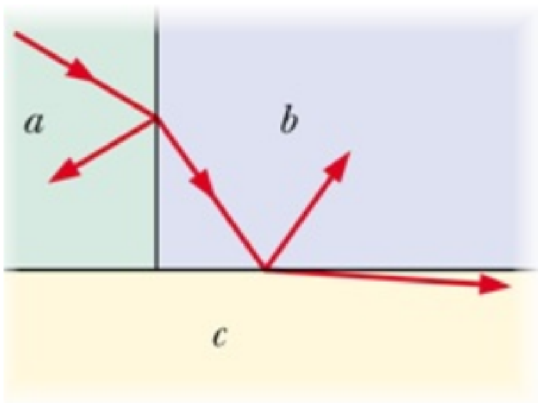
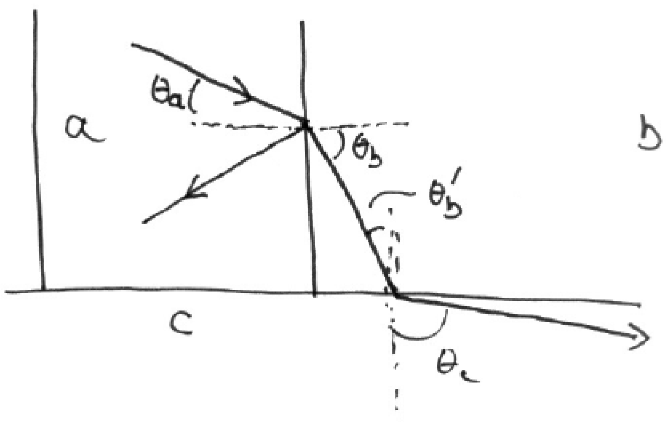
The figure shows rays of monochromatic light passing through three materials *a*, *b*, and *c*. Rank the materials according to index of refraction, greatest first.





From trigonometry, if $\theta_1 > \theta_2$ then $\sin\theta_1 > \sin\theta_2$ for $\theta_1 < \pi/2$ and $\theta_2 < \pi/2$. Since we only care about $\theta_i < \pi/2$, this holds for all angles we will deal with.

Observe: $\theta_a < \theta_b$, so $\sin\theta_a < \sin\theta_b$. From Snell's Law:

$$n_a \sin\theta_a = n_b \sin\theta_b$$

$$\underbrace{\frac{\sin\theta_a}{\sin\theta_b}}_{<1} = \underbrace{\frac{n_b}{n_a}}_{<1}$$

both sides must be <1, so $\boxed{n_b < n_a}$

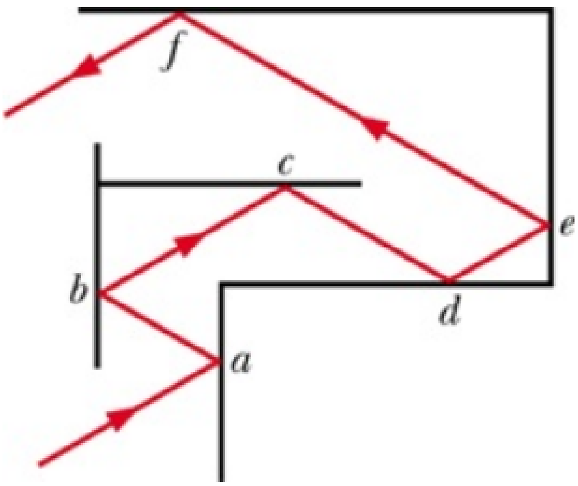
Observe: $\theta_b' < \theta_c$

$$n_b \sin\theta_b' = n_c \sin\theta_c \rightarrow \underbrace{\frac{\sin\theta_b'}{\sin\theta_c}}_{<1} = \underbrace{\frac{n_c}{n_b}}_{<1}$$

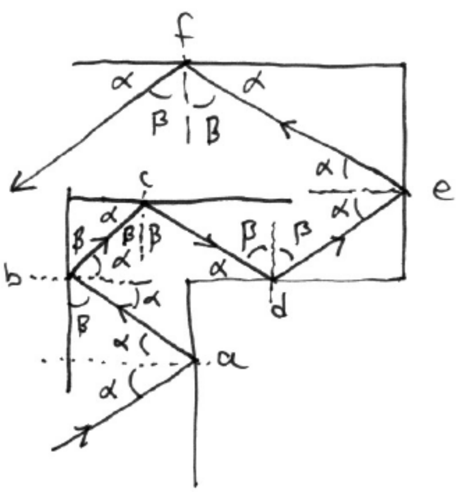
$\boxed{n_c < n_b}$

so $n_c < n_b < n_a$

The figure shows the multiple reflections of a light ray along a glass corridor where the walls are either parallel or perpendicular to one another. If the angle of incidence at point *a* is 30°, what are the angles of reflection of the light ray at points *b*, *c*, *d*, *e*, and *f*?



