ISSUES IN RESUMMATION

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CTEQ Symposium
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Sudakov resummation : resummation of double-log corrections due to multiple soft-gluon emission

Outline :

- Brief overview of basic features of soft-gluon resummation : exponentiation in N-space
- The varieties of Sudakov sensitive processes :
  - e^+e^- event shapes
  - total cross sections in hadron collisions
  - $Q_\perp$ - distributions
  - ⋮
- Some issues in the application of resummed formulae to the calculation of different classes of physical processes
  - "applicability" of x-space resummation formulae

- Some phenomenological applications
  - e^+e^- event shapes and renormalization-scale dependence — Turnock-Trentadue, Webber + S.C.
  - hadronic collisions : • heavy-flavour production
    • jet production — Mangano-Nason, Trentadue + S.C.

i.e. all-order summation of soft-gluon contributions to QCD cross sections

in recent years, it has become a topic of broad interest

- **on the theoretical side**:

  our confidence in resummed calculations has highly increased

  - resummation techniques has been extended beyond leading-logarithmic (LL) order

    in particular, using resummed calculations to next-to-leading logarithmic (NLL) accuracy, one can

    i) introduce meaningful definition (say $\overline{MS}$) of strong coupling $\alpha_s(Q)$ → relevance for α_s determinations

    ii) investigate theoretical accuracy by studying renormalization-scale dependence → as in NLO calculations

    iii) match resummed predictions with full NLO calculations → extend applicability of perturbative QCD to larger kinematic regions

  - many different observables in many different processes have been evaluated

    i) $p_\perp$-distributions, total cross sections in hadron collisions (DY, DIS), event shapes and jet rates in e^+e^- → all known to NLL accuracy — Kodaira-Trentadue, Collins, Soper, Sterman Webber, Turnock, S.C. ...

    ii) NLL results for multiparton ($n \geq 3$) processes just begun to appear — Contopanagos, Laenen Kidonakis, Sterman (Maugano, Nason, Trentadue, Webber, S.C.)

- on the phenomenological side :

many experimental results sensitive to soft-gluon resummation have become available :

⋮

high-precision e^+e^- data from LEP and SLC strongly demand understanding of Sudakov region → fig.

⋮

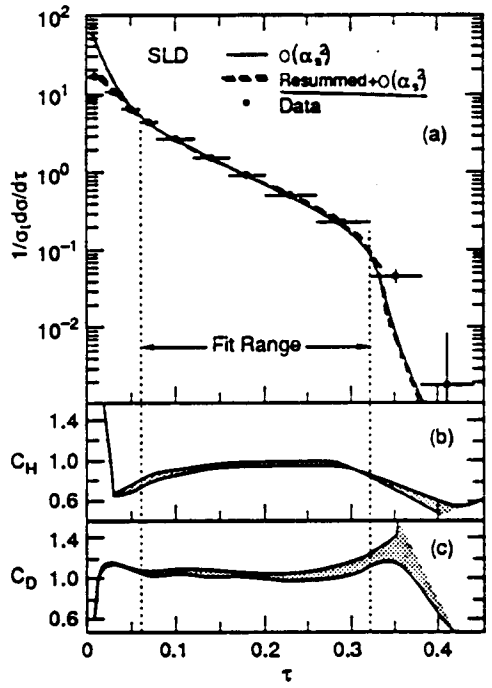
last, but not least,

high-E_T tail of jet cross sections at Tevatron requires carefull investigations of soft-gluon corrections → fig.

