Mechanics Laboratory 1105, Spring 2025

Prof. Dr. T. Neumann, Prof. R. Guarino

https://www.physics.smu.edu/tneumann/110X_Spring2025/

Make-up lab — Resistive Forces on an Incline Max. points: 49

Resistive forces are ubiquitous, opposing the motion or attempted motion of objects. When an object like a block slides, we distinguish between static friction (preventing motion) and kinetic friction (opposing motion). However, when an object like a cart with wheels rolls, the situation is different. Static friction still acts at the contact points of the wheels to prevent slipping and must be overcome to initiate rolling. Once the cart is rolling without slipping, the primary resistive force is *rolling resistance*. This force arises from factors like the deformation of the wheels and the track surface, and friction in the axles and bearings. It is generally much smaller than kinetic friction for a sliding object of similar mass and normal force, and it may depend on factors like the normal force pressing the cart onto the track.

Inclined planes provide a classic context for studying forces. When a cart rests on an incline, gravity pulls it down the slope, while the normal force acts perpendicularly. Rolling resistance acts parallel to the surface, opposing the direction of motion. By carefully balancing the forces acting on the cart using a pulley and hanging mass system, we can experimentally determine the magnitude of the rolling resistance force and investigate its potential dependence on the normal force.

In this experiment, you will investigate the resistive forces acting on a cart placed on an inclined dynamics track. A cord attached to the cart runs over a pulley, allowing a hanging mass to apply tension. By measuring the hanging mass required to maintain constant velocity both up and down the incline at various angles and for different cart masses, you will determine the magnitude of the rolling resistance force and explore its relationship with the normal force. This experiment provides a hands-on application of Newton's laws of motion and the concepts of force resolution and resistive forces in rolling motion.

Your preparation: Work through before coming to the lab

- Prepare for the lab by thoroughly reading this worksheet, focusing on the measurement and analysis procedures. Understand the concepts of static friction (as the force preventing initial motion), rolling resistance, forces on an inclined plane, normal force, and Newton's laws. Photos of the equipment and further introductory material will be available on the course website: https://www.physics.smu.edu/tneumann/110X_Spring2025/schedule-mech/
- Note any questions you have and ask your instructor at the beginning of the lab.
- Review the following topics in Halliday, Resnick, and Walker [1]:
  - **Forces and Newton's Laws:** Newton's First and Second Laws ($\Sigma\vec{F} = m\vec{a}$), concept of equilibrium ($\vec{a} = 0$).

- **Specific Forces:** Gravity ($W = mg$), Normal Force (N), Tension (T).
- **Friction:** Static friction (f_s prevents motion), Kinetic friction (f_k opposes sliding motion), Rolling Resistance (f_{roll} opposes rolling motion). Understand the direction of these forces and that f_{roll} might depend on N.
- **Inclined Planes:** Resolving the force of gravity into components parallel ($mg \sin\theta$) and perpendicular ($mg \cos\theta$) to the incline. Relationship between the perpendicular component and the normal force.
- **Free-Body Diagrams (FBDs):** Importance of drawing FBDs to identify all forces acting on an object.
- **Applying Newton's Laws with FBDs:** Setting up and solving equations for forces in perpendicular directions (usually parallel and perpendicular to the incline).

Pre-lab: Upload to Canvas before coming to the lab

A reminder: Upload your answers as a PDF to Canvas before the lab begins (uploads close 30 minutes prior).

Pre-lab 1 — 9 points

1. (1 point) Define static friction (as the force preventing initial motion) and rolling resistance (as the force opposing rolling motion). In which direction do these forces generally act relative to the motion or impending motion? (Note: We won't use the simple μN formulas for rolling resistance here).
2. (2 points) **Newton's Laws in Equilibrium:** Consider two books, Book B (mass m_B) resting on top of Book A (mass m_A). Book A rests on a horizontal table, which is supported by the Earth. Assume the table itself has negligible mass compared to the books.
   (a) Draw three separate Free-Body Diagrams (FBDs): one for Book B, one for Book A, and one for the Earth. On each FBD, draw and label all forces acting *on* that specific object due to its interactions with the other objects (A, B, Earth, or transmitted through the table). Use clear labels, e.g., W_B for the weight of B, N_{AB} for the normal force exerted *by A on B*, $F_{g,BE}$ for the gravitational force exerted *by B on Earth*, etc.
   (b) Identify all the action-reaction pairs according to Newton's Third Law among the forces you drew in part (a). Remember that action-reaction pairs act on *different* objects and are equal in magnitude and opposite in direction.
   (c) Determine the magnitude of each force shown on your FBDs in terms of the given masses (m_A, m_B) and the acceleration due to gravity, g.
   (d) What is the net force acting on Book B? What is the net force acting on Book A? What is the net force acting on the Earth *due to its interactions with the books (and the negligible-mass table) only*? Explain your reasoning.

