left[2\beta_1\alpha_s(Q)\right]^n$$

In general for an observeable:

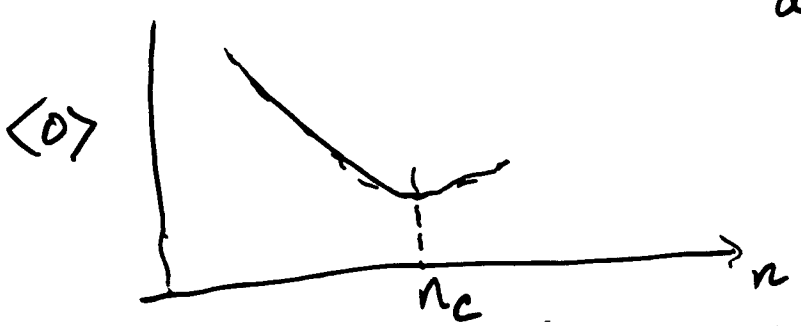
$$\langle O\rangle \sim O_{tree}\sum n!\left(\frac{\beta_1}{p}\right)^n \alpha_s^{\,n} \equiv \sum C_n \alpha_s^{\,n}$$

reason for this divergence: perturbation theory diverges at Landau Pole $k_\perp \sim \Lambda$

Perturbation series expected to be an asymptotic expansion:

If we truncate series at $n = m$, then error is bounded by last term retained.

Fixed order contributions $C_n \alpha_s^{\,n}$ decrease till they reach $n = n_c$ after which increase.



Perturbative calculations make sense only upto n_c. Truncate at $n = n_c$.

The effect of the tail $\sim C_{n_c} \alpha_s^{n_c}$

$\sim \left(\frac{\Lambda^2}{Q^2}\right)^p$ (for $n > n_c$)

The terms with $n > n_c$ push us to momentum scales $\underline{\underline{k^2 < \Lambda^2}}$.

Equivalently we can keep all orders but remove the contribution from large orders $^{(n>n_c)}$ by some prescription like principal value. Various prescriptions differ by terms of order $\left(\frac{\Lambda^2}{Q^2}\right)^p$.

In this way we get a signal of power corrections from large orders of perturbation theory.

For thrust, leading power correction $\sim \frac{\Lambda}{Q}$.

IR renormalon is the divergence of Perturbation series.

Coeff. of $(\Lambda^2/Q^2)^p$ not determined.

For processes where OPE is valid, renormalons in agreement:

Ex: $e^+e^- \to$ hadrons:

Power corrections $\sim \frac{\langle G_{\mu\nu}^2 \rangle}{Q^4}$ Mueller Zakharov

DIS $\sim \frac{\Lambda^2}{Q^2}$ Mueller

However for event shapes, Drell-Yan;... There is no OPE and renormalons only way to probe $\left(\frac{\Lambda^2}{Q^2}\right)^p$.

A variation:

At 1 loop renormalons not the only way to probe Infrared sensitive region.

One can give gluon a mass & look for terms non-analytic in λ^2:

$\frac{\lambda}{Q}$, $\frac{\lambda^2 \ln \lambda^2}{Q^2}$, ...

non-analytic terms only come from IR region

Correspondance: $\lambda^2 \ln \lambda^2 = c \Lambda^2_{\overline{MS}}$ (unknown)

Limitations of Renormalons

1. Renormalons respect symmetries of the lagrangian: Cannot produce $\langle \bar{q}q \rangle \neq 0$.

2. Renormalons are "target blind". Ex:

i) $\langle p|G^2|p\rangle_{ren} = \langle 0|G^2|0\rangle_{ren}$

ii) In DIS: renormalons determined by divergences in Coeff. functions* in the OPE. "Genuine" higher twist corrections reflect correlations between partons inside target.

iii) Renormalons give no direct indication of confinement, ex., string formation.

* "hard" partonic crossections

Power corrections & KLN Theorem.

Power corrections arise due to infared sensitive contributions ($\frac{\lambda}{Q}$, $\frac{\lambda^2 \ln\lambda}{Q^2}$, $\frac{\lambda^4 \ln\lambda}{Q^4}$, ...)

If one considers an inclusive enough quantity, $\ln\lambda$ terms cancell (IR safety) (Ex. $e^+e^- \to$ hadrons). CERTAIN Power corrections also subject to cancellations like logs?

FOR Inclusive processes NOT only $\ln\lambda$ terms but also terms $\sim \lambda/Q$ are cancelled (Ex. Drell-Yan) (1loop: Banneka, Braun)

KLN Theorem is the general Principle behind this cancellation. (V. Zakharov, RA) Cancellations continue till one reaches condensate terms.

