Mechanics Laboratory 1105, Spring 2025

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`https://www.physics.smu.edu/tneumann/110X_Spring2025/`

Lab 5 — Simple harmonic motion Max. points: 52

Your preparation: Work through before coming to the lab

- Your need to thoroughly prepare for the lab by reading and understanding the measurement and analysis procedures on the worksheet! Photos of the equipment and further introductory material will be made available on `https://www.physics.smu.edu/tneumann/110X_Spring2025/schedule-mechanics/`.
- Collect all your questions and ask your instructor at the beginning of the lab.
- Work through chapter 15 "Oscillations" of Halliday, Resnick, and Walker [1] as required by . Focus especially on these topics: Understand the definition of Simple harmonic motion (SHM), restoring force, and equilibrium position. Understand Hooke's Law and how it leads to SHM in a spring-mass system. Study the displacement function $x(t) = x_m \cos(\omega t + \phi)$ and understand amplitude, angular frequency, phase constant, period and frequency. Understand how velocity and acceleration are related to displacement in SHM and their phase relationships. Briefly read about the concept of energy conservation in SHM.

Pre-lab: Upload to Canvas before coming to the lab

A reminder: Upload your answers as a text document (exported as PDF) to Canvas before the lab begins (Canvas uploads are no longer possible 30 minutes before the lab starts!).

Pre-lab 1 7 points

- (1 point) Define simple harmonic motion (SHM) in your own words. What are the key characteristics of SHM?
- (1 point) For a mass-spring system undergoing SHM, what is the restoring force? Write down the formula for the restoring force and explain each term.
- (1 point) Sketch a graph of displacement vs. time for an object in SHM. Label the amplitude and period on your sketch.
- (1 point) How are the velocity and acceleration of an object in SHM related to its displacement? Describe their phase relationships.
- (2 points) The period of oscillation for a mass-spring system is given by $T = 2\pi\sqrt{\frac{m}{k}}$. Explain what each symbol represents and how the period is related to mass m and spring constant k.

- (1 point) What is damping in the context of oscillations? How does damping affect the amplitude of oscillations over time?

Lab measurements and report: submission by end of class

A reminder: All measurements must be fully documented . The final report must be uploaded to Canvas *by the end of the class* exported as PDF with plots and tables from Excel embedded as images. Canvas will stop accepting uploads 10 minutes after the class ends. If you have not fully completed your report, you must upload the documents as far as you have completed them for grading.

In this lab, you will investigate simple harmonic motion using a Smart Cart, a vertical track, and a spring. You will perform two sets of measurements: first, to study basic SHM and determine the spring constant, and second, to investigate how varying mass affects the oscillations and to determine the spring constant using a different method.

Measurement 1 *Simple Harmonic Motion and spring constant determination* 6 points

In this first measurement, you will set up the apparatus and perform a base measurement of simple harmonic motion with the smart cart to determine the spring constant from a single oscillation period.

Equipment: Smart cart (with position, velocity, acceleration sensors), vertical track, spring (suitable for SHM with the cart), level, C-clamp, meter stick or ruler, capstone software, mass scale.

1. (2 points) **Initial vertical track and cart setup:**
   (a) Place the track in a near-vertical position, leaning it against the table in the lab. Ensure the angle from vertical is small, just enough to allow the cart to roll along the track.
   (b) Spring attachment: Hang the spring from the top of the vertical track.
   (c) Measure and record the mass of the cart m_{cart}.
   (d) Cart placement and wheel check: Carefully place the smart cart such that its wheels are positioned within the grooves of the track and attach it to the free end of the spring. Ensure the cart hangs vertically and moves smoothly in the grooves.
   (e) Confirm that the cart's wheels are properly positioned within the grooves of the track, allowing for smooth rolling motion.
   (f) Use a level to ensure that the *side* of the track, where the cart wheels roll, is aligned as vertically as possible.
   (g) Lower one track foot until it rests on the table level.
   (h) Use a C-clamp to securely fasten the track to the table, ensuring it remains stable.
   (i) Describe in your report how you set up the track and attached the spring and cart. Detail how you ensured the track was near-vertical and stable, how you aligned the rolling side vertically, and how you attached the spring and cart. Mention the tools used (level, C-clamp) and the steps you took.

2. **Software and sensor setup:** Power on the smart Ccart and connect to Capstone software. Select position, velocity, and ccceleration measurements at a data rate of 50 Hz or more.

3. **Resetting sensors at equilibrium:** Allow the cart to hang freely until it reaches equilibrium. Reset the sensors in Capstone with the cart at equilibrium.

4. **Measurement procedure: Initial displacement.** Gently pull the cart down by approximately 10 cm from its equilibrium position.

5. **Measurement procedure: Initiate sscillation and data recording.**

   (a) (2 points) **Smooth oscillation initiation and description:** Release the cart smoothly to start oscillations. Describe in your report how you attempted to release the cart smoothly.

   (b) (2 points) **Immediate data recording and duration:** Start data recording in Capstone immediately upon release and record data for at least 5-6 complete oscillations. Save the data run with a descriptive name (e.g., "SHM_BaseMass"). Note the run name in your report.

Measurement 2 *Investigating the effect of mass on oscillations* 10 points

In this measurement, you will investigate how changing the mass attached to the spring affects the period of oscillation, and use this to determine the spring constant in a second way.