3. (1 point) Now consider the main experiment setup: a cart of mass M resting on a plane inclined at an angle θ to the horizontal (no pulley or hanging mass yet). Draw a free-body diagram showing all forces acting on the cart (gravity, normal force). Resolve the gravitational force into components parallel and perpendicular to the incline. Label these components in terms of M, g, θ. What is the magnitude of the normal force N in this case?

4. (1 point) Imagine the cart from question 3 is connected by a cord over a pulley at the top of the incline to a hanging mass m. Draw two separate free-body diagrams: one for the cart M on the incline and one for the hanging mass m. Assume the system is moving at constant velocity. Include tension T.

5. (1 point) For the setup in question 4, assume the cart M is moving *up* the incline at a constant velocity. Add the force of rolling resistance (f_{roll}) to the FBD for the cart M. Write down the equations resulting from applying Newton's Second Law ($\Sigma F = 0$) in the directions parallel and perpendicular to the incline.

6. (1 point) Repeat question 5, but this time assume the cart M is moving *down* the incline at a constant velocity. How does the direction of the rolling resistance force change? Write down the corresponding $\Sigma F = 0$ equations.

7. (1 point) From your equations in Q5 and Q6, express the normal force N in terms of M, g, θ. Then substitute $T = mg$ (since the hanging mass is also moving at constant velocity) to get two equations relating the hanging masses (m_{up} for upward motion, m_{down} for downward motion) to M, θ, and the rolling resistance force f_{roll}.

8. (1 point) If you measure m_{up} and m_{down} for a given M and θ, how could you combine your two equations from Q7 to solve for the magnitude of the rolling resistance force, f_{roll}? (Show the algebraic step).

Lab measurements and report: submission by end of class

A reminder: Document all measurements fully. Upload your final report (PDF with embedded plots/tables) to Canvas *by the end of class* (uploads close 10 minutes after). Submit your work even if incomplete.

Measurement 1 *Setup and Initial Measurements* 1 point

This section guides you through setting up the inclined track and measuring the basic parameters.

Equipment: Dynamics track, cart, pulley with clamp, rod stand, string, mass hanger, set of masses, standard masses (100 g, 200 g, 400 g), digital compass/protractor, balance.

1. **Measure Cart Mass:** Use the balance to measure the mass of the cart, M. Record this value with its uncertainty (δM).

2. **Assemble Setup:** Attach the pulley to the high end of the dynamics track. Elevate this end using the rod stand to create an incline. Place the digital compass/protractor flat on the table

to measure the angle of inclination θ of the track relative to the horizontal. Ensure the track is stable and not tilted sideways.

3. **Attach Cord and Hanger:** Cut a length of string long enough to connect the cart (placed near the bottom of the track) over the pulley to the mass hanger hanging below the pulley. Attach the string securely to the cart and the hanger. Ensure the string runs parallel to the track and does not rub against anything other than the pulley wheel.

4. (1 point) **Record Initial Parameters:** Create a table in your lab notes or spreadsheet to record the mass of the cart $M \pm \delta M$.

Measurement 2 *Constant Velocity Measurements vs Angle* 13 points

Here you will measure the hanging mass required to make the cart move at a constant velocity up and down the incline for several different angles. This allows determination of the rolling resistance force and a consistency check.

1. **Set Angle θ_1:** Set the track to the first angle, $\theta_1 = 15°$. Measure and record the angle θ accurately using the digital compass/protractor placed on the track surface. Note the uncertainty in your angle measurement ($\delta\theta$).