Find angles of reflection at b, c, d, e, f.



At every reflection from a vertical surface, the angle of reflection is α; from a horizontal surface, it is β. $\alpha + \beta + \frac{\pi}{2} = \pi$, since we have a bunch of right-triangles

at point a, $\alpha = 30°$. So at:

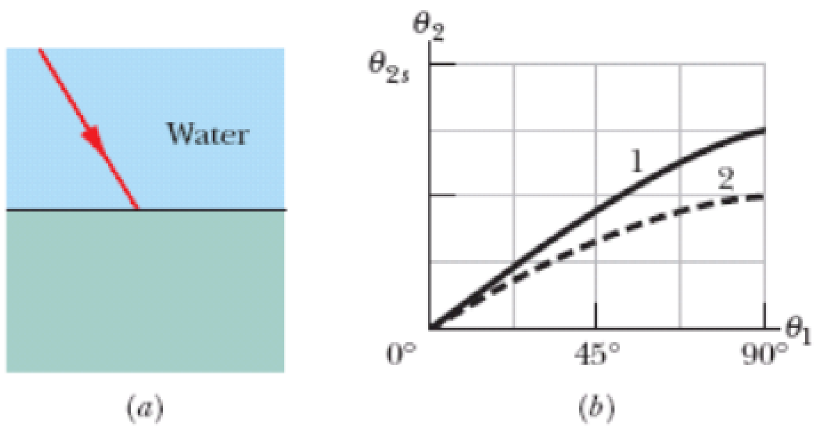
b, e: $\alpha = 30°$ = angle of reflection

at points c, d, f:

$$\beta = \pi - \frac{\pi}{2} - \alpha = \frac{\pi}{2} - \alpha = 60°$$

= angle of reflection.

In Figure (*a*), a light ray in water is incident at angle θ_1 on a boundary with an underlying material, into which some of the light refracts. There are two choices of underlying material. For each, the angle of refraction θ_2 versus the incident angle θ_1 is given in Figure (*b*). The vertical axis scale is set by $\theta_{2s} = 74°$. Without calculation, determine whether the index of refraction of **(a)** material 1 and **(b)** material 2 is greater or less than the index of water (n = 1.33). What is the index of refraction of **(c)** material 1 and **(d)** material 2?



Part a

We are to determine whether the index of refraction of material 1 is greater or less than that of material 1 without calculation. Let us consider total internal reflection. This occurs when light goes from a medium with HIGH index of refraction to one of LOW index of refraction. We see that the char sweeps all possible incidence light ray angles from 0 – 90 degrees. When light achieves the critical angle, we know that θ_2 will be 90-degrees. Yet, from the chart we see that light NEVER hits the critical angle... so there must not be one. We can conclude that material 1 has a higher index of refraction than water.

Part b

Same reasoning as part a.

Part c

To determine the exact index of refraction of material 1, we can use a particular point on the graph. For instance, we observer that when θ_1 = 90-degrees, $\theta_2 = 0.75 * \theta_{2s}$. Thus according to Snell's Law:

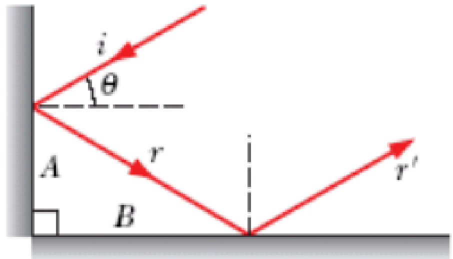
$$(1.33)\sin\theta_1 = n_2 \sin\theta_2 \rightarrow n_2 = \frac{1.33}{\sin\theta_2}$$

