Mechanics Laboratory 1105, Spring 2025

Prof. Dr. T. Neumann, Prof. R. Guarino

https://www.physics.smu.edu/tneumann/110X_Spring2025/

Lab 10 — Measuring the gravity of earth Max. points: 52

Your preparation: Work through before coming to the lab

- Prepare for the lab by thoroughly reading and understanding the measurement and analysis procedures on this worksheet. Photos of the equipment and further introductory material will be made available on https://www.physics.smu.edu/tneumann/110X_Spring2025/schedule-mechanics/.
- Collect all your questions and ask your instructor at the beginning of the lab.
- Work through and review the following topics in Halliday, Resnick, and Walker [1]:
  - **Motion along a straight line:** Kinematic equations for constant acceleration, focusing specifically on free fall motion (where acceleration $a_y = -g$ or displacement $h = \frac{1}{2}gt^2$ for objects dropped from rest).
  - **Oscillations:** Simple Harmonic Motion (SHM), the physical principles of a simple pendulum, the formula for the period of a simple pendulum ($T = 2\pi\sqrt{L/g}$), and the importance of the small-angle approximation.

Pre-lab: Upload to Canvas before coming to the lab

A reminder: Upload your answers as a text document (exported as PDF) to Canvas before the lab begins (Canvas uploads are no longer possible 30 minutes before the lab starts!).

Pre-lab 1 — 8 points

1. (2 points) Starting from the standard kinematic equation for vertical motion ($y(t) = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$) and considering the setup for the g-ball experiment (where h is the drop height and t is the fall time from rest), derive the specific linear relationship between the plotted quantities (h on the y-axis and t^2 on the x-axis) as specified in 'Analysis of g-ball measurements', step 1(b). What physical quantity does the *slope* of this plot represent in terms of g?
2. (2 points) Starting from the formula for the period of a simple pendulum, $T = 2\pi\sqrt{L/g}$, show how you would rearrange it to obtain the linear relationship between the quantities plotted in 'Analysis of Pendulum Measurements', step 1(b) (T^2 on the y-axis vs. L on the x-axis). What expression, in terms of g, represents the *slope* of this line?
3. (1 point) In both the g-ball and Pendulum measurements, multiple time trials are taken. The procedure specifies calculating the *standard deviation of the mean* as the uncertainty in the

average time ($\Delta\bar{t}$). Why is this statistical measure used here to represent the uncertainty of the *average* time, rather than, for example, the standard deviation of the individual trials?

4. (2 points) The analysis for both experiments involves transforming the data (using t^2 or T^2) to create a linear plot and then finding the slope via linear regression. Briefly explain why this process of *linearization* (plotting transformed variables to get a straight line) is a particularly useful technique for determining the value of g from the experimental data, compared to calculating g individually from each data point and averaging.

5. (1 point) For the pendulum experiment, the procedure requires measuring the time for 10, 20, and 30 oscillations for each length. What is the primary advantage of measuring the time for a larger number of oscillations (e.g., 30) compared to a smaller number (e.g., 10 or even 1) when determining the period T?

Lab measurements and report: submission by end of class

A reminder: All measurements must be fully documented. The final report must be uploaded to Canvas *by the end of the class* exported as PDF with plots and tables from Excel embedded as images. Canvas will stop accepting uploads 10 minutes after the class ends. If you have not fully completed your report, you must upload the documents as far as you have completed them for grading.

Measurement 1 *g-Ball Measurements* 12 points

Equipment: g-ball (with embedded stopwatch), meter stick, soft landing pad, right-angle bracket, wooden mounting block, two 4-inch spring clamps, level, spreadsheet software.

1. (2 points) **Setup:**
   (a) Attach the wooden mounting block to the table. Attach the 2-meter stick to the mounting block using the two 4-inch spring clamps. Attach the right angle bracket at the desired height.
   (b) Use the level to ensure the 2-meter stick is vertical (plumb). Adjust the right-angle bracket as needed to achieve this.
   (c) Place the soft landing pad on the floor directly below the bottom of the 2-meter stick.
2. (10 points) **Height and Time Measurements:**
   (a) Using the meter stick, adjust the position of the g-ball so that the drop height (h) from the **bottom of the g-ball** to the **top of the landing pad** is at the first target height: $0.50\,\mathrm{m}$.
   (b) Estimate and record the uncertainty in your height measurement (Δh) for this height. Consider the precision of the meter stick and any difficulty in aligning the measurement.
   (c) Record this height and its uncertainty in your spreadsheet.
   (d) **Time Measurements for this Height:**
      (i) Set the g-ball to zero by pressing the ”push” switch.

(ii) Hold the g-ball at the measured height ($0.50\,\mathrm{m}$), ensuring the bottom of the ball is at the correct height.

(iii) Release the g-ball, allowing it to fall freely onto the landing pad.

(iv) Record the time (t) displayed on the g-ball's stopwatch in your spreadsheet.

(v) Repeat steps (ii)-(iv) at least *five* times, recording the time for each trial.

(vi) Calculate the *average* time of fall ($\bar{t}$) for this height from your five trials.

(vii) Calculate the *standard deviation of the mean* of the time measurements for this height. This will serve as your uncertainty in the time ($\Delta\bar{t}$).

(e) Repeat steps (a)-(d) for the remaining target heights: $0.75\,\mathrm{m}$, $1.00\,\mathrm{m}$, $1.25\,\mathrm{m}$, $1.50\,\mathrm{m}$, and $1.75\,\mathrm{m}$.