Processes that assume some precision measurements ("Exclusive"), like event-shapes, do receive $\frac{1}{Q}$ corrections.

Some Similarity with spectrum of Soft Photons in QED:

spectrum $\sim e^2 \frac{d\omega}{\omega}$ which is universal

Integrating over photon energies (inclusive case) $\Rightarrow$ $\ln \lambda$ divergence which is cancelled due to Bloch-Nordsieck. Mechanism.

"Universality" of $1/Q$ Corrections in event shapes not at the same level of accuracy.

$\frac{1}{Q}$ Corrections to event shape variables

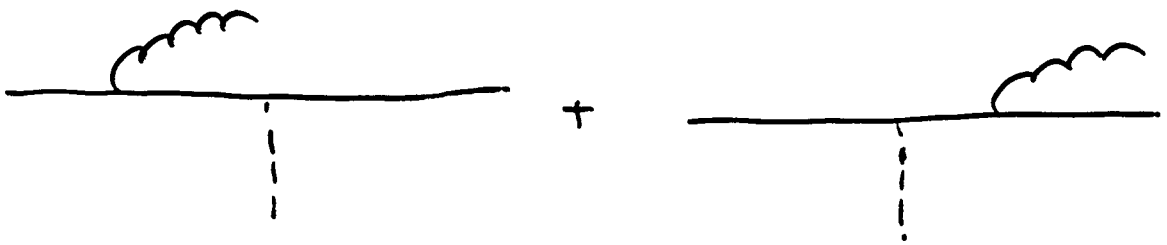
Represent leading power corrections:
$\sim 10\%$ w.r.t. $\mathcal{O}(\alpha_s)$ Predictions.
Comparable to $\mathcal{O}(\alpha_s^2)$ at $Q \sim M_Z$.

– No OPE for event shapes, unlike σ_{Tot}.
Renormalons instramental in Providing Theoretical basis for $1/Q$ Corrections.

Thrust:

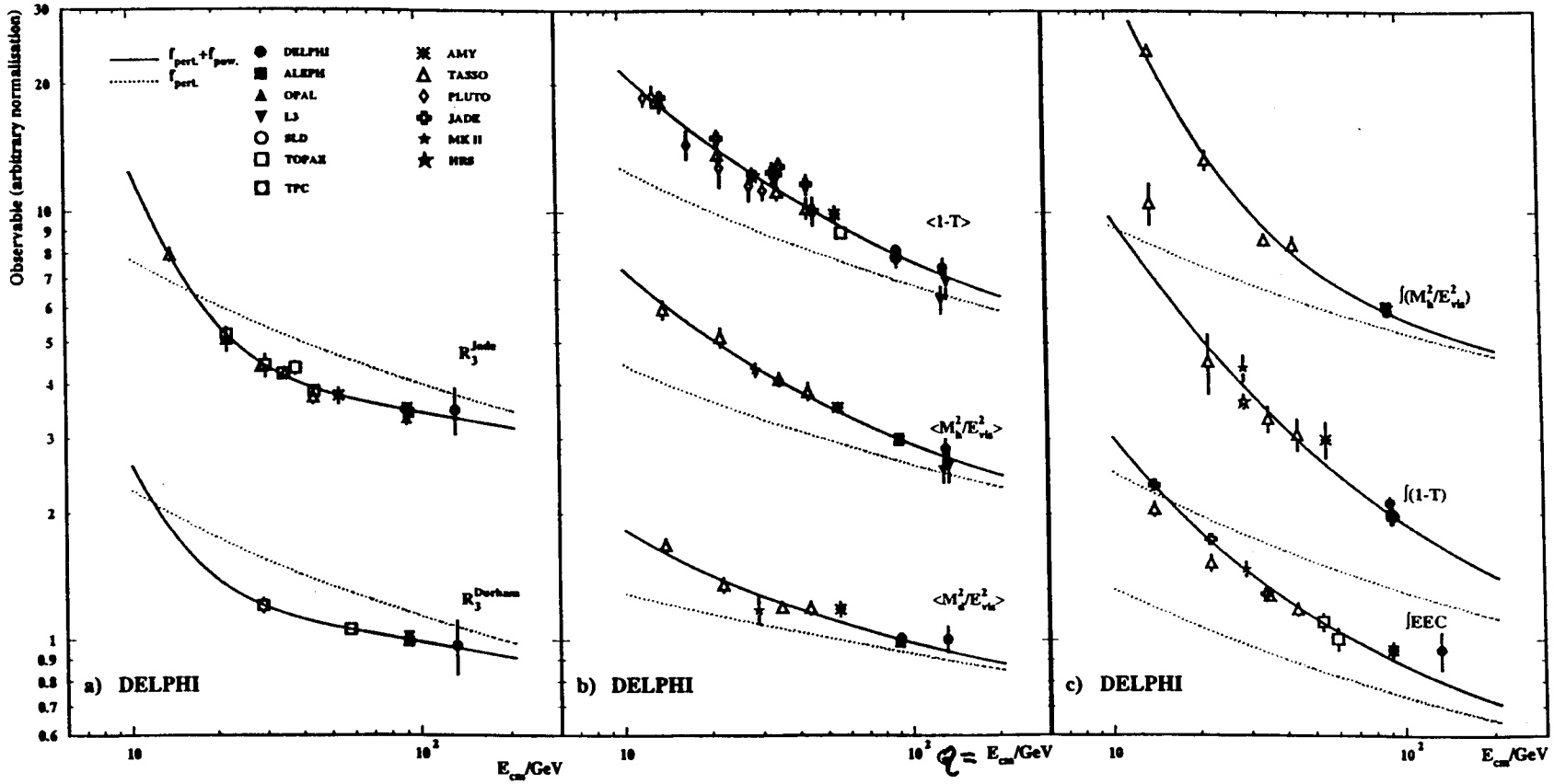
$$T = \max_{\{\hat{n}\}} \frac{\sum |\vec{p}_a \cdot \hat{n}|}{\sum |\vec{p}_a|} \quad \left(\frac{1}{2} < T < 1\right)$$

