Mechanics Laboratory 1105, Spring 2025

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https://www.physics.smu.edu/tneumann/110X_Spring2025/

Lab 11 — Hooke's Law and Simple Harmonic Motion Max. points: 55

Your preparation: Work through before coming to the lab

- Prepare for the lab by thoroughly reading and understanding the measurement and analysis procedures on this worksheet. Photos of the equipment and further introductory material will be made available on https://www.physics.smu.edu/tneumann/110X_Spring2025/schedule-mechanics/.
- Collect all your questions and ask your instructor at the beginning of the lab.
- Work through and review the following topics in Halliday, Resnick, and Walker [1]:
  - **Hooke's Law:** Understand the relationship between the force exerted by a spring and its displacement from equilibrium ($F = -kx$). Note that F typically refers to the *restoring* force exerted *by* the spring, while in this lab we often consider the *external applied* force, $F_{\text{applied}} = +k\Delta x$.
  - **Elastic Potential Energy:** How energy is stored in a stretched or compressed spring.
  - **Simple Harmonic Motion (SHM):** Characteristics of SHM, such as amplitude, period, frequency, and angular frequency.
  - **Oscillations of a Mass-Spring System:** Derive the equations for the period and frequency of a mass-spring oscillator ($T = 2\pi\sqrt{m/k}$).
  - **Effect of Spring Mass:** Understand that a real spring has mass, which contributes to the inertia of the oscillating system. Introduce the concept of *effective mass* (m_{eff}) of the spring. For vertical oscillations, it's often approximated as $m_{\text{eff}} \approx \frac{1}{3}m_{\text{spring}}$. The total oscillating mass is then $m_{\text{total}} = m_{\text{total added}} + m_{\text{hanger}} + m_{\text{eff}}$. However, for the analysis method used here (plotting T^2 vs $m_{\text{total added}}$), the hanger mass becomes part of the intercept, so we often group it with the effective spring mass: $m_{\text{total}} = m_{\text{total added}} + (m_{\text{hanger}} + m_{\text{eff}})$.
  - **Energy Conservation in SHM:** How potential and kinetic energy are exchanged in a mass-spring system.

Pre-lab: Upload to Canvas before coming to the lab

A reminder: Upload your answers as a text document (exported as PDF) to Canvas before the lab begins (Canvas uploads are no longer possible 30 minutes before the lab starts!).

Pre-lab 1 6 points

1. (1 point) State Hooke's Law in your own words. Explain the physical meaning of the spring constant, k. What are its units?
2. (1 point) Derive the formula for the period (T) of an *ideal* (massless) mass-spring oscillator, starting from Newton's second law and Hooke's Law.
3. (1 point) What is simple harmonic motion (SHM)? Give two examples of systems that exhibit SHM.
4. (1 point) Explain how energy is conserved in an oscillating mass-spring system. Describe the exchange between kinetic and potential energy during one cycle of oscillation.
5. (1 point) Why does the mass of the spring affect the period of oscillation? Explain the concept of effective mass (m_{eff}) and state the commonly used approximation for a vertically oscillating spring. How does the formula for the period change when considering the spring's mass?
6. (1 point) In this experiment, you will determine the spring constant k using two methods: static extension and dynamic oscillation. Briefly outline how each method allows you to find k, considering the spring's mass in the dynamic case.

Lab measurements and report: submission by end of class

A reminder: All measurements must be fully documented. The final report must be uploaded to Canvas *by the end of the class* exported as PDF with plots and tables from Excel embedded as images. Canvas will stop accepting uploads 10 minutes after the class ends. If you have not fully completed your report, you must upload the documents as far as you have completed them for grading.

Measurement 1 *Static Extension Measurements* 11 points

Equipment: Spring, spring hanger, set of masses, PASCO wireless motion sensor, clamp stand, balance (for measuring spring and hanger mass), spreadsheet software, plumb bob.