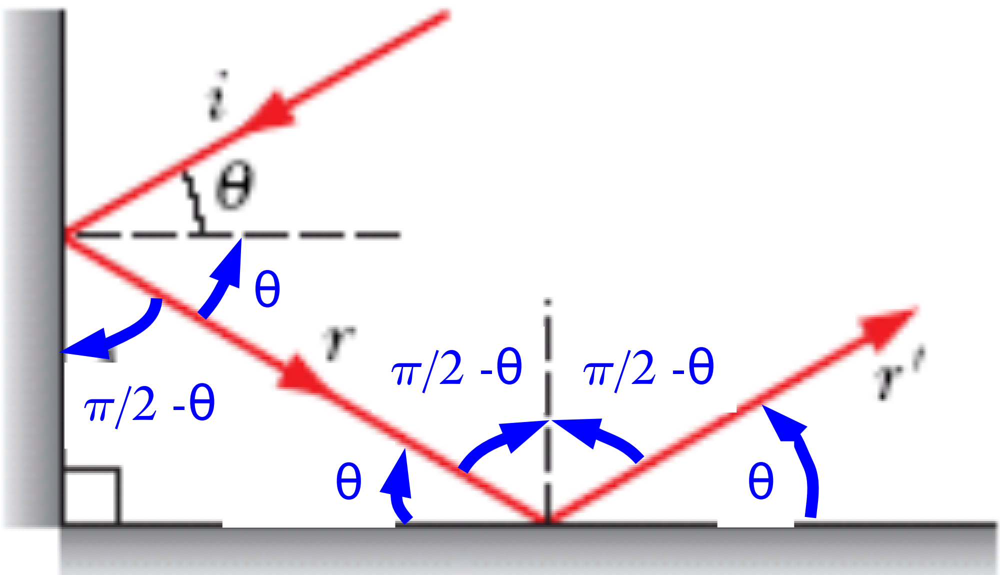
Plugging in numbers we find n_2 = 1.48. This confirms our reasoning in part a.

Part d

Similar to part c, we notice that when θ_1 = 90-degrees, $\theta_2 = 0.50 * \theta_{2s}$. Thus n_2 = 2.03.

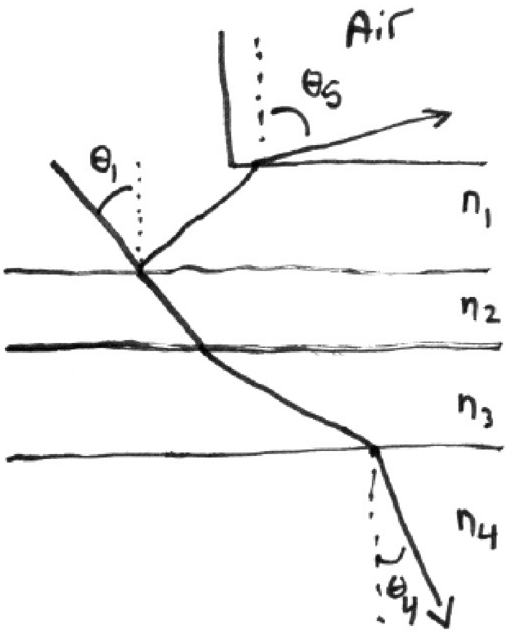
The figure shows light reflecting from two perpendicular reflecting surfaces *A* and *B*. Find the angle between the incoming ray *i* and the outgoing ray *r*′.





Apply the law of reflection at each reflection, correctly using the normal to each surface.

Following the geometry all the way through, we find that the angle of the outgoing ray, with respect to the horizontal, is the same as the angle of the incoming ray with respect to the horizontal . . . thus they are are a ZERO relative angle to one another.



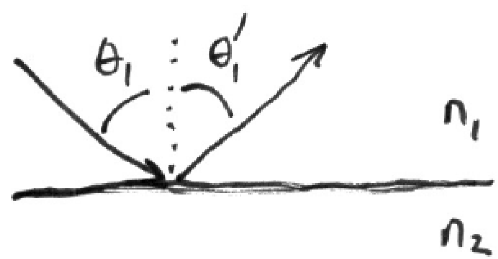
Find θ_4 and θ_5

→ given $\theta_1 = 42.0°$
$n_1 = 1.30$
$n_2 = 1.40$
$n_3 = 1.32$
$n_4 = 1.45$

at boundary between n_1, n_2 we have reflection and refraction.