2. **Uphill Motion (Constant Velocity):**

   (a) Place the cart near the bottom of the incline. Add mass to the hanger until the cart, when given a gentle push *up* the incline, continues moving *up* at an approximately constant velocity (judge by eye).

   (b) Refine the measurement: Find the minimum mass ($m_{up,min}$) that keeps it moving up without slowing down significantly, and the maximum mass ($m_{up,max}$) before it noticeably accelerates upwards.

   (c) (2 points) Record the best estimate for the hanging mass required for constant uphill velocity as $m_{up} = (m_{up,min} + m_{up,max})/2$. Record the uncertainty as $\delta m_{up} = (m_{up,max} - m_{up,min})/2$. Don't forget to include the mass of the hanger itself.

3. **Downhill Motion (Constant Velocity):**

   (a) Place the cart near the top of the incline. Adjust the mass on the hanger (you will likely need less mass than for uphill motion) until the cart, when given a gentle push *down* the incline, continues moving *down* at an approximately constant velocity.

   (b) Refine the measurement: Find the range of masses ($m_{down,min}, m_{down,max}$) that allow approximately constant downward velocity. $m_{down,min}$ is the mass below which it clearly accelerates downwards, $m_{down,max}$ is the mass above which it slows down or stops.

   (c) (2 points) Record the best estimate $m_{down} = (m_{down,min} + m_{down,max})/2$ and its uncertainty $\delta m_{down} = (m_{down,max} - m_{down,min})/2$.

4. (6 points) **Repeat for Different Angles:** Repeat steps 1-3 for the following angles: $20°, 25°, 30°, 35°, 40°, 45°$. Ensure the track remains stable at each angle. (1 point per angle set).

5. (3 points) **Data Table (Constant Velocity vs Angle):** Organize all your constant velocity measurements in a neat table. Columns should include: Angle θ, $\delta\theta$, m_{up}, δm_{up}, m_{down}, δm_{down}. Remember to use SI units (kg for mass, degrees or radians for angle as appropriate for calculations). Ensure the table is clearly labeled and includes units in column headers.

Measurement 3 *Dependence on Normal Force* 2 points

Here you will investigate if the rolling resistance force depends on the normal force by adding specific masses to the cart at a fixed angle.

1. **Choose Fixed Angle:** Select one of the angles used previously, preferably one in the middle range (e.g., $\theta = 30°$). Record this fixed angle $\theta_{fixed} \pm \delta\theta$.
2. **Measurements with Base Mass M:** You already have the measurements for m_{up} and m_{down} (with uncertainties $\delta m_{up}, \delta m_{down}$) for the cart mass M at this angle from the "Constant Velocity Measurements vs Angle" section. Record these values again clearly as the first trial for this part.
3. **Prepare Added Masses:** Obtain standard masses of approximately $100\,\mathrm{g}$, $200\,\mathrm{g}$, and $400\,\mathrm{g}$. Measure the actual mass of each ($M_{add,1}, M_{add,2}, M_{add,3}$) using the balance and record their uncertainties ($\delta M_{add,1}$, etc.).
4. (2 points) **Measurements with Added Masses:** At the same fixed angle θ_{fixed}, perform the following steps:
   (a) Securely place the first added mass ($M_{add,1} \approx 100\,\mathrm{g}$) on the cart. Calculate the total mass $M_{total,1} = M + M_{add,1}$ and its uncertainty $\delta M_{total,1}$. Repeat the constant velocity procedures to find the hanging masses $m_{up,1} \pm \delta m_{up,1}$ and $m_{down,1} \pm \delta m_{down,1}$.
   (b) Replace the first added mass with the second ($M_{add,2} \approx 200\,\mathrm{g}$). Calculate $M_{total,2} = M + M_{add,2}$ and $\delta M_{total,2}$. Repeat the constant velocity measurements to find $m_{up,2} \pm \delta m_{up,2}$ and $m_{down,2} \pm \delta m_{down,2}$.
   (c) Replace the second added mass with the third ($M_{add,3} \approx 400\,\mathrm{g}$). Calculate $M_{total,3} = M + M_{add,3}$ and $\delta M_{total,3}$. Repeat the constant velocity measurements to find $m_{up,3} \pm \delta m_{up,3}$ and $m_{down,3} \pm \delta m_{down,3}$.
5. **Data Table (Normal Force Dependence):** Create a table summarizing the measurements for this part. Include rows for each trial (Base Cart, +100g, +200g, +400g). Columns should include: Trial Description, Total Cart Mass $M_{total} \pm \delta M_{total}$, $m_{up} \pm \delta m_{up}$, $m_{down} \pm \delta m_{down}$. Ensure the fixed angle θ_{fixed} is clearly noted.