2-jet



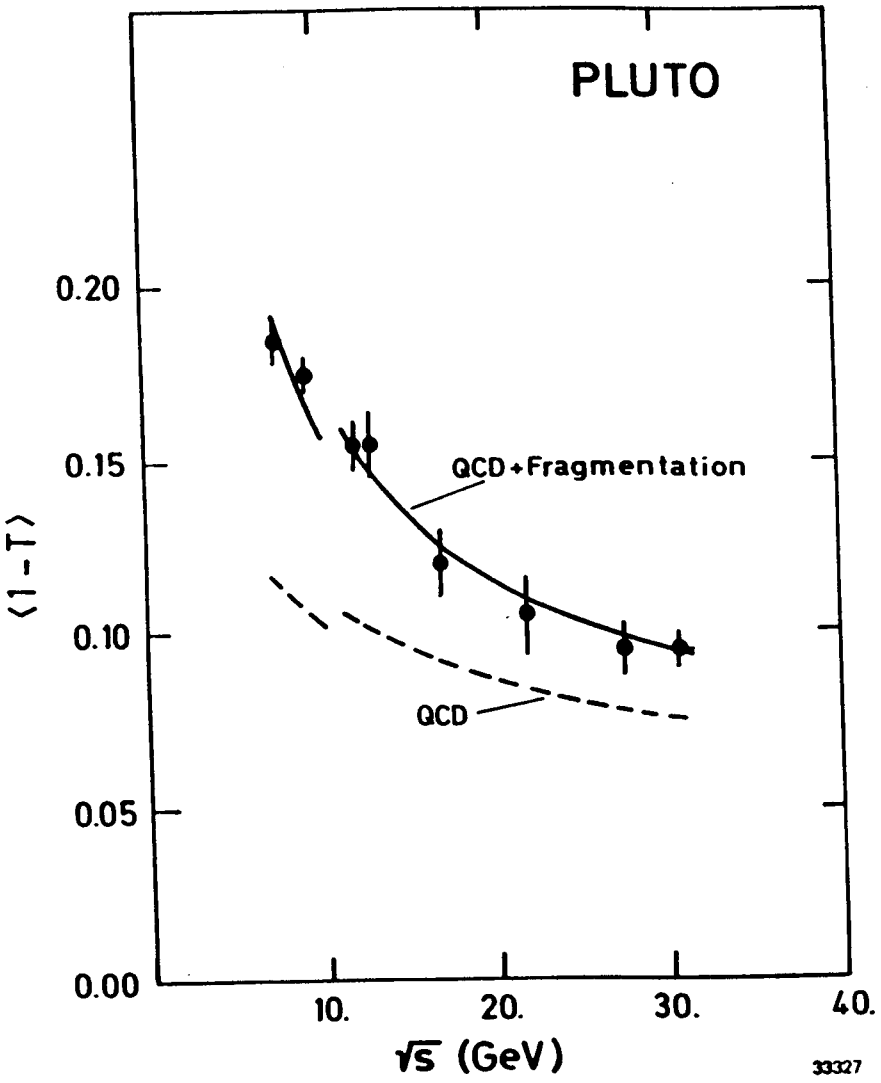
$$\langle 1-T \rangle_{1/Q} \simeq \frac{4}{Q} \int_0^{\sim Q} dk_\perp \frac{\alpha_s(k_\perp^2) C_F}{\pi}$$

due to soft gluons
Come from region $T \to 1$.