Measurement procedure:

1. (2 points) **Add mass 1:** Obtain a known mass (e.g., $25\,\mathrm{g}$ or $50\,\mathrm{g}$). Record the value of the added mass m_1. Carefully attach this mass securely to the smart cart.

2. **Reset sensors at new equilibrium (mass 1):** Allow the cart with added mass m_1 to hang freely until it reaches the new equilibrium position. Reset the sensors in Capstone with the cart at this new equilibrium.

3. (3 points) **Data run with added mass 1:** Perform the SHM experiment again (pull down and release, record 5-6 oscillations) with the added mass m_1 (after resetting sensors). Save this data run as "MassRun_Mass1". Note the run name in your report.

4. (2 points) **Add mass 2:** Obtain a second known mass m_2 (you can use the same mass value as m_1 or a different value). Record the value of the *additional* mass m_2. Attach this second mass to the smart cart, in addition to the first mass m_1. The total added mass is now $m_1 + m_2$.

5. **Reset sensors at new equilibrium (mass 2):** Allow the cart with total added mass $m_1 + m_2$ to hang freely until it reaches the new equilibrium position. Reset the sensors in Capstone with the cart at this new equilibrium.

6. (3 points) **Data run with added mass 2:** Repeat the SHM experiment with both added masses (total added mass $m_1 + m_2$), after resetting sensors. Save this data run as "MassRun_Mass2". Note the run name in your report.

Analysis 1 *Analysis of simple harmonic motion data (base mass)* 15 points

In this analysis section, you will analyze the data from Measurement 1 to understand simple harmonic motion and determine the spring constant.

1. (2 points) **Position vs. time plot (base mass):** Create and include a position vs. time plot for "SHM_BaseMass" run, with labeled axes (Capstone, or Excel). Describe the shape (sinusoidal).
2. (4 points) **Period (T) and amplitude (A) (base mass):** Determine and report the average period T_{base} and amplitude A_{base} from the position vs. time plot for the base mass run, with units. Calculate the uncertainty ΔT, which is the standard deviation of the mean. Explain your methods.
3. (1 point) **Damping observation (base mass):** Describe the damping observed in the base mass oscillation.
4. (4 points) **Spring constant k from base mass period:** Calculate the spring constant k_{base} using the base mass m_{cart} and the period T_{base}. Calculate the uncertainty in Δk_{base}, based on the uncertainty in ΔT. Show your derivation and report k_{base} and its uncertainties with units.
5. (4 points) **Velocity and acceleration plots (base mass):** Create and include velocity vs. time and acceleration vs. time plots for "SHM_BaseMass", with labeled axes. Describe the shapes and phase relationships between position, velocity, and acceleration graphs. Explain consistency with SHM theory.

Analysis 2 *Analysis of simple harmonic motion with varying mass* 14 points

In this analysis section, you will analyze the data from Measurement 2 and determine the spring constant in a different way.

1. **Period determination for different masses:** (4 points)
   (a) For each data run ("MassRun_BaseMass", "MassRun_Mass1", "MassRun_Mass2"), create a position vs. time plot.
   (b) Determine the period of oscillation for each run by averaging over a few cycles. Record the periods T_{base}, T_{mass1}, T_{mass2} with units and uncertainties (standard deviation of the mean).
   (c) Calculate the total mass for each run: $m_{\text{total}} = m_{\text{cart}} + m_{\text{added}}$. For the base mass run, $m_{\text{added}} = 0$, so $m_{\text{total}} = m_{\text{cart}}$.
   (d) Create a table in your report with columns for run name, total mass, measured period (T) and period uncertainty. Include values for all your mass runs.
2. **Graph of period squared vs. total mass:** (4 points)
   (a) Calculate the square of the period T^2 for each run.
   (b) Create a scatter plot of Period Squared (T^2) (y-axis) versus Total Mass (m_{total}) (x-axis). Include this plot in your report with labeled axes and units.
3. **Spring constant from graph slope:** (4 points)

(a) According to theory, $T^2 = (\frac{4\pi^2}{k})m$. Therefore, the slope of your T^2 vs. m_{total} graph is approximately $\frac{4\pi^2}{k}$.

(b) Estimate the slope of your T^2 vs. m_{total} graph. You can do this by choosing two points on your plotted data that appear to represent a reasonable line and calculating the slope: Slope = $\frac{\Delta(T^2)}{\Delta(m_{\text{total}})}$.

(c) From the estimated slope, calculate the spring constant k_{massVar} using the relation $k_{\text{massVar}} = \frac{4\pi^2}{\text{Slope}}$. Report the value of k_{massVar} with its unit (N/m). Show your slope estimation and calculation in the report.

4. **Comparison of spring constants:** (2 points)

(a) Compare the spring constant k_{massVar} obtained from the varying mass experiment (Measurement 2) with the spring constant k_{base} calculated from the base mass measurement (Measurement 1).

(b) Do the values agree reasonably? Discuss possible reasons for any differences between the two values. Which method for determining the spring constant do you think is more reliable, and why?

Learning Outcomes

- Understand the principles of simple harmonic motion (SHM) in a spring-mass system.
- Experimentally observe and measure SHM using a smart cart and vertical track.
- Determine the period and amplitude of oscillations and analyze damping effects.
- Calculate the spring constant from measured oscillation periods.
- Interpret position, velocity, and acceleration graphs for SHM and understand their relationships.

References

[1] D. Halliday, R. Resnick, and J. Walker. *Fundamentals of Physics*. Fundamentals of Physics. John Wiley & Sons.