Analysis 1 *Data Analysis and Interpretation* 24 points

In this section, you will analyze your data using free-body diagrams and Newton's laws to determine the rolling resistance force and investigate its properties.

Part 1: Rolling Resistance vs Angle and Consistency Check

1. (2 points) **Free-Body Diagrams (Constant Velocity):**

(a) Draw a clear FBD for the cart (M) when it is moving at constant velocity *up* the incline. Include gravity (Mg), normal force (N), tension (T_{up}), and rolling resistance (f_{roll}). Ensure rolling resistance opposes motion.

(b) Draw a clear FBD for the cart (M) when it is moving at constant velocity *down* the incline. Include gravity (Mg), normal force (N), tension (T_{down}), and rolling resistance (f_{roll}). Note the change in direction for f_{roll}.

2. (2 points) **Applying Newton's Laws (Constant Velocity):**

(a) For the *uphill motion* FBD, apply Newton's Second Law ($\Sigma F_x = 0, \Sigma F_y = 0$) using coordinates parallel (x) and perpendicular (y) to the incline. Show that $N = Mg\cos\theta$ and $T_{up} - Mg\sin\theta - f_{roll} = 0$.

(b) For the *downhill motion* FBD, apply Newton's Second Law similarly. Show that $N = Mg\cos\theta$ (again) and $T_{down} - Mg\sin\theta + f_{roll} = 0$.

(c) Substitute $T_{up} = m_{up}g$ and $T_{down} = m_{down}g$ into your parallel force equations to derive the following relationships (valid for any specific M and θ):

$$m_{up}g = Mg\sin\theta + f_{roll} \tag{1}$$

$$m_{down}g = Mg\sin\theta - f_{roll} \tag{2}$$

3. (6 points) **Calculating Rolling Resistance vs Angle:** Use the data from the "Constant Velocity Measurements vs Angle" section.

(a) Starting from equations (1) and (2) derived in step 2(c), subtract the downhill equation (2) from the uphill equation (1) to show that the rolling resistance force f_{roll} can be calculated as:

$$f_{roll} = \frac{(m_{up} - m_{down})g}{2}$$

(Assume $g = 9.81\,\mathrm{m/s^2}$ or use a locally measured value if available).

(b) Create a data table showing θ, $m_{up} \pm \delta m_{up}$, $m_{down} \pm \delta m_{down}$, and the calculated value of f_{roll} for each angle. Propagate uncertainties to find δf_{roll} for each angle (Hint: $\delta f_{roll} \approx \frac{g}{2}\sqrt{(\delta m_{up})^2 + (\delta m_{down})^2}$).

(c) Calculate the average value of f_{roll} from your measurements across all angles (using the base cart mass M). Estimate the uncertainty in this average value (e.g., using the standard deviation of your calculated f_{roll} values, or the average of the individual δf_{roll}). Report your final result for the average f_{roll} with units (Newtons).

(d) Discuss your results for f_{roll} vs angle. Is the force reasonably constant across the different angles, or does it show a trend? Keep in mind that the normal force $N = Mg\cos\theta$ changes with angle. Compare the magnitude of f_{roll} to the weight component $Mg\sin\theta$ at a typical angle.

4. (4 points) **Consistency Check:** Use the data from the "Constant Velocity Measurements vs Angle" section (base cart mass M).

(a) Add equations (1) and (2) together. Show that this yields: $(m_{up} + m_{down})g = 2Mg \sin\theta$, which simplifies to $m_{up} + m_{down} = 2M \sin\theta$. Notice that f_{roll} cancels out.