→ reflection
– use Law of Reflection

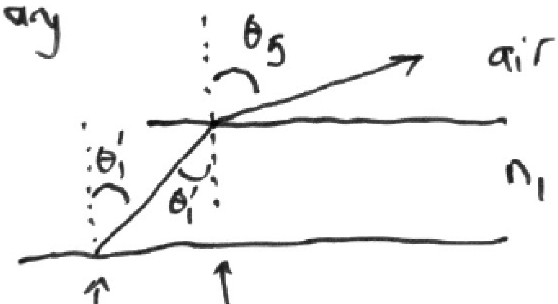
$$\theta_1 = \theta_1'$$



→ refraction at n_1, air boundary

$$n_1 \sin\theta_1' = n_{air} \sin\theta_5$$

↑ 1.000



normals are parallel, so interior angles are similar

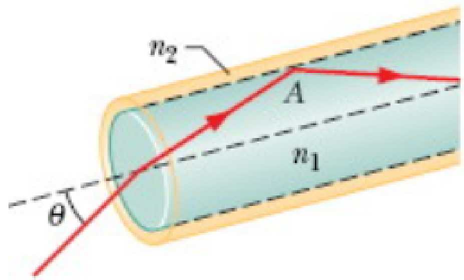
$$\sin\theta_5 = n_1 \sin\theta_1'$$

$$\theta_5 = \arcsin\left[n_1 \sin\theta_1'\right]$$

$$= \arcsin\left[0.0166\right]$$

$$= 0.953 \text{ rad} = \boxed{54.6°}$$

The figure depicts a simplistic optical fiber: a plastic core (n_1 = 1.56) is surrounded by a plastic sheath (n_2 = 1.50). A light ray is incident on one end of the fiber at angle θ. The rays is to undergo total internal reflection at point A, where it encounters the core-sheath boundary. (Thus there is no loss of light through that boundary.) What is the maximum value of θ that allows total internal reflection at A?



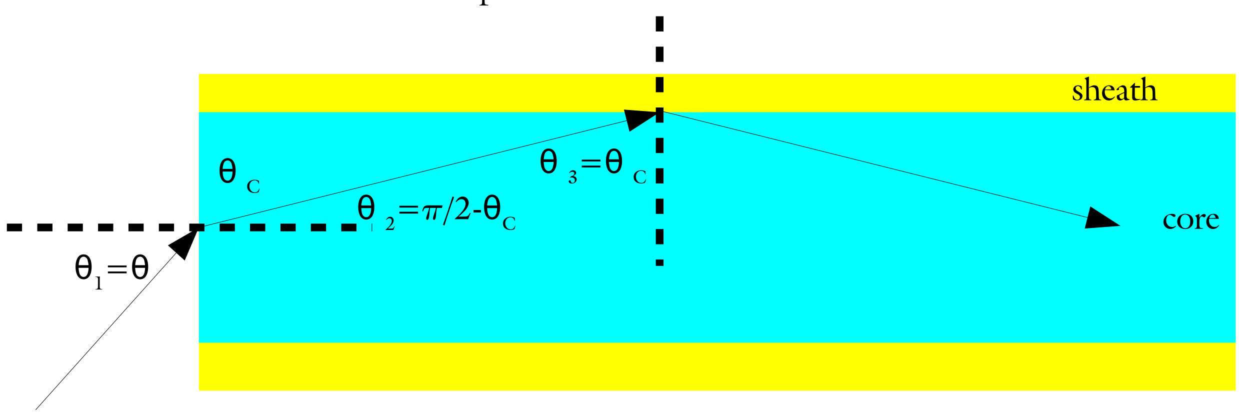
We have a two-part refraction problem. Light refracts FIRST when it enters the fiber and passes through the air-core boundary. The SECOND refraction occurs at the core-sheath boundary, and winds up being a total internal reflection. We want to know the maximum θ such that this total internal reflection will occur.

Let us first determine the critical angle for the core-sheath refraction. This is given in Snell's Law via:

$$\theta_C = \sin^{-1}\frac{n_2}{n_1} = 1.293\ \text{rad} = 75.06\ \text{deg}$$

So we now know the minimum angle, with respect to the normal to the core-sheath boundary, that light has to enter before total internal reflection occurs. Beyond that angle, reflection is 100%.

Consider the side-view of the above picture:



For light to enter the core-sheath boundary at the critical angle (or greater), it must have made an angle with respect to the normal of the air-core boundary of $\pi/2 - \theta_C$. If this is unclear, keep in mind that the normals to each of these surfaces are at different angles with respect to the horizontal, and you MUST use the angle between the NORMAL and the ray to do the calculations. Consider the geometry shown above.

We apply Snell's Law again at the air-core boundary to solve for the maximum θ that can lead to this total internal reflection.

$$\theta_3 > \theta_C = \sin^{-1}\frac{n_2}{n_1}$$

$$n_{\text{air}}\sin\theta = n_1\sin\theta_2 = n_1\sin(\pi/2 - \theta_3)$$

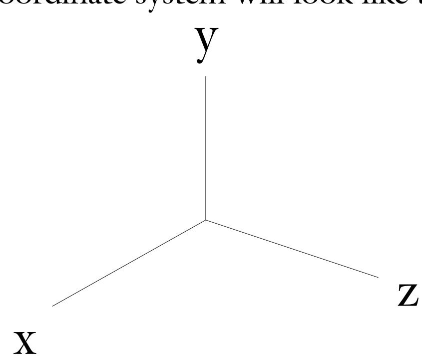
We need only rearrange, solve for θ, and employ the inequality. Keep in mind that as θ increases in value from 0 to 90-degrees, so does sinθ increase in value from 0 to 1. This is important when using the inequality.

$$\theta = \sin^{-1}\left(n_1\sin\left(\pi/2 - \sin^{-1}\left(\frac{n_2}{n_1}\right)\right)\right)$$

So the maximum angle of incident light needed to achieve the minimum angle for total internal reflection is θ = 0.443rad = 25.37 degrees. Beyond that, total internal reflection will not occur.