1. (1 point) **Measure Spring and Hanger Mass:**
   (a) Measure the mass of the spring (m_{spring}) using the balance and record it with its uncertainty.
   (b) Measure the mass of the spring hanger (m_{hanger}) using the balance and record it with its uncertainty.
2. **Setup:**
   (a) Set up the clamp stand securely on the lab table. Place the PASCO wireless motion sensor on the table roughly where the spring will oscillate above it.
   (b) Use a plumb bob to find the correct attachment point on the clamp stand's horizontal rod. Hang the plumb bob from the rod and adjust its position along the rod until the bob hangs directly over the center of the motion sensor. Mark this position on the rod.

(c) Suspend the spring from the marked position on the rod. Attach the spring hanger to the bottom of the spring.

(d) Briefly hang the plumb bob from the bottom center of the *hanger* to confirm it is still centered over the motion sensor. Make minor adjustments to the sensor's position on the table if needed.

(e) Start the PASCO Capstone software and pair the motion sensor.

3. (10 points) **Static Extension Measurements:**

(a) Ensure the motion sensor is measuring distance correctly. You might zero it now or use the first position as a reference.

(b) With only the spring hanger attached to the spring (no *additional* mass), wait for the system to come to rest. Record this equilibrium position using the motion sensor. This position reading serves as the reference (x_0) for calculating extensions caused by *added* masses.

(c) Add $50\,\mathrm{g}$ to the spring hanger. Let the *total added mass* be $m_{\text{total added}} = 50\,\mathrm{g}$. Wait until the system is at rest.

(d) Record the new position (x) of the bottom of the spring hanger using the motion sensor.

(e) Calculate the extension of the spring ($\Delta x = x - x_0$) caused by the *total added* mass ($m_{\text{total added}}$).

(f) Add an additional $50\,\mathrm{g}$ to the hanger (so $m_{\text{total added}}$ is now $100\,\mathrm{g}$). Wait until the system is at rest.

(g) Repeat steps (d) to (f), increasing $m_{\text{total added}}$ each time by $50\,\mathrm{g}$, up to $400\,\mathrm{g}$ in total.

(h) Estimate and record the uncertainty of your total added mass measurements ($\Delta m_{\text{total added}}$).

(i) Estimate and record the uncertainty of your position measurements from the motion sensor (consider fluctuations or sensor precision).

(j) Create a table with columns for: Total Added Mass ($m_{\text{total added}}$), Added Weight ($F_{\text{added}} = m_{\text{total added}} g$), measured position ($x$), and elongation ($\Delta x = x - x_0$). Include uncertainties.

Analysis 1 *Analysis of Static Measurements* 8 points

Note: For the static analysis, we plot the added force versus the resulting extension relative to the equilibrium position with the hanger already attached. The initial stretch due to the spring's own weight and the hanger's weight establishes the initial equilibrium position (x_0), but does not affect the slope k determined from how the spring stretches further under added load.

1. (3 points) **Data plotting and linear regression:**

(a) Calculate the force exerted by each *total added* mass ($m_{\text{total added}}$) on the spring, using $F_{\text{added}} = m_{\text{total added}} g$. Calculate the uncertainty $\Delta F_{\text{added}} = (\Delta m_{\text{total added}}) g$.

(b) Create a plot of added force (F_{added}) (on the y-axis) versus spring extension (Δx) (on the x-axis). Include error bars for both F_{added} and Δx.

(c) Perform a linear regression (fit a straight line) to your data. Display the equation of the best-fit line (including slope and intercept with uncertainties) and the R-squared value on the plot. Include a screenshot of this plot in your report.

2. (3 points) **Calculation of k (Static Method):**

   (a) From the equation of the best-fit line on your F_{added} vs. Δx plot, identify the numerical value of the *slope* and its uncertainty.

   (b) According to Hooke's Law, the applied force is proportional to the extension it causes ($F_{\text{applied}} = k\Delta x$). Since you plotted the *added* force (F_{added}, due to the total added mass) against the resulting *additional* extension (Δx) relative to the hanger's equilibrium position, the slope of this graph directly represents the spring constant, k. Record this value as k_{static} with its uncertainty Δk_{static} (which is the uncertainty of the slope).