(b) Create a data table with calculated values for $\sin\theta$ and $(m_{up} + m_{down})$ for each angle. Include uncertainties for $(m_{up} + m_{down})$ by adding the uncertainties: $\delta(m_{up} + m_{down}) = \delta m_{up} + \delta m_{down}$.

(c) Plot $(m_{up} + m_{down})$ versus $\sin\theta$. Include error bars for the y-values. Perform a linear fit that is forced through the origin (or check if the intercept is consistent with zero).

(d) According to the theory, this plot should be a straight line passing through the origin with a slope equal to $2M$. Compare the slope obtained from your fit to $2M$ (using your measured value of M). Calculate the percentage difference. Does this check support the validity of your measurements and force analysis? Discuss the quality of the agreement and any discrepancies, considering the uncertainties.

Part 2: Dependence of Rolling Resistance on Normal Force

5. (4 points) **Analysis of Normal Force Data:** Use the data collected in the "Dependence on Normal Force" measurement section (at fixed angle θ_{fixed}).

   (a) For each trial (different total cart mass M_{total}), calculate the normal force acting on the cart: $N = M_{total} g \cos\theta_{fixed}$. Calculate the uncertainty δN (primarily from δM_{total}, assume θ_{fixed} is well-known for this calculation or propagate its uncertainty if significant).

   (b) For each trial, calculate the rolling resistance force using the corresponding m_{up} and m_{down} values: $f_{roll} = \frac{(m_{up} - m_{down})g}{2}$. Calculate the uncertainty $\delta f_{roll} \approx \frac{g}{2}\sqrt{(\delta m_{up})^2 + (\delta m_{down})^2}$.

   (c) Create a summary table showing M_{total}, $N \pm \delta N$, and $f_{roll} \pm \delta f_{roll}$ for each trial.

   (d) Plot f_{roll} versus N. Include error bars for both f_{roll} (vertical) and N (horizontal).

   (e) Discuss your plot in detail. Does the rolling resistance force f_{roll} appear to be constant within uncertainties, or does it show a clear dependence on the normal force N? If it depends on N, describe the relationship (e.g., approximately linear?). Fit a line if appropriate. What does this suggest about the physical mechanisms causing rolling resistance in your setup (e.g., deformation vs. axle friction)?

Part 3: Discussion and Conclusion

6. (1 point) **Overall Rolling Resistance Discussion:** Discuss your findings about the rolling resistance force f_{roll}. Refer to both the average value found in Part 1 and the dependence (or lack thereof) on normal force found in Part 2. Is its magnitude reasonable for a dynamics cart?

7. (4 points) **Sources of Error:** Discuss the main sources of uncertainty and potential systematic errors in your experiment across all parts. Quantify the likely impact where possible. Consider:

   - Accuracy of angle measurement ($\delta\theta$). How might error in θ affect the f_{roll} vs N plot?
   - Uncertainty in masses (M, M_{added}, m_{up}, m_{down}). How significant was the range needed to determine constant velocity ($\delta m_{up/down}$)? How does this compare to balance precision?

- Judging constant velocity by eye. How could this be improved? Estimate the potential error introduced.
- Friction and mass of the pulley. Revisit the hint: Pulley friction increases needed m_{up} and decreases needed m_{down}. How does this affect the calculated $f_{roll} = (m_{up} - m_{down})g/2$? Does this systematic error affect the *conclusion* about whether f_{roll} depends on N? Why or why not? (Hint: Does the pulley friction itself likely change significantly when M changes but $m_{up/down}$ don't change drastically?)
- Track condition (straightness, levelness, cleanliness).
- String alignment (parallel to track).
- Axle friction/bearing quality in the cart wheels (a primary component of f_{roll}, potentially dependent on N).
- Secure placement of added mass (did it shift?).

8. (1 point) **Conclusion:** Summarize your key findings, including your best estimate for the average rolling resistance force f_{roll} (from Part 1), your conclusion about its dependence on normal force (from Part 2), and the result of the consistency check (from Part 1). Briefly state whether your results support the application of Newton's laws to this system, considering the experimental uncertainties and potential errors.

References

[1] D. Halliday, R. Resnick, and J. Walker. *Fundamentals of Physics*. Fundamentals of Physics. John Wiley & Sons.