$\langle 1-T \rangle \propto 1/Q \sim \frac{1\,GeV}{Q}$ at $Q \sim 100$ GeV

Figure 8: Energy dependence of event shape observables using the cuts defined in Table 2. The curves are results of the fits to equations (1-3). The corresponding parameters are given in Table 3. In b) the fits with $C_2 = 0$ are displayed. The dotted lines correspond to the pure second order perturbative prediction, $\langle f_{pert} \rangle$, the full curves represent the sum of the perturbative and power terms, $\langle f_{pert} \rangle + \langle f_{pow} \rangle$.



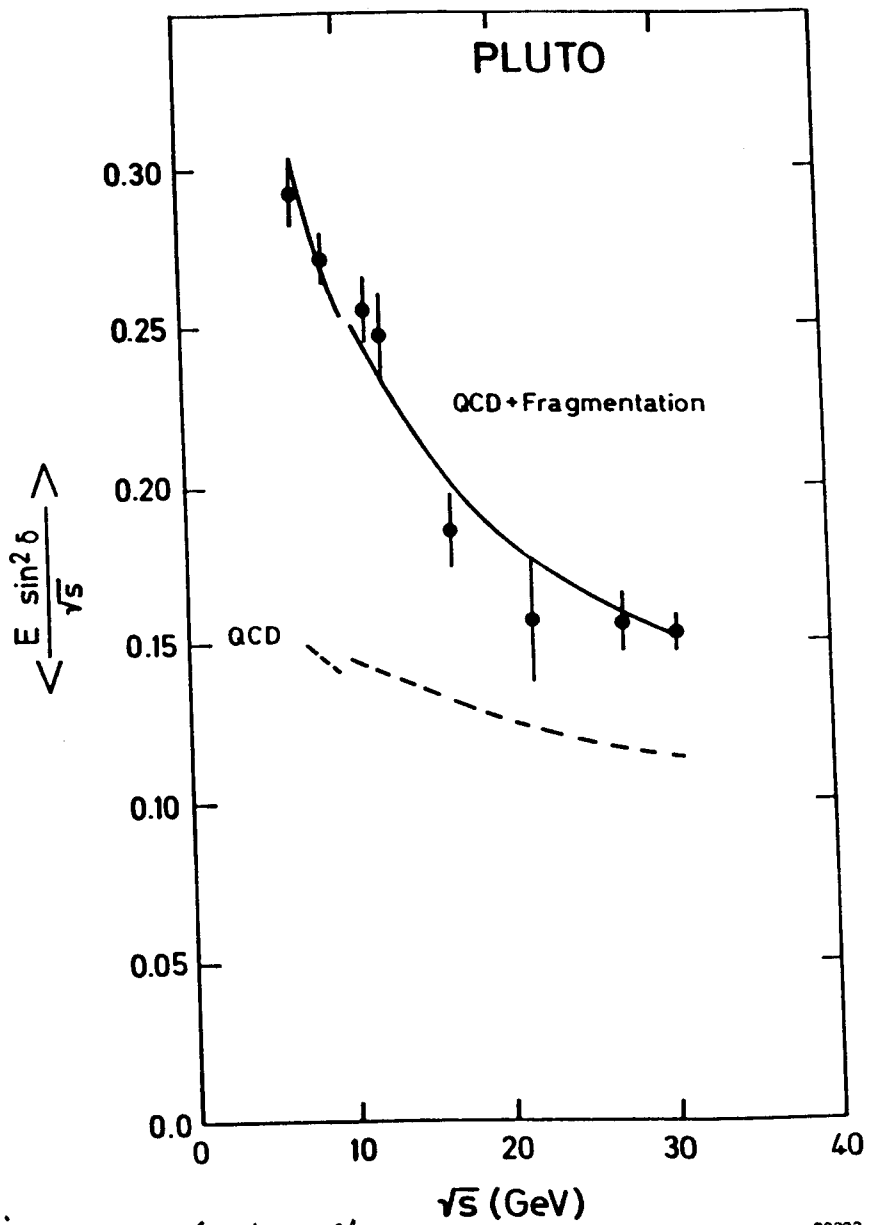


Fig. 43

From F. Barreiro, Fortschr. Phys. 34 (1986) 503 Fig. 44

All Powers of $\alpha_s(k_\perp^2)$ equally important.

Argue: (V. Zakharov, R.A., G. Sterman, G. Korchemski)

Near 2-Jet limit:

$$\langle 1-T\rangle_{\frac{1}{Q}} = R_T \frac{1}{Q} \cdot \int \frac{dk_\perp^2}{k_\perp^2} A(\alpha_s(k_\perp^2))\, k_\perp$$

↑ (in log approx, Just the universal Sudakov anomalous dim.

