Mechanics Laboratory 1105, Spring 2025

Prof. Dr. T. Neumann, Prof. R. Guarino

https://www.physics.smu.edu/tneumann/110X_Spring2025/

Lab 4 — Newton's laws of motion Max. points: 56.5

Learning Objectives

- Understand the concepts of normal force, parallel force (component of gravity along the incline), and friction.
- Apply Newton's second law to analyze the motion of an object on an inclined plane.
- Learn to experimentally measure forces and accelerations using a smart cart.
- Compare experimental measurements with theoretical predictions and analyze discrepancies.
- Understand and estimate uncertainties in measurements and calculations.

Your preparation: Work through before coming to the lab

- Review chapter 5 in Halliday, Resnick, and Walker [1] about Newton's laws: Focus on understanding Newton's first, second, and third Laws. Pay close attention to the definition of force, mass, and inertia. Understand how forces are vectors and how to find the net force. Section 5-6 (Newton's second law and gravity) is particularly relevant for understanding gravitational force and weight. Also pay attention to how free-body diagrams are constructed and used to analyze forces. Practice drawing free-body diagrams for different scenarios, including objects on inclined planes. Carefully study examples and explanations related to objects on inclined planes. Understand how gravity is decomposed into components along and perpendicular to the incline.
- Review chapter 6 in Halliday, Resnick, and Walker [1]: Study "Friction" and "The Properties of Friction". Understand the difference between static and kinetic friction, and the concept of coefficients of friction. Pay attention to how friction force depends on the normal force and the surface properties.
- Prepare by reviewing the lab measurement and report section below and the equipment overview and further introductory material on https://www.physics.smu.edu/tneumann/110X_Spring2025/schedule-mechanics/.

Pre-lab: Upload to Canvas before coming to the lab

A reminder: Upload your answers as a text document (exported as PDF) to Canvas before the lab begins (Canvas uploads are no longer possible 30 minutes before the lab starts!).

Pre-lab 1 6.5 points

1. (1 point) State Newton's second law of motion in your own words. Explain what each symbol in the equation $F = ma$ represents, including their units.
2. (2 points) A cart is placed on an inclined plane. The gravitational force acts vertically downwards. Describe how the gravitational force can be resolved into two components: one component parallel to the inclined plane and one component perpendicular (normal) to the inclined plane. Which component is responsible for pulling the cart down the incline if friction is negligible?
3. (1.5 points) Draw a free-body diagram for a cart at rest on an inclined plane. Include the following forces: gravitational force (weight), normal force, and static friction. Clearly label each force vector and indicate the direction of each force.
4. (1 point) Imagine the cart in question 3 starts to move down the inclined plane. In what direction does the force of kinetic friction act? What factors might influence the magnitude of the kinetic friction force between the cart and the track?
5. (1 point) What are the standard SI units for mass, acceleration, and force? If you measure mass in grams and acceleration in cm/s^2, will the force you calculate using Newton's second law be in Newtons? Explain why or why not, and if not, how would you convert it to Newtons?

Lab measurements and report: submission by end of class

A reminder: All measurements must be fully documented . The final report must be uploaded to Canvas *by the end of the class* exported as PDF with plots and tables from Excel embedded as images. Canvas will stop accepting uploads 10 minutes after the class ends. If you have not fully completed your report, you must upload the documents as far as you have completed them for grading.

This worksheet is designed to explore Newton's second law of motion using a cart on an inclined track. You will use a wireless smart cart to measure forces and accelerations, and compare these measurements to theoretical predictions. This experiment will help you understand how forces act on an object on an incline, and how these forces relate to the object's motion according to Newton's laws.

Measurement 1 *Forces on a static cart on an inclined track* 21 points

In this experiment, we investigate the normal and parallel components of the gravitational force acting on a *static* cart positioned on an inclined track. You will use a scale to directly measure the normal force exerted by the track on the cart, and a wireless force sensor to measure the force component parallel to the inclined plane.

1. (3 points) Start with the track at zero degrees (horizontal). Place the cart on the track, ensuring the scale is under the cart to measure the normal force and the parallel force sensor is connected but not under load. Record the reading on the scale as the normal force and the reading from the parallel force sensor. Draw a free-body diagram of the forces acting on the cart in this

situation. What values do you expect for the normal and parallel forces in this case based on your free-body diagram?

2. (4 points) Ensure the parallel force sensor is connected to the cart such that it can measure forces *parallel* to the incline. With the track still horizontal, zero the parallel force sensor. This step calibrates the sensor to read zero when no parallel force is applied. Now, raise the track to incline angles of approximately $10, 20, 30, 40,$ and 50 degrees. For each angle:

   - Record the incline angle.
   - Record the reading from the scale as the measured normal force [N].
   - Record the reading from the parallel force sensor as the measured parallel force [N].

   Organize your measurements in a table with columns for:

   - Incline angle [degrees]
   - Normal force (measured) [N]
   - Parallel force (measured) [N]
   - Uncertainty in angle [degrees] (estimate)
   - Uncertainty in normal force [N] (estimate based on scale precision or fluctuations)
   - Uncertainty in parallel force [N] (estimate based on sensor precision or fluctuations)

   Estimate the uncertainties in your angle measurements and in both force measurements. Include these uncertainty estimates in your table. Pay attention to units.