SLD

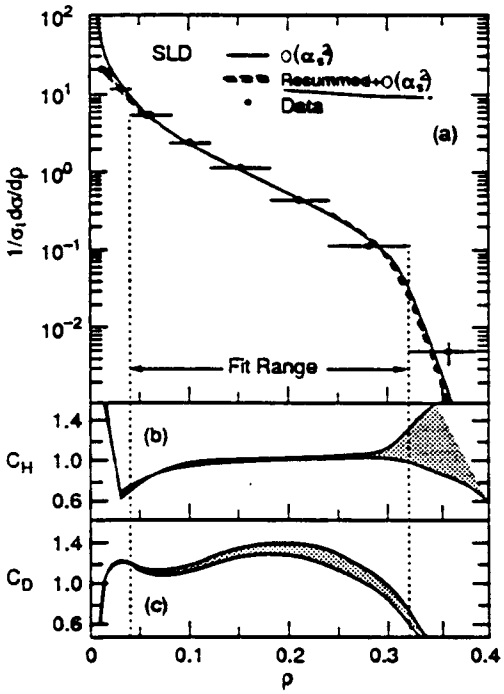
renormalization scale : $\mu^2 = Q^2$

$\alpha_s = 0.118 \pm 0.009$

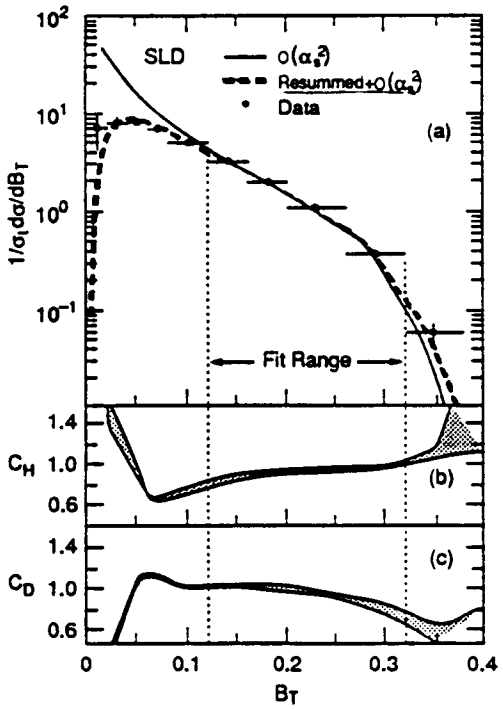


THRUST

$\alpha_s = 0.114 \pm 0.008$

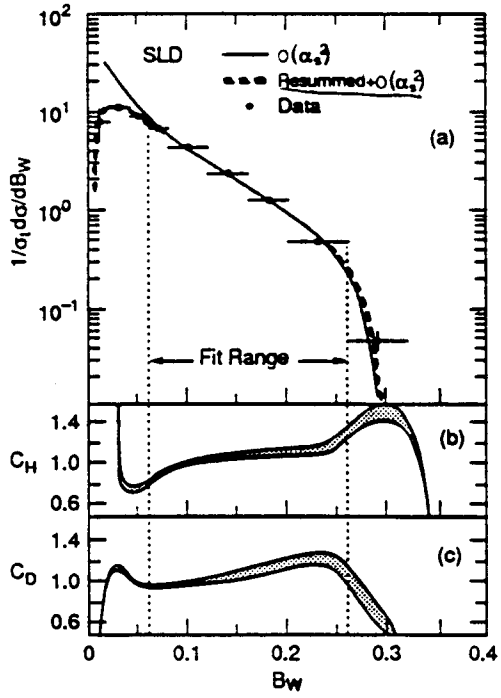


HEAVY JET MASS



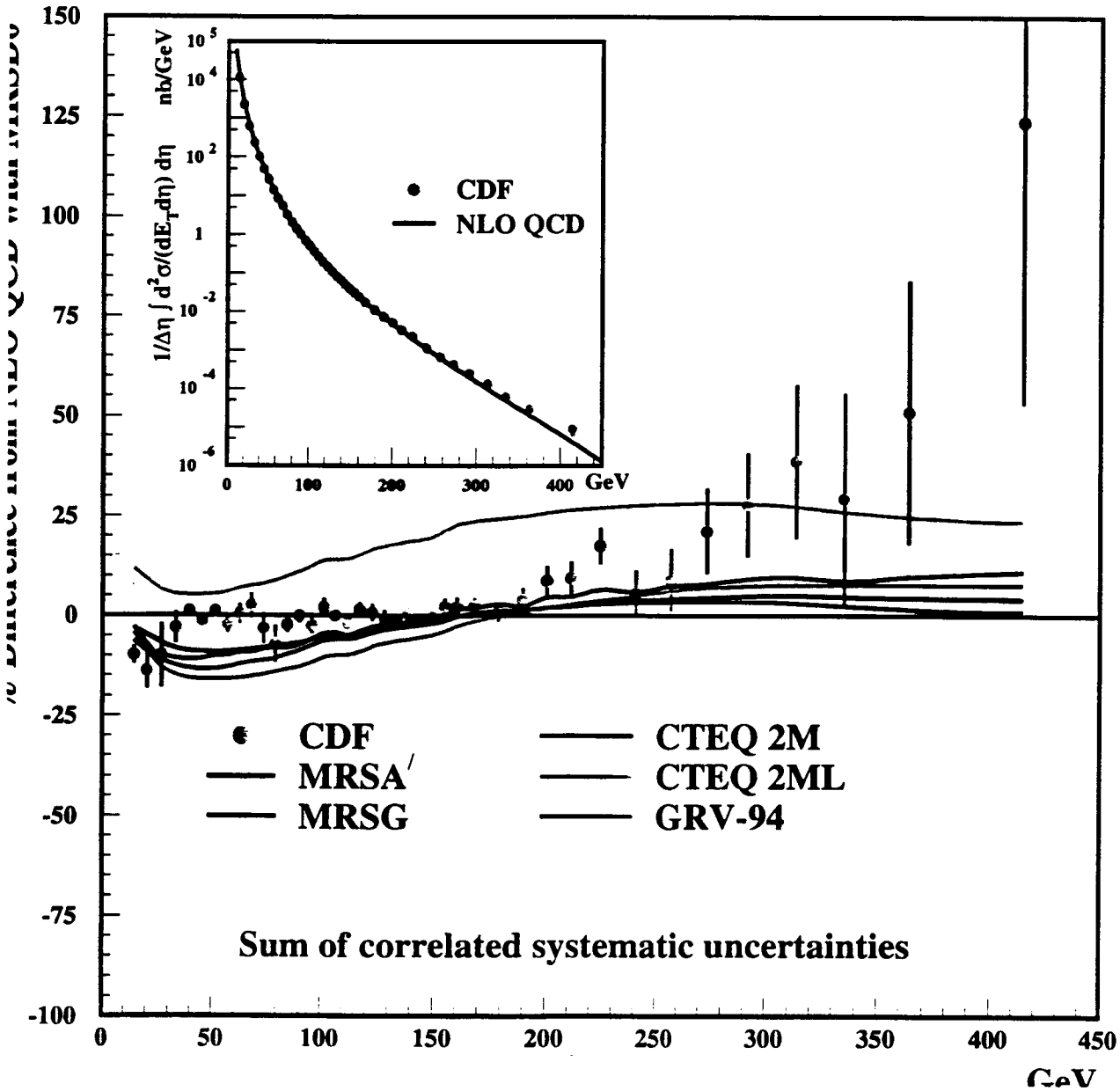
TOTAL JET BROADENING

$\alpha_s = 0.11? \pm 0.00?$

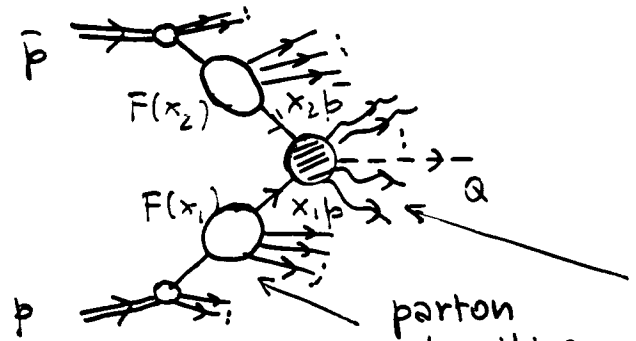


WIDE JET BROADENING

$\alpha_s = 0.107 \pm 0.005$


150
125
100
75
50
25
0
-25
-50
-75
-100
0
50
100
150
200
250
300
350
400
450
GeV
CDF
MRSA′
MRSG
CTEQ 2M
CTEQ 2ML
GRV-94
Sum of correlated systematic uncertainties
CDF
NLO QCD
$1/\Delta\eta \int d^2\sigma/(dE_T d\eta)\, d\eta$ nb/GeV
10^5
10^4
10^2
1
10^{-2}
10^{-4}
10^{-6}
0
100
200
300
400 GeV