$R_T = 2.$

Relation to other event shapes?

2 Jet: Kinematics ⇒

$$1-T \approx \frac{M_1^2}{Q^2} + \frac{M_2^2}{Q^2}$$

$k_\perp$?

$$M_{Jet}^2 \sim (E + \langle k_\perp\rangle)^2 - p^2$$
$$\sim 2E\langle k_\perp\rangle$$
$$\Rightarrow \frac{M_{Jet}^2}{Q^2} \sim \frac{\langle k_\perp\rangle}{Q}$$

$\frac{1}{Q}$ ~ mean hadronization correction to Jet masses: "Tube model"

Similar Kinematic Configuration also appear to contribute to $1/Q$ corrections C-parameter, $\left\langle \frac{\sum E_i \sin^2\theta_i}{Q} \right\rangle$ (Dasham Brown S. Ellis Love)

and σ_L (?)

$$\langle C \rangle_{1/Q} \sim \frac{3\pi}{Q} \int \frac{dk_\perp^2}{k_\perp^2} A(\alpha_s(k_\perp^2))\, k_\perp$$

$$\left\langle \frac{E\,sin^2\delta}{Q} \right\rangle_{\frac{1}{Q}} \sim \frac{\pi}{Q} \int \frac{dk_\perp^2}{k_\perp^2} A(\alpha_s(k_\perp^2))\, k_\perp$$

$$\left\langle \frac{\sigma_L}{\sigma_T} \right\rangle_{1/Q} \sim \frac{3\pi}{8Q} \int \frac{dk_\perp^2}{k_\perp^2} \overset{?}{A}(\alpha_s(k_\perp^2))\, k_\perp$$

$A = \frac{C_F}{\pi}\alpha_s$ Lowest order

A = univ. Sudakov anom. dim. in Log. appx.

Since looking at same e^+e^- process assume that all non-perturbative effects are just the same type! Universality!

$$\langle O \rangle_{1/Q} = \frac{R_0}{Q} \cdot E_{soft}.$$

(Webber, Dokshitzer, V. Zakharov, R.A. G. Sterman, G. Korchemsky)

	R_0	Coeff$\left(\frac{1}{Q}\right)$(Expt.) E(Gev)	E_{soft} (Gev)
$\langle B_0 \rangle$			
$\langle 1-T \rangle$	2 (2)	0.77 ± .07 (Delphi)	0.4
$\langle C \rangle$	3π (3π)		
$\left\langle \frac{E\,sin^2\delta}{Q} \right\rangle$	π (π)	1.20 ± 0.25 (Pluto)	0.4
$\left\langle \frac{\sigma_L}{\sigma_T} \right\rangle$	$\frac{3\pi}{8}$ $(\pi/2)$ (Tube-model)	~ 0.8 (opal)	~ 0.5

$$\langle O \rangle = \langle O \rangle_{PT} + \frac{C_1}{Q} + \frac{C_2}{Q^2}$$

Observable		C_1(GeV)	C_2(GeV^2)	$\alpha_s(M_Z)$	χ^2/ndf
R_3^{Jade}	($y_{cut} = 0.08$)	-3.59 ± 0.55	61.3 ± 7.6	0.123 ± 0.002	8.0/13
R_3^{Durham}	($y_{cut} = 0.04$)	-2.53 ± 3.15	31.0 ± 28.4	0.137 ± 0.019	1.8/3
$\langle 1-T \rangle$		0.67 ± 0.20	1.0 ± 2.0	0.126 ± 0.004	43.8/26
		0.77 ± 0.07	0.0 (fixed)	0.125 ± 0.002	44.1/27
$\left\langle \frac{M_h^2}{E_{vis}^2} \right\rangle$		0.76 ± 0.26	-2.9 ± 3.3	0.116 ± 0.006	6.3/9
		0.54 ± 0.08	0.0 (fixed)	0.121 ± 0.002	7.1/10
$\left\langle \frac{M_d^2}{E_{vis}^2} \right\rangle$		0.03 ± 0.15	2.0 ± 1.8	0.100 ± 0.006	5.6/6
		0.0 (fixed)	2.4 ± 0.5	0.101 ± 0.002	5.7/7
		0.19 ± 0.04	0.0 (fixed)	0.094 ± 0.002	6.9/7
$\int (1-T)$	$T < 0.8$	0.36 ± 0.03	0.9 ± 1.18	0.120 (fixed)	19.2/9
$\int \frac{M_h^2}{E_{vis}^2}$	$\frac{M_h^2}{E_{vis}^2} > 0.1$	0.05 ± 0.03	9.6 ± 0.9	0.120 (fixed)	7.1/5
$\int$ EEC	$\lvert\cos\theta\rvert < 0.5$	1.26 ± 0.05	4.6 ± 0.8	0.120 (fixed)	66/12

Table 3: Fits to the mean values and integrals of event shape variables at all available energies.