The figure shows a coordinate system in front of a flat mirror, with the x axis perpendicular to the mirror. Draw the image of the system in the mirror. **(a)** Which axis is reversed by the reflection? **(b)** If you face a mirror, is your image inverted (top for bottom)? **(c)** Is it reversed left and right (as commonly believed)? **(d)** What then is reversed?

We know that plane mirrors make upright, virtual images out of their original objects and preserve the height of the images. Thus, the mirror coordinate system will look like this:



The vertical axis (y) is preserved. The z-axis is also preserved – it pointed to the right in the object, and it still points to the right in the image. However, the x-axis, pointing INTO the mirror surface, has an image that points OUT OF the mirror's surface. This one has been reversed.

Part (a)

The x-axis is the only one reversed by the mirror.

Part (b)

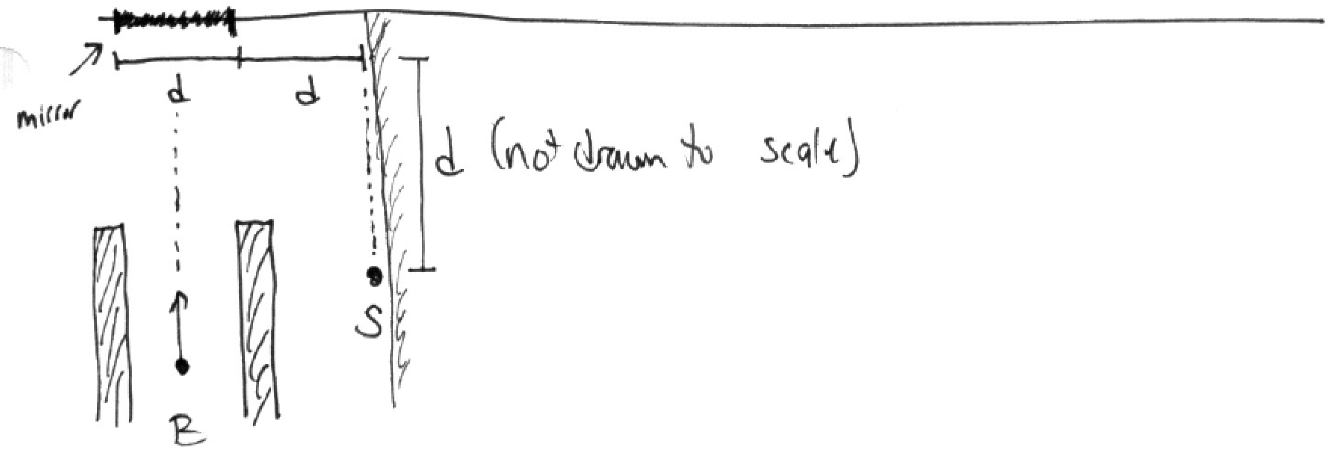
If you face a mirror, your image is PRESERVED top-to-bottom. It is NOT reversed vertically, as the y-axis is not reversed above.

Part (c)

In fact, you are not reversed left-to-right either. This is a common myth and misunderstanding of plane mirrors. When you point right, your image also points to the right in your coordinate system. So your image is not reversed left-to-right.

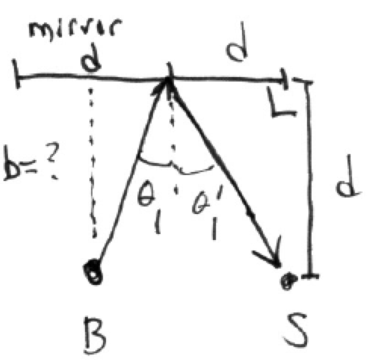
Part (d)

What is reversed is front-to-back. If you point straight at a mirror, your image points in the exact opposite direction, pointing behind you in your coordinate system.



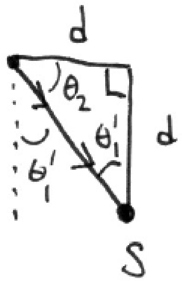
If burglar (B) sneaks toward mirror dead-center, how close to the mirror will the burglar be when the security guard (S) first spots her?