For instance : DRELL-YAN process



$(p+\bar{p})^2 = S$

$\tau = \frac{Q^2}{S}$ inelasticity variable

hard subprocess : partonic cross section

parton densities

QCD factorization theorem

$$\sigma(Q^2,\tau) = \int_0^1 dz \int_0^1 dx_1 dx_2 \; F(x_1,Q^2)\, F(x_2,Q^2)\, \delta(\tau - x_1 x_2 z)\, \Delta(z,\alpha_s(Q^2))\, \hat{\sigma}_0(Q^2)$$

momentum conservation

$1-z$: energy-fraction radiated in hard subprocess

radiative-correction factor

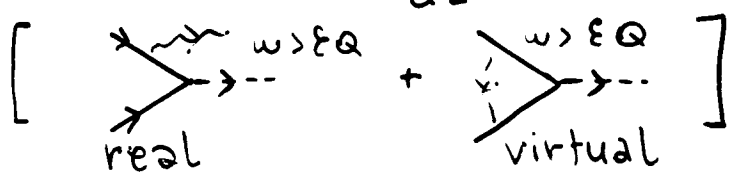
radiative-correction factor : computable as power series in α_s

$$\Delta(z,\alpha_s) = \delta(1-z) + \alpha_s \Delta^{(1)}(z) + \dots$$

consider soft-gluon contribution ↕ $1-z = \omega/Q$

$$\alpha_s \Delta^{(1)}(z) = \frac{dW_1(z)}{dz} + \dots$$

real and virtual contributions are separately divergent



ε : unphysical cutoff on gluon energy

single-gluon emission probability

$$\frac{dW_1(z)}{dz} = 2a \lim_{\varepsilon\to 0}\left[\frac{1}{1-z}\ln\frac{1}{1-z}\,\Theta(1-z-\varepsilon) - \delta(1-z)\int_0^{1-\varepsilon} \frac{dz'}{1-z'}\ln\frac{1}{1-z'}\right]$$

logarithmic bremsstrahlung spectrum

logarithic collinear spectrum

the + distribution :

$$\int_0^1 dz\, f(z)\left(\frac{1}{1-z}\ln\frac{1}{1-z}\right)_+ \equiv \int_0^1 dz\, \frac{f(z)-f(1)}{1-z}\ln\frac{1}{1-z}$$

$$= 2a\left(\frac{1}{1-z}\ln\frac{1}{1-z}\right)_+$$

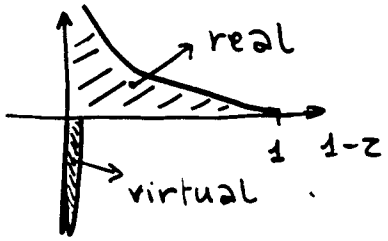
well-defined distribution

$a = C_i\, \alpha_s/\pi$

process-dependent coefficient

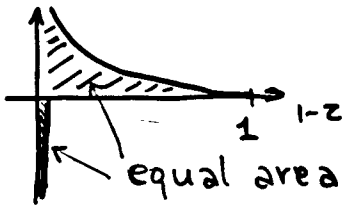
finite emission probability for $\varepsilon \to 0$

cross section



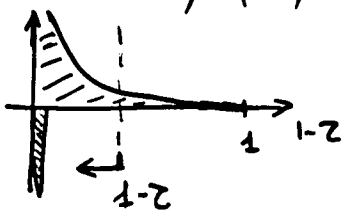
$$\left(\frac{1}{1-z}\ln\frac{1}{1-z}\right)_+$$

i) finite for $\varepsilon \to 0$ (soft-gluon cancellation) and in particular



$$\int_0^1 dz \left(\frac{1}{1-z}\ln\frac{1}{1-z}\right)_+ = 0$$

ii) for any physical value of z ($z \neq 0$)



$$\alpha_s \int_z^1 dz \left(\frac{1}{1-z}\ln\frac{1}{1-z}\right)_+ = -\alpha_s \int_0^z \frac{dz}{1-z}\ln\frac{1}{1-z} =$$

$$= -\frac{1}{2}\alpha_s \ln^2(1-z)$$

⬇

hadronic threshold → for $z \to 1$: real emission strongly suppressed
➡ virtual term unbalanced
➡ large correction factor expected $\boxed{\alpha_s \ln^2(1-z)}$

in general :

$$\Delta(z,\alpha_s) = \delta(1-z) + \sum_{n=1}^{\infty} \alpha_s^n \sum_{m=0}^{2n-1} C_{nm} \left(\frac{1}{1-z}\ln^m\frac{1}{1-z}\right)_+$$

soft-gluon corrections are dominant at threshold but their actual size depends on

- coefficient C_{nm} (process-dependent)
- x-shape of parton densities

⇒ soft-gluon effects can be substantial also before approaching the region $z \to 1$

⬇

worthwhile to study the transition toward $z \to 1$ by evaluating soft-gluon effects to higher-orders and possibly resumming them to all-orders in α_s

QCD PROCESSES SENSITIVE TO SUDAKOV RESUMMATION

Different classes of QCD processes :

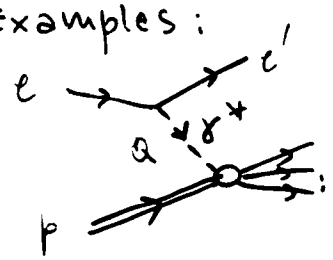
- hadronic collisions at threshold
- e^+e^- event shapes in two jet region
- Θ_1-distributions
- ⋮

- **Hadronic collisions at threshold** : inelasticity variable (measures distance from hadronic threshold)

$$x = \frac{Q^2}{S}$$

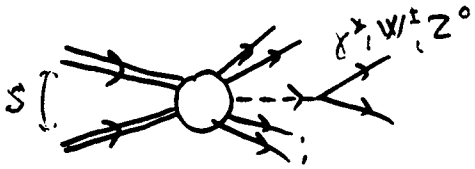
(Q^2: hard-scattering scale; S: centre-of-mass energy)

Examples :



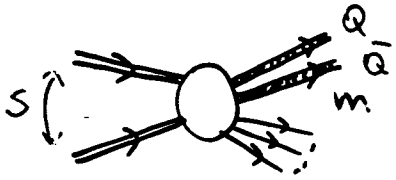
DIS deep-inelastic lepton-hadron scattering

x : $x_{Bj} = \frac{|Q^2|}{2p\cdot Q}$ (Bjorken variable)



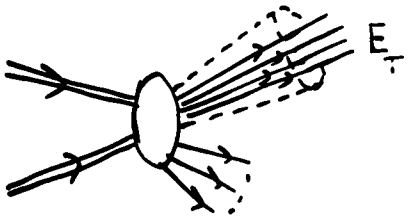
DY

x : $z = Q^2/S$



HQ heavy-quark production

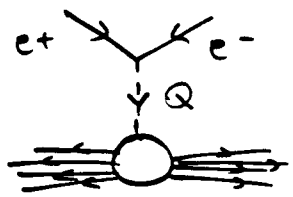
x : $\rho = \frac{4m^2}{S}$



High-E_T jets

x : $x_T = \frac{2E_T}{\sqrt{S}}$

- **e^+e^- event shapes in two-jet region** :

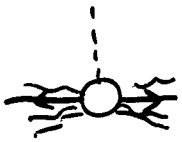


← 2 highly collimated jets

Examples :