$\langle 1-T \rangle = .0678 \pm .0002$ ($Q = 91.29$ev

$= .0616 \pm .0034$ ($Q = 133$gev

$\int (1-T)$, $T < 0.8 = .0130 \pm .0005$

(excludes 2 jets). $Q = 91.2$ge

If mean hadronyation correction to jet mass:

$$\langle M^2_{had} \rangle = \lambda Q$$

$\lambda \simeq 0.5$ GeV at LEP (Webber)

Sets scale of $1/Q$ Corrections at "2-jet".

[Near 2-jet limit]

$$\langle C \rangle^{IR}_{2jet} = 3 \left\langle \frac{\Sigma E_i \sin^2 \delta_i}{Q} \right\rangle \quad \text{(nason)}$$

3-jet crossection also gives $1/Q$ but down by $\alpha_s(Q)$.
Gluon mass estimate: Webber & Dokshitzer

Uncertainities:

1) $\alpha_s(k_\perp^2)$: scale important: ex:

$\alpha_s(k^2)$ where $k^2 =$ off-shellness

gives no $1/Q$. However this does not correctly reproduce $\ln^R(1-T)$, $\ln^{p-1}(1-T)$ terms in next to leading order.

However no way to rule out for example

$\alpha_s(2k_\perp^2)$. OK for similar processes.

2. (Nason + Seymour):

At level of 4 parton final state:

two types of $1/Q$ terms.

(1) "Sudakov type"

(2) Cannot be associated with $\alpha_s(k_\perp^2)$ prescr

↑

Violate "universality"

They are small compared to universal term (~10%). Subleading?

$\alpha_s(Q^2)$?

Enhancement ?

"Frozen Coupling" ?

σ_L and Power Corrections to Fragmentation Functions.

$$e^+e^- \to h(p)X \qquad x = \frac{2p\cdot q}{q^2}$$

$$\frac{1}{\sigma_{tot}}\frac{d\sigma}{dx\,d\cos\theta} = \frac{3}{8}(1+\cos^2\theta)F_T(x) + \frac{3}{4}\sin^2\theta\, F_L(x) + "F_A"$$

$$F_L(x) = \sum_{\substack{\text{Partons}\\ i}} \int_x^1 \frac{dz}{z}\; C_L^{(i)}(z,\alpha_s,Q)\; D_i^h(x/z)$$

$C_L^{(i)}$: Partonic cross section "short distance"

D_i^h: Parton Fragmentation function.

$$\frac{\sigma_L}{\sigma_{tot}} = \sum_h \frac{1}{2}\int_0^1 dx\cdot x\; F_L(x)$$

$$= \frac{1}{2}\sum_i \int_0^1 dz\cdot z\cdot C_L^{(i)}(z)$$

$$\sigma_{tot} = \sigma_0\left[1 + \frac{\alpha_s}{\pi} + 1.411\left(\frac{\alpha_s}{\pi}\right)^2 + \cdots\right]$$

$$\sigma_L = \sigma_0\left[\frac{\alpha_s}{\pi} + 8.5\left(\frac{\alpha_s}{\pi}\right)^2 + \frac{A}{Q} + \cdots\right]$$

large compared to σ_{Tot}.