For S to spot B, the following must occur:



Light will make its greatest angle of reflection if it strikes the mirror's <u>edge</u> and bounces to the security guard – that's the earliest B can be <u>spotted</u>.

we have <u>two</u> <u>triangles</u>:



we can solve for θ_1', since

$$\tan\theta_1' = \frac{d}{d} = 1$$

$$\theta_1' = 45^\circ$$

Only upright-virtual (UV) or inverted-real (IR) images are ever possible for a <u>spherical</u> mirror.

I_1 = upright-real (UR)

I_2 = inverted-real (IR)

I_3 = upright-virtual (UV)

I_4 = inverted-virtual (IV)

which are not representing <u>possible</u> images?

I_1, I_4

↙ UR ↘ IV

↑ not allowed for <u>spherical</u> <u>mirrors</u> ↑

of the possible images, which would be due to a <u>concave</u> mirror?

possible: I_2, I_3

concave makes both UV and IR, depending on p and f relationship. So

I_2, I_3 both due to concave

which possible images could be due to convex mirror?

possible: I_2, I_3

convex only makes UV images, so **only I_3**

which possible images are virtual?

I_3 (UV)

which possible images involve negative magnification?

$m < 0$ means inverted

I_2 (IR)

Now we have either a diverging or converging lens.

I_1 : upright - virtual (UV)

I_2 : inverted - virtual (IV)

I_3 : upright - real (UR)

I_4 : inverted - real (IR)

diverging thin lenses only make UV images, while converging thin lenses can make UV or IR images, depending on the relationship of $\underline{f}$ and $\underline{p}$.

<u>which images represent possible images?</u>

I_1, I_4

<u>of the possible, which are due to converging lens?</u>

I_1, I_4

<u>of the possible, which are due to <u>diverging</u> lens?</u>

only UV, so I_1 only

<u>of the possible, which are virtual?</u>

I_1 only

<u>of the possible, which involve negative magnification?</u>

$I_4 \rightarrow \underline{IR}$, so $M<0$

A concave shaving mirror has a radius of curvature of +30.4 cm. It is positioned so that the (upright) image of a man's face is 1.85 times the size of the face. How far is the mirror from the face?

This is a problem about a CONCAVE shaving mirror (a spherical mirror with a positive focal length, f, and also a positive radius of curvature, R), and the image that forms due to that image. An object (a male face) is positioned so that the magnification factor is $|m|=1.85$. We are to determine the distance of the mirror from the face (the object distance).

We know that m = -i/p and |m| = h'/h. We don't know the image distance or the object distance. But, we can use the mirror equation to eliminate one of the unknowns in favor of the known radius of curvature and thus the known focal length. The focal length is determined by:

$$f=\frac{1}{2}R=15.2\,\mathrm{cm}$$

We can then relate the unknown image distance, i, to the focal length and the object distance we want to find:

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f}\rightarrow i=\frac{fp}{p-f}$$

We can then plug that into the magnification equation. We are told the image is upright, so m > 0. Thus:

$$m=\frac{-i}{p}=\frac{-f}{p-f}$$

$$m(p-f)=-f$$

$$p=f\frac{(m-1)}{m}=6.98\,\mathrm{cm}$$

Spherical mirrors. Object *O* stands on the central axis of a spherical mirror. For this situation object distance is p_s = +24 cm, the type of mirror is concave, and then the distance between the focal point and the mirror is 13 cm (without proper sign). Find **(a)** the radius of curvature *r* (including sign), **(b)** the image distance *i*, and **(c)** the lateral magnification *m*. Also, determine whether the image is **(d)** real or virtual, **(e)** inverted from object *O* or noninverted, and **(f)** on the *same* side of the mirror as *O* or on the *opposite* side.

This is a problem about a spherical mirror. We are given a variety of parameters and need to calculate things about the mirror and its images. We have the object distance, p = +24cm. The mirror is concave, so it must have a positive radius of curvature and a positive focal length. We are told the focal point is 13cm from the mirror, but not given explicitly the sign. We can figure that out, though. ;-)

Part a

The radius of curvature is related to focal length for a spherical mirror via f = (1/2)R. So, R = 2f. We know f, and we inferred the sign (f>0) from this being a concave spherical mirror. Thus: R = 2(13cm) = 26cm.

Part b

The image distance will be obtained from the mirror equation, 1/i + 1/p = 1/f. Thus i = (fp)/(p-f) = 28cm. Does this make sense? Well, it's a real image. Our object distance was 24cm, GREATER than the distance to the focal length. A concave mirror can make real images when the object is placed beyond the focal length, so this makes physical sense.

Part c

The lateral magnification, including is sign, is given by m = -i/p = -1.2. We get an inverted, enlarged image.

Part d

This is real image, since i > 0.