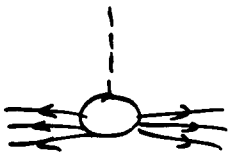
THRUST (maximizes longitudinal momentum)

$$T = \max_{\underline{n}} \frac{\sum_i |\underline{p}_i \cdot \underline{n}|}{\sum_i |\underline{p}_i|}$$



$T \to 1$

heavy jet mass



$m_H > m_L$

$$\rho = \frac{m_H^2}{Q^2}$$

$1 - \rho \to 1$

⋮

- $Q_\perp$-distributions

e.g. $Q_\perp$-distribution in DY process

energy-energy correlations in e^+e^-

⋮

Dokshitzer-Dyakonov-Troyan
Parisi-Petronzio
Kodaira-Trentadue
Collins-Soper
Altarelli-Ellis-Martinelli-Greco

⋮

Consider again DY process

- convenient to introduce **N-moments** of cross section

$$\sigma_N(Q^2) \equiv \int_0^1 dz\, z^{N-1}\, \sigma(Q^2, z)$$

(likewise, for any other function in x-space : $g_N \equiv \int_0^1 dx\, x^{N-1} g(x)$)

x-convolution of cross section (related to energy conservation) ➡ exact factorization in N-space

$$\sigma_N(Q^2) = \hat{\sigma}_0(Q^2)\; F_N^2(Q^2)\; \Delta_N(\alpha_s(Q^2))$$

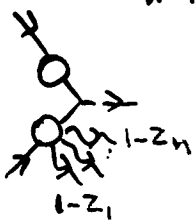
$\hat{\sigma}_0(Q^2)$: N-moments of parton densities ; $\Delta_N(\alpha_s(Q^2))$: soft-gluon terms

$$\left(\frac{1}{1-z}\, z^{n-1}\, \frac{1}{1-z}\right)_+ \longrightarrow z^n \ln N \qquad \text{i.e.} \quad \boxed{z \to 1 \sim N \to \infty}$$

- soft-gluon resummation

$$\Delta(z, \alpha_s) = \delta(1-z) + \sum_{n=1}^{\infty} \int_0^1 dz_1 \dots dz_n\, \frac{d\mathcal{W}_n(z_1,\dots,z_n)}{dz_1 \dots dz_n}\, \mathcal{D}_{PS}(z; z_1, \dots, z_n)$$

multi-gluon amplitude



$$\frac{d\mathcal{W}_{(n)}(z_1,\dots,z_n)}{dz_1 \dots dz_n} \overset{\text{soft limit}}{\simeq} \frac{1}{n!} \prod_{i=1}^{n} \frac{d\mathcal{W}_{(1)}(z_i)}{dz_i}$$

phase space that defines physical cross section

dynamical factorization (multiple emission is uncorrelated)

phase space (total cross section with only constraint of energy conservation)

$$\mathcal{D}_{PS}(z; z_1, \dots, z_n) = \delta(z - z_1 \cdots z_n) \longrightarrow z_1^{N-1} \cdots z_n^{N-1}$$

NOT FACTORIZED ; N-space ; FACTORIZED

$$\Delta_N(\alpha_s) \simeq \sum_{n=0}^{\infty} \frac{1}{n!} \left[\prod_{i=1}^{n} \int_0^1 dz_i\, \frac{d\mathcal{W}_1(z_i)}{dz_i}\, z_i^{N-1} \right] = \exp\left\{ \int_0^1 dz\, z^{N-1}\, \frac{d\mathcal{W}_1(z)}{dz} \right\}$$

physical basis for exponentiation : dynamics ⊕ kinematics* ; factorization ⟺ exponentiation

(there are no physical basis for exponentiation directly in x-space)

*that's why some jet definitions may be preferred

• generalized exponentiation theorem

previous argument is exhaustive in QED

$$\Delta_N(\alpha_s) = \exp\left\{ 2a \int_0^1 dz\, \frac{z^{N-1}-1}{1-z} \ln \frac{1}{1-z} \right\} =$$

$$\underset{N\to\infty}{=} \exp\left\{ -a \ln^2 N + O(\ln N) \right\}$$

dominant large-N behaviour ⟵ double-log (DL) approximation

in QCD, corrections to dynamic factorization of multi-gluon amplitudes have to be taken into account

nonetheless ➡ ALL-ORDER EXPONENTIATION STILL VALID (Sterman / Trentadue + S.C.)

$$\boxed{\Delta_N(\alpha_s) = \exp\left\{ \ln N\, f_1(\alpha_s \ln N) + f_2(\alpha_s \ln N) + \alpha_s f_3(\alpha_s \ln N) + \dots \right\}}$$

LEADING LOGS $\alpha_s^n (\ln N)^{n+1}$ — NEXT-TO-LEADING LOGS $\alpha_s^n (\ln N)^n$ — ....

formally down by a power of α_s

exponentiation theorem allows us to define an improved (resummed) perturbative expansion

example DY : $\ln N\, f_1(\alpha_s \ln N) = a \ln^2 N \cdot \left[\frac{2\lambda + (1-2\lambda)\ln(1-2\lambda)}{2\lambda^2} \right]$, $\lambda = \beta_0 \alpha_s \ln N$

• remark about Landau singularity :

most of QCD corrections taken into account by running coupling effects $\alpha_s \to \alpha_s(q_\perp^2)$

$$\ln \Delta_N(\alpha_s) \simeq \frac{C}{\pi} \int_0^1 dz\, \frac{z^{N-1}-1}{1-z} \int_{(1-z)^2 Q^2}^{Q^2} \frac{dq^2}{q^2} \alpha_s(q^2)$$

$$\simeq \frac{C}{\pi} \int_0^{1-e^{-r\varepsilon/N}} \frac{dz}{1-z} \int_{(1-z)^2 Q^2}^{Q^2} \frac{dq^2}{q^2} \alpha_s(q^2) + \text{all-order subleading terms}$$

LL + NLL

one may argue about renormalon contributions

(Alvero-Contopanagos-Sterman, Beneke-Braun, Marchesini-Dokshitzer-Webber)

no renormalons: only branch cut for [illegible]

RESUMMED FORMULAE TO PHYSICAL PROCESSES

① : - resummed formulae are provided in N-space (in general, properly defined conjugate space)

$$\sigma_N = \int_0^1 dz\, z^{N-1}\, \sigma(z)$$ ← experimentally we do not know $\sigma(z)$ for $0 \le z \le 1$

- need to go back to x-space

$$\Delta_N \to \Delta(z)$$

in phenomenological applications

② : - resummed formulae in N-space are sensitive to Landau singularity in α_s

↓

introduce regularization prescription without spoiling main perturbative features