$$\sigma_{Tot} = \sigma_L + \sigma_T$$

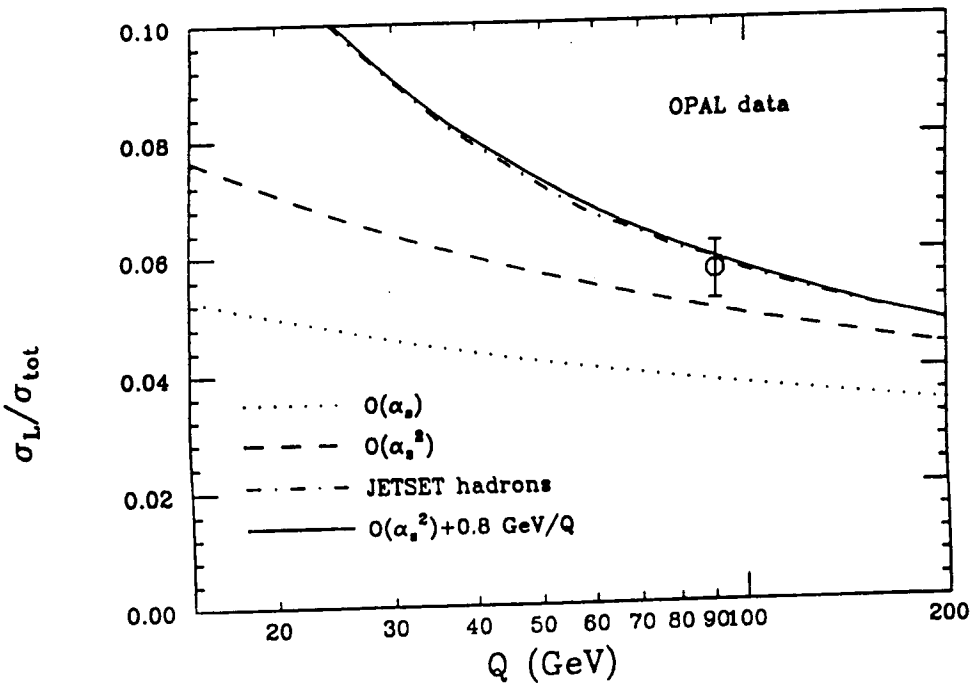
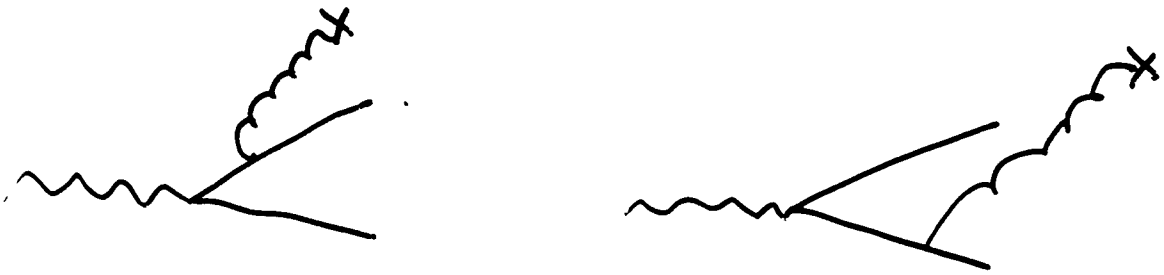


Figure 1: Longitudinal fraction of the e^+e^- hadronic cross section.

Only C_L^g Contributes to $\frac{1}{Q}$ term (i.e gluon fragmentation)



Soft gluon region as before:

$A = \frac{3\pi}{8} E_{soft.}$ renormalon chain: $\alpha_s \to \alpha_s(k_L)$

$A = \frac{\pi^2}{8} \cdot \lambda$ gluon mass.

massive gluon method less reliable?
since $\sigma_L|_{1/Q}$ controlled by gluon fragmentation

$z C_L^g(z)$ is very singular as $z \to 0$

$$z C_L^g(z) \sim \frac{\lambda^2}{Q^2 z^2} + \frac{\lambda^4}{Q^4 z^4} + \cdots$$

integral promotes $\lambda^2/Q^2 \longrightarrow \lambda/Q$

Webber + Dasgupta; Beneke, Braun, Magnea.

HIGHER TWIST ($\frac{1}{Q^2}$) IN DIS.

These can be adressed within the Standard OPE. Renormalons in this case unify evaluation of Coeff. functions and of matrix elements.

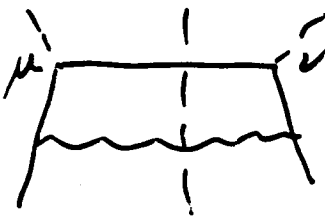
For example if we are interested in power corrections to moments of structure functions, specifically F_3:

$\int F_3(x)\,dx$ is relevant to GLS.

$$F_3 = \int_x^1 \frac{dz}{z}\, C(z,Q^2)\, q(x/z, Q^2)$$

$C(z,Q^2)$: Coeff. function "partonic"

$q(x/z, Q^2)$: quark distribution

From 

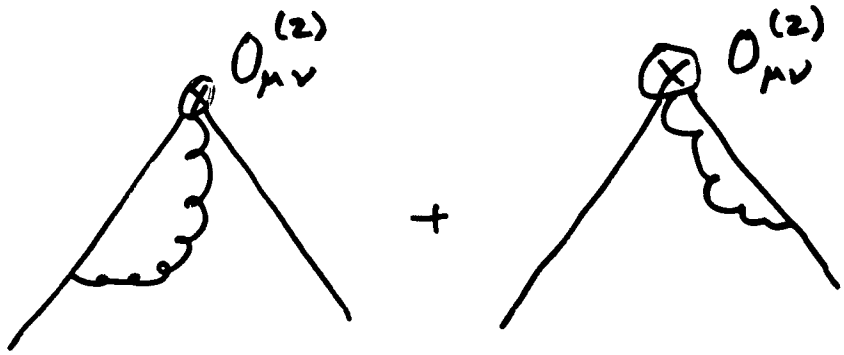
, etc. one can find power corrections by looking for IR sensitive terms like $\lambda^2 \ln \lambda / Q^2$.