3. (2 points) **Discussion:**

   (a) Discuss whether your graph supports Hooke's law (i.e., is it linear? Does it pass close to the origin?). Comment on the R-squared value.

   (b) Discuss potential sources of systematic and random error in the static measurement (e.g., reading the sensor, masses not exact, spring not perfectly Hookean, alignment).

Measurement 2 *Dynamic Oscillation Measurements* 10 points

Equipment: Same as static measurements (Spring, hanger, masses, motion sensor, clamp stand, balance, spreadsheet software).

1. (10 points) **Oscillation Measurements:**

   (a) **Setup**: Use the same setup as in the static measurements (spring, hanger, sensor aligned). Ensure you have the measured mass of the spring (m_{spring}) and hanger (m_{hanger}) recorded.

   (b) Attach a mass $m_{\text{total added}} = 50\,\text{g}$ to the spring hanger.

   (c) Gently pull the mass down a small distance (e.g., $2\,\text{cm}$ to $3\,\text{cm}$) from its equilibrium position.

   (d) Release the mass smoothly to initiate vertical oscillations. Avoid any sideways motion or swinging.

   (e) Use the PASCO wireless motion sensor and the Capstone software to record the position of the mass as a function of time.

   (f) Record data for a sufficient duration to capture at least 10-15 complete oscillations, allowing for accurate period determination.

   (g) Repeat steps (b)-(f), increasing $m_{\text{total added}}$ in steps of $50\,\text{g}$ up to $250\,\text{g}$ ($m_{\text{total added}} = 50, 100, 150, 200, 250\,\text{g}$).

Analysis 2 *Analysis of Oscillation Measurements* 20 points

Note: For the dynamic analysis, the total mass involved in the oscillation is the sum of the total added mass, the hanger mass, and the effective mass of the spring ($m_{total} = m_{total\ added} + m_{hanger} + m_{eff}$), where $m_{eff} \approx \frac{1}{3}m_{spring}$. The analysis method relates T^2 to $m_{total\ added}$.

1. (5 points) **Period Determination:**
   (a) For each total added mass ($m_{\text{total added}}$), examine the plot of position versus time from the Capstone data.
   (b) From each position-time graph, accurately determine the period (T) of the oscillation. The most reliable method is often to measure the total time for a large number of full oscillations (e.g., $N = 10$) and calculate $T = \text{Total Time}/N$. Estimate the uncertainty in your period measurement (ΔT), considering factors like the precision of time measurement and identifying complete cycles.
   (c) Calculate the square of the period (T^2) and its uncertainty ($\Delta(T^2) \approx |2T\Delta T|$) for each total added mass.
   (d) Create or update a table to include $m_{\text{total added}}$, T, ΔT, T^2, and $\Delta(T^2)$.

2. (5 points) **Data Plotting and Linear Regression (T^2 vs $m_{\text{total added}}$):**
   (a) The theoretical relationship is $T = 2\pi\sqrt{\frac{m_{\text{total added}}+m_{\text{hanger}}+m_{\text{eff}}}{k}}$. Squaring this gives: $T^2 = \frac{4\pi^2}{k}(m_{\text{total added}} + m_{\text{hanger}} + m_{\text{eff}}) = \left(\frac{4\pi^2}{k}\right) m_{\text{total added}} + \left(\frac{4\pi^2(m_{\text{hanger}}+m_{\text{eff}})}{k}\right)$.
   (b) This equation is in the form $y = ax + c$, where $y = T^2$, $x = m_{\text{total added}}$, the slope $a = \frac{4\pi^2}{k}$, and the y-intercept $c = \frac{4\pi^2(m_{\text{hanger}}+m_{\text{eff}})}{k}$.
   (c) Create a plot of T^2 (y-axis) versus *total added* mass $m_{\text{total added}}$ (x-axis). Include error bars for T^2 (error bars for $m_{\text{total added}}$ might be negligible, but consider them).
   (d) Perform a linear regression (fit a straight line) to your T^2 vs $m_{\text{total added}}$ data. Display the equation of the best-fit line (including slope and intercept with their uncertainties) and the R-squared value on the plot. Include a screenshot of this plot in your report.