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① - back to x-space *

Can we possibly exploit formal equivalence

$$N \sim \frac{1}{1-x} \quad \text{(valid to LL order)}$$

to (systematically) perform resummation directly in x-space?

i.e. $\Delta(z) = \exp\left\{ \ln \Delta_{N=\frac{1}{1-z}} + \dots \right\}$

* based on

$$\frac{1}{n!}\alpha_s^n \ln^{2n} N = \int_0^1 dz\, z^{N-1} \left\{ \underbrace{\frac{1}{(n-1)!}\left(\frac{2\alpha_s^n \ln^{2n-1}(1-z)}{1-z} \right)_+}_{\text{LL}} + \underbrace{O\left(\left(\frac{\alpha_s^n \ln^{2n-2}(1-z)}{1-z} \right)_+ \right)}_{\text{NLL}} + \dots \right\}$$

Warning

- to any fixed order n., logarithmic hierarchy only if $\alpha_s \ln \frac{1}{1-z} \lesssim 1$

and, most important,

- after all-order summation : series ↔ series + series + ...

$$\underbrace{\sum_{n.}^{(\infty)} (LL)}_{N\text{-space}} = \underbrace{\sum_{n.}^{(\infty)} (LL)}_{x\text{-space}} + \underbrace{\sum_{n.}^{(\infty)} (NLL)}_{x\text{-space}} + \dots \to$$ are these series convergent?

COMPARING: e^+e^- event shapes

cross sections in hadron collisions

- e^+e^- event shapes : e.g. Thrust distribution

$$\frac{1}{\sigma}\frac{d\sigma}{dT} = -\frac{d}{dT}\sum^{I}(T,\alpha_s(Q)) + \dots \equiv \Delta^{(T)}(T,\alpha_s) + \dots$$

$\sum^{I}$: soft; $\Delta^{(T)}$: radiative factor that includes soft-gluon effects

$$\Delta^{(T)}(T,\alpha_s) = \int \frac{dN}{2\pi i}\, T^{N-1}\, \Delta_N^{(T)}(\alpha_s) \leftarrow \text{N-moments}$$

- cross sections in hadron collisions : e.g. DY

$$\tilde{\sigma}(z,Q^2) = \hat{\sigma}_0(Q^2) \int_\tau^1 \frac{dz}{z}\, \Delta(z,\alpha_s) \underbrace{\int_0^1 \frac{dx_1}{x_1}\frac{dx_2}{x_2} F(x_1,Q^2) F(x_2,Q^2)\, \delta\left(1-\frac{\tau}{x_1 x_2 z}\right)}_{\text{parton luminosity}}$$

because of Sudakov exponentiation in N-space:

thrust $\Delta_N^{(T)}(\alpha_s)$ ← analogous → $\Delta_N(\alpha_s)$ DY

BUT:

in T-distribution ⟶ we are interested in $\Delta^{(T)}(T,\alpha_s)$ for $T \to 1$ but $T < 1$

in DY process:
- $\Delta(z,\alpha_s)$ is always integrated up to $z=1$ ($\tau \le z \le 1$) also if τ is far from threshold
- Sudakov effects included in part also in parton distributions

in particular: $|a| = 2C_F\alpha_s/\pi$

$$\Delta_N^{(T)}(\alpha_s) = \exp\left\{-|a|\ln^2 N + \dots\right\}$$ Sudakov suppression

$$\Delta_N(\alpha_s) = \exp\left\{+|a|\ln^2 N + \dots\right\}$$ (anti-)Sudakov enhancement

Incidentally,

$$\Delta_N^{(T)}(\alpha_s) = \exp\left\{E_N(\alpha_s) + \text{NNLL}\right\} \qquad \Delta_N(\alpha_s) = \exp\left\{-E_N(\alpha_s) + \text{NNLL}\right\}$$

↖ equal up to NLL accuracy ↗

focus on [illegible]

recall pert. exp.:

$\Sigma(z,\alpha_s) = \delta(1-z) + \frac{dw_1}{dz}(z) + \dots$

ε unphysical cutoff on gluon energy

$\frac{dw_1(z)}{dz} = 2a\left(\frac{1}{1-z}\ln\frac{1}{1-z}\right)_+ =$

$= 2a \lim_{\varepsilon\to 0}\left[\frac{\theta(1-z-\varepsilon)}{1-z}\ln\frac{1}{1-z} - \delta(1-z-\varepsilon)\frac{1}{2}\ln^2\varepsilon\right]$

$$\Delta_N(\alpha_s) = \exp\left\{-a\ln^2 N + \dots\right\}$$

? ↓ x-space

$$\Delta(z,\alpha_s) = \lim_{\varepsilon\to 0} -\frac{d}{dz}\left\{\theta(1-z-\varepsilon)\, e^{-a\ln^2(1-z)+\dots}\right\}$$

↓ in agreement with perturbative exp.

but if $a<0$ (as happen typically in hadron collisions)

$e^{-a\ln^2(1-z)} = e^{+|a|\ln^2(1-z)}$: divergent faster than any power of $1-z$ for $z\to 1$ ⇒ not integrable at $z=1$:

the unphysical cutoff ε cannot any longer be removed without producing singularities

tracing back the origin of the divergence :

- x-space exponentiation sistematically neglects subleading terms related to momentum conservation

  ➡ violating infinitely many times energy conservation one can build up a (kinematical) divergence

$$\int_0^1 dz\, e^{-a\ln^2(1-z)} = \sum_{n=0}^{\infty}\frac{(-a)^n}{n!}\int_0^1 dz\,\ln^{2n}(1-z) = \sum_{n=0}^{\infty}\frac{(-a)^n}{n!}(2n)! \xrightarrow[n\gg 1]{} (-4a)^n\, n!$$

neglected subleading logs have similar divergences and the total is finite ← if $a<0$: same-sign factorial divergence

- performing cut-off regularization of divergent x-space formula → introduce power-like effects

$\left(\frac{\Lambda}{Q}\right)^p$ ← process dependent and typically < 1

$p \sim \frac{\pi\beta_0}{2C_F} = \begin{cases} 0.36 & q\bar{q}\ (\overline{\text{MS}}) \\ 0.16 & gg\ (\overline{\text{MS}}) \end{cases}$

thus dominant w.r.t. true non-perturbative effects $\left(\frac{\Lambda}{Q}\right)^p$, $p \geq 1$ (2 DY)

- can systematically be implemented for e^+e^- event shapes as long as $\alpha_s \ln \frac{1}{1-x} \lesssim 1$ → extend reliability of perturbative calculations in the kinematic region $\alpha_s \ln^2(1-x) > 1$