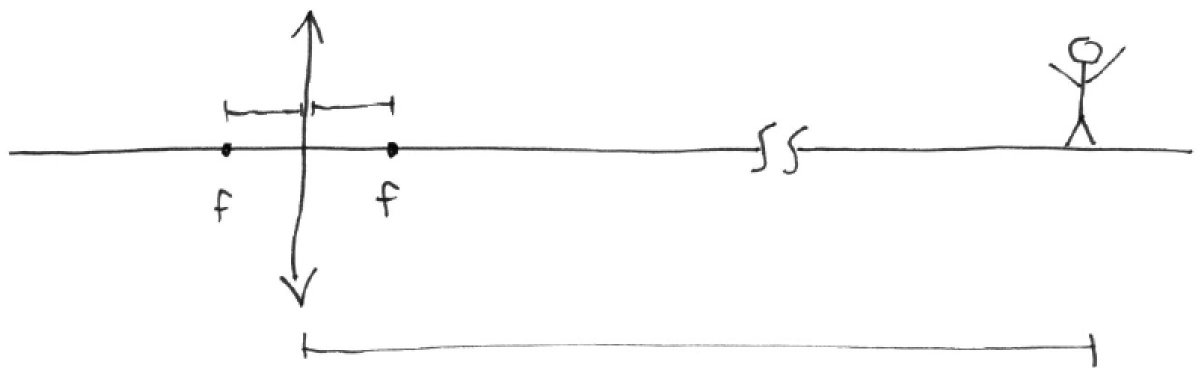
Part e

This is an inverted image, since m < 0.

Part f

This image forms on the SAME side of the mirror as the object, since it's real (i > 0).

movie camera has single lens, $f = 85mm$. Object is a person $35m = p = 35,000\ mm$ away. If $h = 180cm = 1800mm$, what is h' ?



→ use lens equation to get i.

$$\frac{1}{i} + \frac{1}{p} = \frac{1}{f} \rightarrow i = \frac{pf}{p-f} = \frac{(35,000mm)(85mm)}{(35,000mm - 85mm)}$$

$$= \frac{2,975 \times 10^6\ mm^2}{34,915\ mm}$$

$$= \boxed{85.2mm}$$

Makes sense ... $35,000 - 85 \approx 35,000$,

so $i = \frac{pf}{p-f}$ when $f << p$

$$\approx \frac{pf}{p} = f$$

now, $m = -\frac{i}{p}$ and $|m| = \frac{h'}{h}$, so $|m| = \left|\frac{i}{p}\right| = \frac{h'}{h}$ and thus

$$h' = h \cdot \left|\frac{i}{p}\right| = \boxed{4.4\ mm}$$ → good! has to project onto small CCD or film, so this makes sense!

You produce an image of the Sun on a screen using a thin lens whose focal length is 21.1 cm. What is the diameter of the image in millimeters? (Take the radius of the Sun to be 6.96×10^8 m and its distance to Earth to be 1.5×10^{11} m.)

This is a problem about a thin lens. We are given its focal length, f = 21.1cm. It's positive, so this is a CONVERGING thin lens. Real images will form on the opposite side of the lens from where the object is located, as is true for all lens systems. We are told we use it to produce an image of the Sun on a screen. The object distance is given, $p = 1.5 \times 10^{11}$m. We are also given the radius of the Sun.

We know that we can determine the image distance given the object distance and the focal length. Using the lens equation:

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f}\rightarrow i=\frac{fp}{p-f}=0.211\,\mathrm{m}=21.1\,\mathrm{cm}$$

Does this make sense? Well, the sun is very far away. By the time its rays reach us, those that strike the lens are almost exactly parallel to one another. We know that a lens will appear to focus parallel rays at the focal point... so it makes sense that when we plug everything into the lens equation, the sun's image forms at exactly the focal length.

Let's proceed. The original diameter of the sun was $D = 2R = 1.39 \times 10^9$m. We know that the magnification factor is given by $m = -i/p = -1.41 \times 10^{-12}$. We can apply this to the object diameter to determine the image diameter.

$D' = |m|D = 0.00196\text{m} = 2.0\text{mm}$.

Thin lenses. Object *O* stands on the central axis of a thin symmetric lens. For this situation, each problem in the table (below) gives object distance *p* (centimeters), the type of lens (C stands for converging and D for diverging), and then the distance (centimeters, without proper sign) between a focal point and the lens. Find **(a)** the image distance *i* and **(b)** the lateral magnification *m* of the object, including signs. Also, determine whether the image is **(c)** real or virtual, **(d)** inverted from object *O* or noninverted, and **(e)** on the same side of the lens as object *O* or on the opposite side.

p	Lens	**(a)** *i*	**(b)** *m*	**(c)** R/V	**(d)** I/NI	**(e)** Side
+9.1	D, 16					

This is a problem about a thin, DIVERGING lens. We are given the object distance, p = +9.1cm, and that the focal length is -16cm (we are told DIVERGING, which always means a NEGATIVE focal length). We are to compute a number of other lens and image properties.