3. (3 points) **Calculation of k (Dynamic Method):**
   (a) From the *slope* (a) and its uncertainty (Δa) of your T^2 vs. $m_{\text{total added}}$ plot, determine the spring constant (k_{dynamic}). Use the relation: $a = \frac{4\pi^2}{k_{\text{dynamic}}}$, so $k_{\text{dynamic}} = \frac{4\pi^2}{a}$. Show your calculation.
   (b) Propagate the uncertainty in the slope (Δa) to determine the uncertainty in k_{dynamic} ($\Delta k_{\text{dynamic}}$). Use $\Delta k_{\text{dynamic}} \approx |k_{\text{dynamic}}\frac{\Delta a}{a}| = |\frac{4\pi^2}{a^2}\Delta a|$. Show your error propagation calculation.

4. (3 points) **Determination of Effective Spring Mass (m_{eff}):**
   (a) From the *y-intercept* (c) and its uncertainty (Δc) of your T^2 vs. $m_{\text{total added}}$ plot, and using your calculated k_{dynamic} and measured m_{hanger}, determine the effective mass (m_{eff}) of the

spring. Use the relation: $c = \frac{4\pi^2(m_{\text{hanger}}+m_{\text{eff}})}{k_{\text{dynamic}}}$. Rearrange to solve for m_{eff}: $m_{\text{eff}} = \frac{c \cdot k_{\text{dynamic}}}{4\pi^2} - m_{\text{hanger}}$. Show your calculation.

(b) Propagate the uncertainties in the intercept (Δc), k_{dynamic} ($\Delta k_{\text{dynamic}}$), and hanger mass ($\Delta m_{\text{hanger}}$) to determine the uncertainty in m_{eff} (Δm_{eff}). A simplified approach assuming dominant uncertainty from c and k might be sufficient, or use a more complete propagation: $\Delta m_{\text{eff}} \approx \sqrt{(\frac{k_{\text{dynamic}}}{4\pi^2}\Delta c)^2 + (\frac{c}{4\pi^2}\Delta k_{\text{dynamic}})^2 + (\Delta m_{\text{hanger}})^2}$. Show your error propagation calculation.

(c) Calculate the theoretical effective mass using $m_{\text{eff, theory}} = \frac{1}{3}m_{\text{spring}}$ (using your measured m_{spring}). Calculate its uncertainty $\Delta m_{\text{eff, theory}} = \frac{1}{3}\Delta m_{\text{spring}}$.

(d) Compare your experimentally determined effective mass ($m_{\text{eff}} \pm \Delta m_{\text{eff}}$) with the theoretical value ($m_{\text{eff, theory}} \pm \Delta m_{\text{eff, theory}}$). Calculate the percentage difference and discuss if they agree within uncertainty.

5. (4 points) **Comparison and Discussion:**

(a) Compare the value of $k_{\text{dynamic}} \pm \Delta k_{\text{dynamic}}$ obtained from the oscillation measurements to the value of $k_{\text{static}} \pm \Delta k_{\text{static}}$ obtained from the static measurements.

(b) Do the two values of k agree within the limits of your experimental uncertainty? Calculate the percentage difference and discuss significance.

(c) Discuss the agreement (or disagreement) between your experimentally determined m_{eff} and the theoretical value $\frac{1}{3}m_{\text{spring}}$. What could cause a discrepancy?

(d) Discuss potential sources of systematic and random error in the dynamic measurements (e.g., non-ideal SHM like swinging, air resistance, accuracy of period measurement, sensor limitations, uncertainty in spring/hanger mass).

(e) Briefly discuss any observed damping (decrease in amplitude) in the oscillations. How might significant damping affect your period measurements, if at all? (For small damping, the effect on the period is usually negligible compared to other uncertainties).

References

[1] D. Halliday, R. Resnick, and J. Walker. *Fundamentals of Physics*. Fundamentals of Physics. John Wiley & Sons.