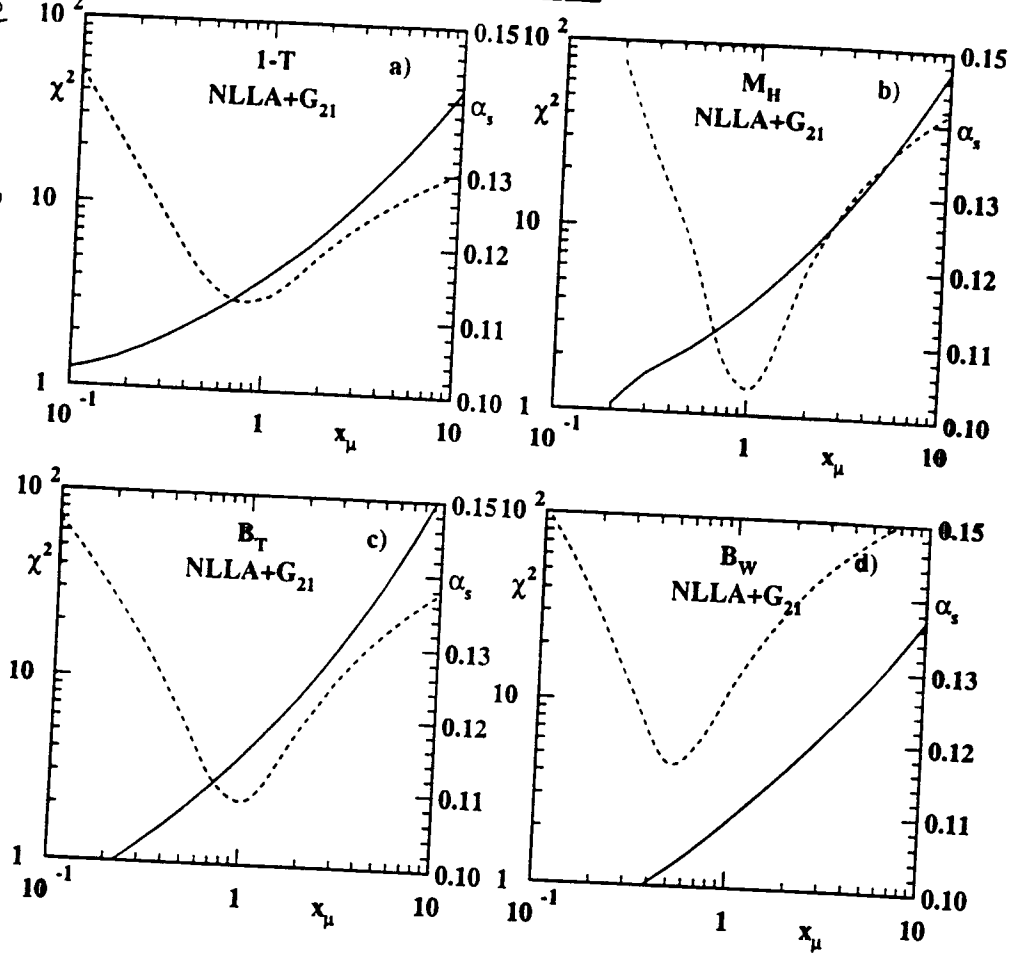
Turnock-Trentadue-Webber + S.C.

- in general, for hadronic collisions, one <u>cannot</u> define a logarithmic hierarcy directly in x-space
  → need resummation procedure in N-space

Mangano-Nason-Trentadue + S.C.

renormalization-scale dependence in fits of e^+e^- event shapes

NLL resummed



$x_\mu = \mu/Q$

Figure 3: Dependence of $\alpha_s(M_{Z^0})$ (solid curves) and χ^2/d.o.f. (dashed curves) on the renormalisation scale parameter x_μ for NLLA+G_{21} fits.

LO

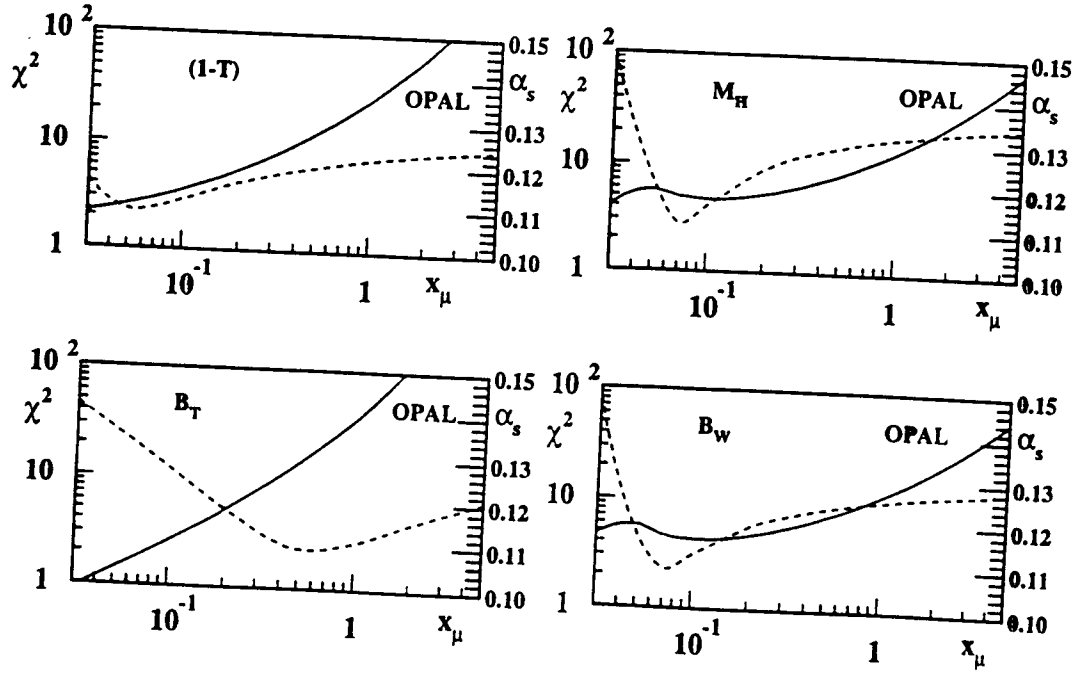


Figure 5: Dependence of $\alpha_s(M_{Z^0})$ (solid curves) and χ^2/d.o.f. (dashed curves) on x_μ for $\mathcal{O}(\alpha_s^2)$ fits to the OPAL data.