Part a

The image distance is merely given by the lens equation:

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f}\rightarrow i=\frac{fp}{p-f}=-5.8\,\text{cm}$$

Part b

Magnification is given by m = -i/p = 0.64.

Part c

This is a virtual image, since i < 0. Diverging thin lenses ALWAYS yield a virtual image, and only a virtual image.

Part d

This image is upright, since m > 0.

Part e

This image forms on the SAME side of the lens as the object. That is the definition of virtual for a thin lens, when i < 0.

A concave mirror has a radius of curvature of 24.0 cm. How far is an object from the mirror if the image formed is **(a)** virtual and 2 times the size of the object, **(b)** real and 2 times the size of the object, and **(c)** real and 1/2 times the size of the object?

We are given a concave mirror and its radius of curvature. Since it's concave, we expect R > 0 and indeed R = 24.0cm. We are to compute various bits of information given some other knowns.

Part a

We are to find p when the image formed is virtual (i < 0) and |m| = 2. We know that m = -i/p. We also know, because it is given, that i < 0. That means that -i > 0, and from this we can determine that m > 0 (since p is always positive). Thus we know that: p = |i/m|.

However, we do not know i. We can use the magnification equation to solve for it:

$$m=-i/p \rightarrow i=-mp=-2p$$

Thus:

$$\frac{1}{i}+\frac{1}{p}=\frac{1}{-2p}+\frac{1}{p}=\frac{1}{2p}=\frac{1}{f} \rightarrow p=(1/2)f=6.0\,\text{cm}$$

(where I utilized that f = (1/2)R, and we were given R.

Part b

If the image is real and again |m| = 2.0, then what? Well, now we know that i = -|i| < 0, since virtual images mean negative image distances. We can again solve for i in terms of m and p:

$$m=-i/p=|i|/p \rightarrow i=|mp|=2p$$

$$\frac{1}{i}+\frac{1}{p}=\frac{1}{2p}+\frac{1}{p}=\frac{3}{2p}=\frac{1}{f} \rightarrow p=(3/2)f=18\,\text{cm}$$

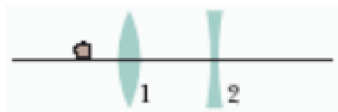
Part c

Repeating this exercise again, we use that i = |-mp| = (1/2)p. Then:

$$\frac{1}{i}+\frac{1}{p}=\frac{2}{p}+\frac{1}{p}=\frac{3}{p}=\frac{1}{f} \rightarrow p=3f=36\,\text{cm}$$

In the figure, a sand grain is 3.4 cm from thin lens 1, on the central axis through the two symmetric lenses. The distance between focal point and lens is 4.9 cm for both lenses; the lenses are separated by 9.8 cm.
(a) What is the distance between lens 2 and the image it produces of the sand grain?

Is that image **(b)** to the left or right of lens 2, **(c)** real or virtual, and **(d)** inverted relative to the sand grain or not inverted?



This is a problem about an object forming one image as its light passes through one lens, and then the first image becoming the object for a SECOND lens, which forms a SECOND image. We then wasn to find the distance between lens 2 and the image it produces of the sand grain. We are given the object distance from lens 1, p1 = 3.4cm, and the focal length of each – the first lens is converging, so f1 = 4.9cm, and the second is diverging, so f = - 4.9cm. The distance between lenses, D, is given, too.

Let's find the image distance for the first thin lens: $1/i + 1/p = 1/f \rightarrow i = (fp)/(p-f) = -11.07$cm.

This tells us that the image distance for lens 1 is 11.1cm TO THE LEFT of the lens, since it's a virtual image and it forms on the same side of the first lens as the object.

To then track the light from this image through lens 2, we need to find the distance from this image to the second lens. This is given by:

$i2 = |i1| + D = 11.1\text{cm} + 9.8\text{cm} = 20.9\text{cm}$

Now we can use the lens equation for lens 2:

$1/i2 + 1/p2 = 1/f2 \rightarrow i2 = (p2 * f2)/(p2 - f2) = -3.96\text{cm}$

This is then the distance between the second lens and the image. The image is to the left of lens 2, it's virtual (because it forms on the same side as the object – the image from lens 1).

What about the magnification? The total magnification is given by $m = m1 * m2 = (-i1/p1)(-i2/p2) > 0$

So it's upright.