$$C(x) = -P_0(x) \ln \epsilon + C_0(x) - C_2(x) \frac{\lambda^2}{Q^2} \ln \lambda^2 + \ldots$$

The same $C_2(x)$ is obtained by considering quark matrix elements of higher twist operators (Shuryak + Vainshtein):

$$O_{\mu\nu} = \frac{2i}{q^2}\epsilon_{\mu\nu\alpha\beta} q_\alpha \left(\bar{q}\gamma_\beta q + \frac{4g}{9q^2}\bar{q}\tilde{G}_{\beta\delta}\gamma_\delta\gamma_5 q \right)$$

↑ leading twist; ↑ higher twist



In I_R region both give same answer.

Webber + Dasgupta: Attempt to explain the x-dependance of structure function of $1/Q^2$ power correction:

$$F(x,Q^2) = q(x,Q^2)\left(1 + \frac{D_2(x,Q^2)}{Q^2}\right)$$

$$D_2(x,Q^2) = \frac{A_2'}{q(x,Q^2)}\int_x^1 \frac{dz}{z} C_2(z)\, q(x/z, Q^2).$$

Critique:
Anomalous dimension of higher twist oper. different from leading twist $\Rightarrow$ new structure functs for higher twist.

Webber + Dasgupta: hep-ph/9604388

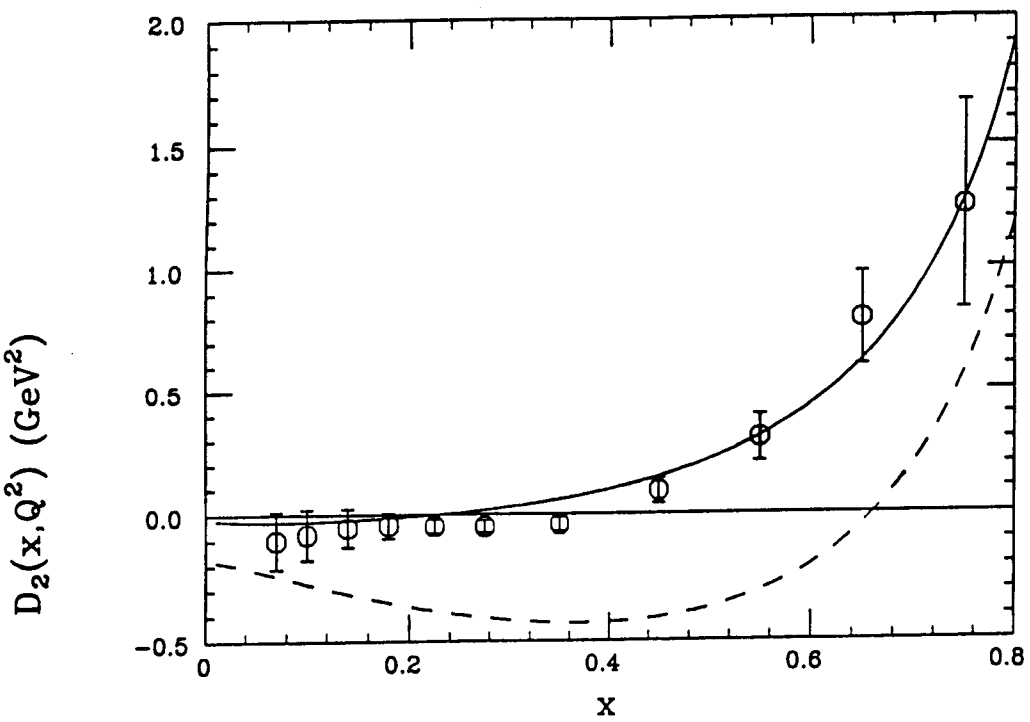


Figure 1: Coefficients of $1/Q^2$ contributions to F_2 (solid), and to F_1 and F_3 (dashed).

Valence quark distribution in ν scatt.

$F = \frac{1}{2}(F^{\nu} + F^{\bar{\nu}})$, $q_v = u_v + d_v$

↑ MRSA parametrization

$Q^2 = 10\ \text{GeV}^2$, $A_2' = -0.2\ \text{GeV}^2$

Conclusions:

Renormalons give us indication of Power corrections.
With additional assumptions one can go further. (universality, ...)
Renormalon calculus still in its infancy.
If universality holds up to experiment, strong reasons for looking for a generalization of OPE (George Sterman's talk)

OPE generalization is the best way to go.