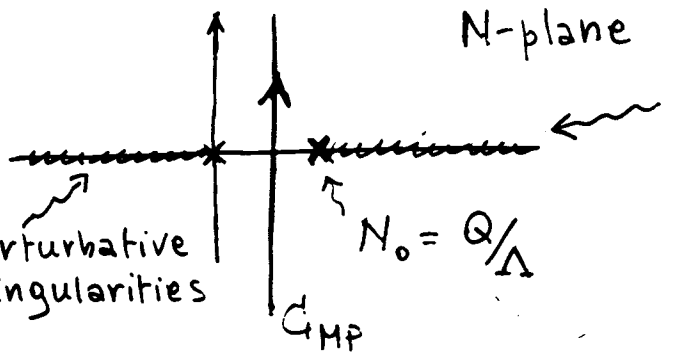
our method : "minimal" prescription

Catani-Mangano-Nason-Trentadue+S.C.

- systematically perform resummation in N-space

$$\ln \Delta_N = \underbrace{\ln N \, f_1(\alpha_s \ln N)}_{LL} + \underbrace{f_2(\alpha_s \ln N)}_{NLL} + \dots$$

$f_i(\alpha_s \ln N)$: well-defined analytic functions of N in the complex plane



- perform (numerical) inversion to x-space using the contour C_{MP} (Landau singularity is left on the right)

$$\Sigma(\tau, Q^2) = \int_{C_{MP}} \frac{dN}{2\pi i} \tau^{-N} F_N^2(Q^2) \Delta_N(Q^2)$$

Main features :

- correct prescription to any fixed-order in perturbation theory
- after resummation, it is equivalent to introduce non-perturbative effects of the type

$$e^{-\text{const.} \frac{Q}{\Lambda}(1-\tau)}$$

for fixed $1-\tau$, suppression is stronger than any power of $1/Q$

"minimal" : affect perturbative properties as least as possible (no cut-off, no factorially growing terms, no power corrections $\to$ in particular, no unjustified $(1/Q)^p$ $p<1$)

➡ interpolate consistently small-x and large-x regions taking into account soft-gluon resummation

eventually, for $x \to 1$ physically (theoretically) motivated treatment of non-perturbative effects should be introduced

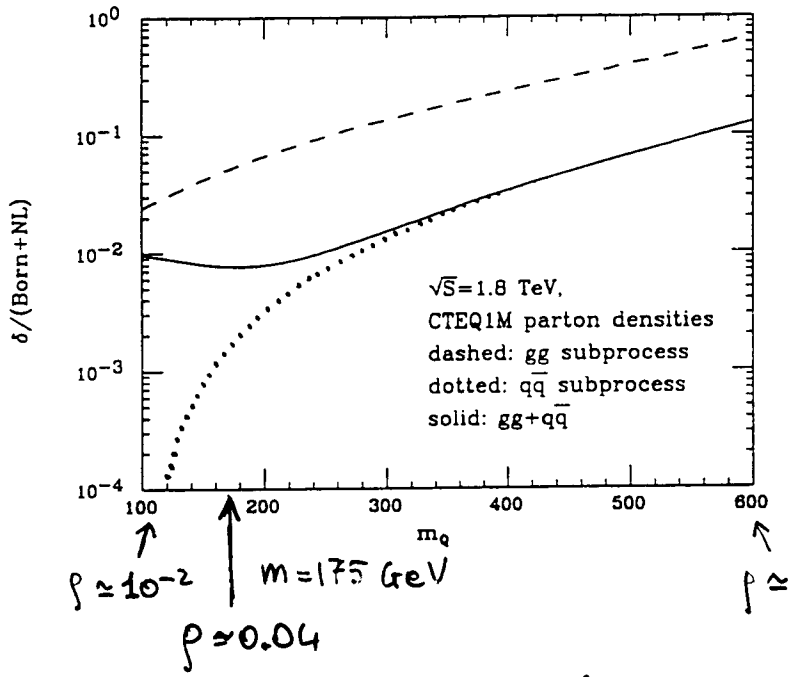
compare : standard NLO calculation

σ_{NLO} = Born + NLO
$O(\alpha_s^2)$ $O(\alpha_s^3)$

vs.

resummation effects beyond NLO
(only leading-log resummation included)

$$\delta = \sigma^{(res)} - \sigma^{(res)}\Big|_{\alpha_s^3}$$



top quark production in $p\bar{p}$ collisions

$$\rho = \frac{4m_t^2}{S}$$

- resummation effect ~ 1% for top production at Tevatron
(Berger-Contopanagos find ~ 10%, similar to Laenen-Smith-van Neerven)

- we conclude :
top quark production at Tevatron reliably estimated by NLO QCD calculation (Nason-Dawson-R.K. Ellis / Beenakker-Kuijf-van Neerven-Smith)

- our updated calculation :

$$\sigma_{t\bar{t}}(1.8\ \text{TeV}) = e^{\frac{175-m_t}{31.8}} \cdot 4.75\ ^{+0.63}_{-0.68}\ \text{pb}$$

$$\sigma_{t\bar{t}}(1.8\ \text{TeV}) = 6.5 \pm ^{+0.90}_{-0.94}\ \text{pb}$$

$(m_t = 175\ \text{GeV})$

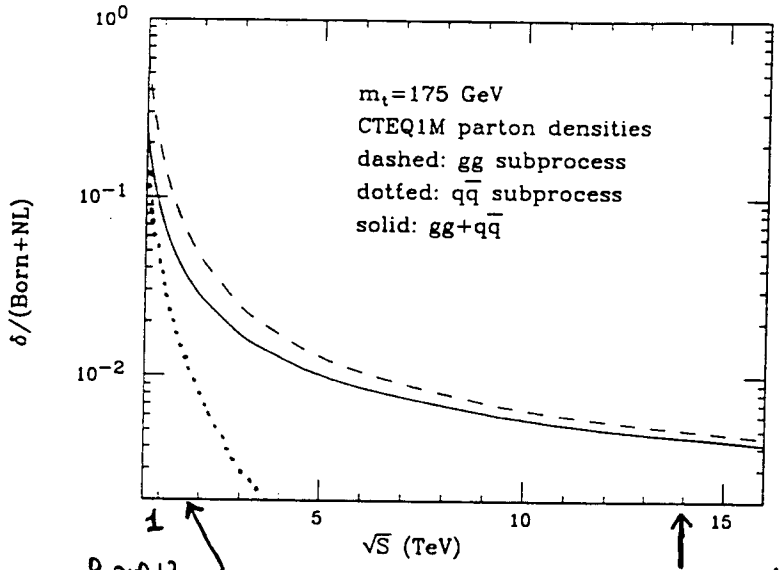
updated parton densities (CTEQ2, MRSA', MRS with variable $\alpha_s = 0.105 \div 0.125$)

scale variation

$$\frac{m_t}{2} < \mu = \mu_R = \mu_F < 2m_t$$

α_s variation gives dominant theoretical uncertainty (typically a factor of 2 larger than scale variation)