Analysis 1 *Analysis of g-ball measurements* 10 points

1. (4 points) **Data plotting and linear regression:**

   (a) In your spreadsheet, create a new column and calculate t^2 (time squared) for each value of the average time, $\bar{t}$.

   (b) Create a plot of h (on the y-axis) versus t^2 (on the x-axis).

   (c) Perform a linear regression (fit a straight line) to your data. Display the equation of the best-fit line and the R-squared value on the plot. Include a screenshot of this plot in your report.

2. (3 points) **Calculation of g:**

   (a) From the equation of the best-fit line on your h vs. t^2 plot, obtain the numerical value of the *slope*.

   (b) Using the equation you derived in the pre-lab, calculate the acceleration due to gravity (g) from the slope of your graph. Show your calculation.

   (c) Propagate the uncertainty in the slope to determine the uncertainty in your calculated value of g (Δg). Show your error propagation calculation.

3. (3 points) **Comparison with accepted value:**

   (a) Compare your calculated value of g (with its uncertainty) to the accepted value of $g = 9.81\,\mathrm{m\,s^{-2}}$.

   (b) Does your measured value agree with the accepted value within the limits of your experimental uncertainty? Explain your reasoning.

   (c) Discuss any potential sources of error that might have contributed to any discrepancy between your measured value and the accepted value.

Measurement 2 *Pendulum Measurements* 10 points

Equipment: Pendulum bracket, steel ball, string, meter stick, stopwatch, spreadsheet software.

1. (10 points) **Pendulum Length and Time Measurements (Iterative Process):**
   (a) Using the meter stick, adjust the length (L) of the pendulum by changing the length of the string. Measure from the pendulum bracket to the center of the steel ball. Set the length to the first target length: $0.50\,\mathrm{m}$.

   (b) Estimate and record the uncertainty in your length measurement (ΔL) for this length. Consider the precision of the meter stick and any difficulty in aligning the measurement.

   (c) Record this length and its uncertainty in your spreadsheet.

   (d) **Time Measurements for this Length:**
      (i) Displace the steel ball from its equilibrium (lowest) position by a small angle (no more than 15 degrees).

      (ii) Release the ball, allowing it to swing freely.

      (iii) Using the stopwatch, measure the time it takes for the pendulum to complete 5 full oscillations. Record this time (t_5) in your spreadsheet.

      (iv) Repeat steps (i)-(iii) at least *three* times, recording the time for each trial.

      (v) Using the stopwatch, measure the time it takes for the pendulum to complete 15 full oscillations. Record this time (t_{15}) in your spreadsheet.

      (vi) Repeat steps (i) and (v) at least *three* times, recording the time for each trial.

      (vii) Using the stopwatch, measure the time it takes for the pendulum to complete 30 full oscillations. Record this time (t_{30}) in your spreadsheet.

      (viii) Repeat steps (i) and (vii) at least *three* times, recording the time for each trial.

      (ix) Calculate the *average* time for 5, 15, and 30 oscillations ($\bar{t}_5$, $\bar{t}_{15}$, $\bar{t}_{30}$) for this length from your three trials.

      (x) Calculate the *standard deviation of the mean* of the time measurements for 5, 15, and 30 oscillations for this length. This will serve as your uncertainty in the time ($\Delta\bar{t}_5$, $\Delta\bar{t}_{15}$, $\Delta\bar{t}_{30}$).

      (xi) Calculate the average period for this length by dividing the average time by the number of oscillations.

   (e) Repeat steps (a)-(d) for the remaining target lengths: $1.00\,\mathrm{m}$, $1.50\,\mathrm{m}$, and $1.80\,\mathrm{m}$.

Analysis 2 *Analysis of Pendulum Measurements* 12 points

1. (6 points) **Data plotting and linear regression:**

(a) For each pendulum length, calculate the average period (T) using each of the three different oscillation counts (5, 15, and 30). You will have three average periods for each length: T_5, T_{15}, and T_{30}.

(b) For each pendulum length, calculate T_5^2, T_{15}^2, and T_{30}^2.

(c) Create three separate plots:

(i) Plot T_5^2 (on the y-axis) versus L (on the x-axis).

(ii) Plot T_{15}^2 (on the y-axis) versus L (on the x-axis).

(iii) Plot T_{30}^2 (on the y-axis) versus L (on the x-axis).

(d) Perform a linear regression (fit a straight line) to your data for each of the three plots. Display the equation of the best-fit line and the R-squared value on each plot. Include a screenshot of these plots in your report.

2. (3 points) **Calculation of g:**

(a) For each plot (T_5^2 vs. L, T_{15}^2 vs. L, and T_{30}^2 vs. L), obtain the numerical value of the *slope* from the equation of the best-fit line.

(b) Using the equation for the period of a simple pendulum, calculate the acceleration due to gravity (g) from the slope of each graph. You will have three values: g_5, g_{15}, and g_{30}. Show your calculation for each.

(c) Propagate the uncertainty in the slope to determine the uncertainty in each calculated value of g (Δg_5, Δg_{15}, and Δg_{30}). Show your error propagation calculation.

3. (3 points) **Comparison with accepted value:**

(a) Compare your calculated values of g_5, g_{15}, and g_{30} (with their uncertainties) to the accepted value of $g = 9.81\,\mathrm{m\,s^{-2}}$.

(b) Do your measured values agree with the accepted value within the limits of your experimental uncertainty? Explain your reasoning.

(c) Discuss any potential sources of error that might have contributed to any discrepancy between your measured values and the accepted value.

(d) Compare the three values of g, and discuss any differences.

References

[1] D. Halliday, R. Resnick, and J. Walker. *Fundamentals of Physics*. Fundamentals of Physics. John Wiley & Sons.