Top quark production in pp collisions



$\rho \simeq 0.12$ 1.8 TeV: effect larger than in $p\bar{p}$ because gluon channel dominates

LHC (14 TeV) $\rho \simeq 6 \cdot 10^{-4}$

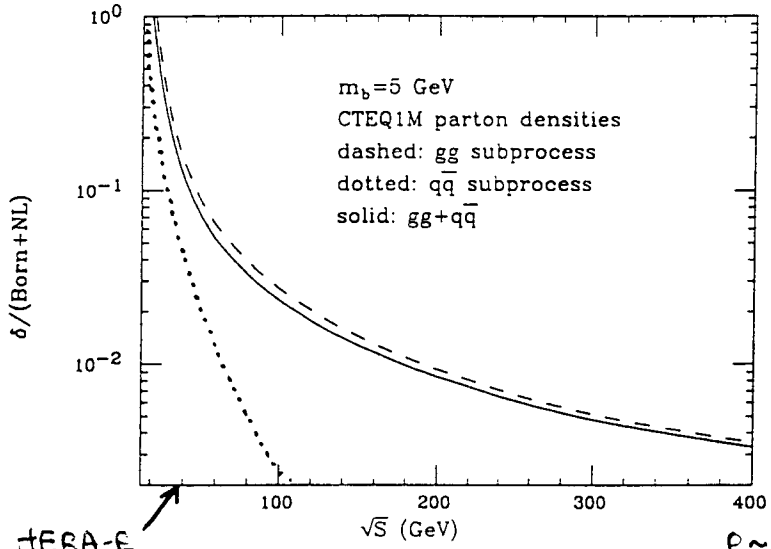
soft-gluon resummation effects are negligible at LHC (small-x effects have to be investigated)

$$\sigma_{t\bar{t}}(14\,\text{TeV}) = 768\ ^{+232}_{-126}\ \text{pb}$$

$m_t = 175$ GeV

(Berger-Contopanagos $\sigma = 760$ pb : fully consistent)

b production in pp collisions



HERA-B
39.2 GeV $\rho \simeq 6 \cdot 10^{-2}$

$\rho \simeq 6 \cdot 10^{-4}$

for fixed ρ, resummation effects are larger than for top production in $p\bar{p}$ collisions :

- gluon channel dominates
- α_s is larger

compensated by larger uncertainty

12% effect at HERA-B

but

theoretical uncertainty in NLO calculation is much larger

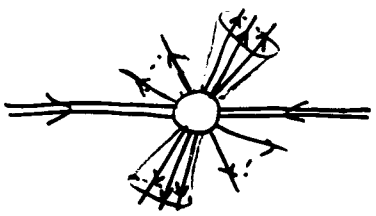
MRSA' $m_b = 4.75$ GeV

$\sigma_{b\bar{b}}(39.2\,\text{GeV}) = 10.5\ ^{+8.2}_{-4.7}$ nb

(Kidonakis-Smith : resummation effects increase cross section by a factor of 2)

- charm production in pp collisions :
  we find 100% effect of soft-gluon at $\rho \sim 0.3$
  but usual NLO calculation has <u>very large</u> theoretical uncertainty

Dijet cross section at Tevatron



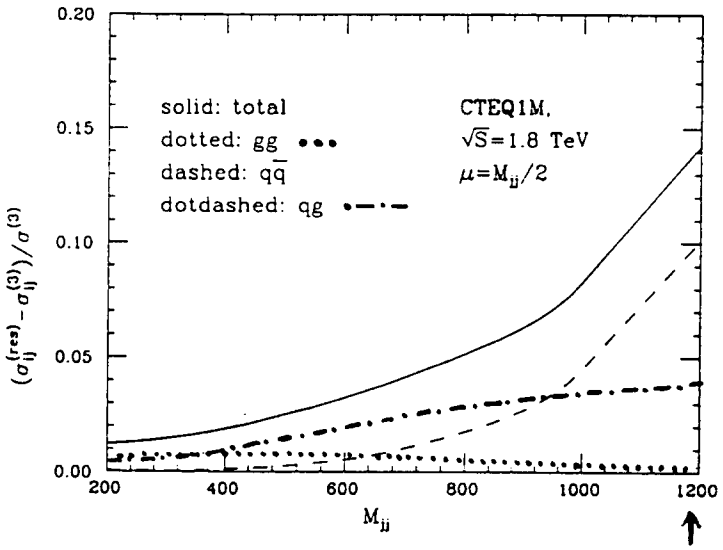
$\frac{d\sigma}{dz}$ dijet invariant mass distribution at large $z = \frac{M_{JJ}}{S}$

(not one-jet inclusive distribution at large E_T; however, excess of events with $M_{JJ} > 500$ GeV w.r.t. LO calculation reported by CDF Coll.)

soft-gluon resummation to leading-log accuracy

$\sigma^{(3)}$: Born + soft-gluon term to first order (included in NLO calculation)

$\sigma^{(res)} - \sigma^{(3)}$: soft-gluon resummation beyond first order



$z \simeq 0.3$

- for $z \simeq 0.3$ we find 10 ÷ 15 % effect
  (one-jet cross section at $E_T \simeq 400$ GeV $\rightarrow$ $x_T \simeq 0.3$)

  our calculation gives only order of magnitude:
  - not used full NLO QCD
  - only leading-log resummation
  - no detailed implementation of experimental cuts
  - for one-jet cross section additional soft-gluon effects in final state (jet broadening) have to be included

➡ further investigations warranted
(including consistently soft-gluon effects in Q^2-evolution of parton densities at large x)

Summary and outlook

- Sudakov resummation can be used to extend applicability of perturbative QCD to larger phase-space regions

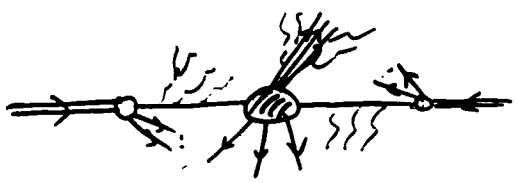
  e.g. e^+e^- event shapes
  hadronic collisions "close" to threshold
  ⋮
  Q_L-distributions

- Some issues still to be addressed

phenomenology → detailed and consistent study of extraction/Q^2-evolution of parton densities by using resummed anomalous dimensions
(soon ?)

theory/phenomenology → interplay of initial- and final-state effects in Sudakov resummation

e.g. relevant in high-E_T jet cross sections



Mangano-Nason-Treutadue + S.C.
(very much in progress)