3. (3 points) Choose one incline angle from your measurements (e.g., 30 degrees). Draw a free-body diagram of the static cart at this incline, showing all forces acting on it: gravitational force, normal force, and the parallel force component of gravity, and any other relevant forces. Indicate which force is measured by the scale and which is measured by the parallel force sensor. Clearly label each force vector and indicate the angle of the incline in your diagram. Are the measured forces consistent with the cart being in equilibrium (net force = zero)?

4. (1 point) Measure the mass of the cart using a scale, and estimate the uncertainty in your mass measurement. Record:

   - Mass of cart [kg]
   - Uncertainty in mass [kg] (estimate)

   Add a mass of approximately $100\,\mathrm{g}$ to the cart. Measure the total mass of the cart with the added mass and its uncertainty.

5. (2 points) Repeat the force measurements from step 2 for incline angles of approximately $10, 20, 30, 40,$ and 50 degrees with the added mass. Record your data in a new table with the same columns as in step 2, but clearly label this data as being ”with added mass”.

6. (4 points) Generate a scatter plot with incline angle on the horizontal axis and magnitude of force on the vertical axis for your data with the added mass. Plot two data series on the same graph:

- Measured Normal Force
- Measured Parallel Force

Label axes clearly, including units. Include vertical error bars representing the uncertainties in your force measurements.

7. (2 points) Calculate the theoretical normal and parallel forces using the measured mass of the cart + added mass. Add two more data series to your plot, which you will have to calculate, including uncertainties:
   - Theoretical magnitude of the parallel component of the gravitational force as a function of the incline angle.
   - Theoretical magnitude of the normal force as a function of the incline angle.

   Clearly distinguish between measured data and theoretical predictions in your plot's legend.

8. (2 points) Briefly comment on the agreement (or disagreement) between your theoretical predictions and experimental measurements for both normal and parallel forces, referring to your plot and error bars. Besides the uncertainty in mass and angle, state one other factor that could contribute to differences between theory and experiment in these static force measurements. Classify this factor as contributing to systematic or random error, and explain your reasoning.

Measurement 2 *Newton's 2nd law for downhill motion of a cart on an incline* 13 points

In this experiment, we study the *downhill motion* of a cart on an incline, exploring Newton's second law. By analyzing the downhill acceleration, we will determine the incline angle.

When the cart moves downhill on the incline, the net force along the incline is F_{down}. Assuming a constant friction force F_f opposing motion and gravitational force *component along the incline* F_g acting downhill:

$$F_{\text{down}} = m|a_{\text{down}}| = |F_g| - |F_f|\,, \tag{1}$$

Here, a_{down} is the acceleration downhill (positive value). We use absolute values for simplicity, keeping in mind the directionality of forces and accelerations.

Set up the cart on the track and raise the track on one end to a height of approximately $5\,\text{cm}$.

1. (1 point) Measure the mass of the cart using a scale, and estimate its uncertainty. Record:
   - Mass of cart [kg]
   - Uncertainty in mass [kg] (estimate)

2. (2 points) Calculate the incline angle $\alpha_{\text{measured},5}$ based on the measured elevation ($5\,\text{cm}$) and the length of the track. Assume the track length is measured between the points where the track is supported, and measure the height from the base to the raised end, considering the initial height of the track at the 'zero elevation' end from the table surface. Determine the uncertainty in the angle $\Delta\alpha_{\text{measured},5}$ by propagating the uncertainties in the measurements of length and elevation. Record:

- Measured elevation [m] ($5\,\mathrm{cm}$)
- Uncertainty in elevation [m] (estimate)
- Length of track [m] (measure and record)
- Uncertainty in length [m] (estimate precision of measurement)
- Calculated incline angle $\alpha_{\text{measured},5}$ [degrees]
- Uncertainty in incline angle $\Delta\alpha_{\text{measured},5}$ [degrees]

Practice releasing the cart from rest at the top of the incline so it moves smoothly downhill.

3. (4 points) Start data recording in Capstone. Release the cart from rest at the top of the incline and let it move smoothly downhill. Stop recording just before the cart reaches the bottom or is stopped. Ensure you obtain smooth position and velocity curves. Save the data run, labeling it clearly as '$5\,\mathrm{cm}$ elevation downhill'. Generate a plot of acceleration vs. time, focusing on the time interval during the *downhill motion* after release and before stopping.
4. (2 points) Now, increase the track elevation to approximately $10\,\mathrm{cm}$. Repeat step 2 to calculate the incline angle $\alpha_{\text{measured},10}$ and its uncertainty $\Delta\alpha_{\text{measured},10}$. Record all measured and calculated values and uncertainties as in step 2, but for $10\,\mathrm{cm}$ elevation.
5. (4 points) With the $10\,\mathrm{cm}$ elevation, repeat step 3 to record another run of the cart going downhill. Save the data as '$10\,\mathrm{cm}$ elevation downhill' and generate a plot of acceleration vs. time for the relevant time interval, as in step 3.

Analysis 1 Now we analyze the data to determine the incline angle from downhill motion. 16 points

1. (2 points) Calculate the magnitude of the gravitational force acting on the cart, $F_g = m \cdot g$, where $g = 9.81 \pm 0.03\,\mathrm{m/s^2}$. Propagate uncertainties from your mass measurement and the uncertainty in g. Is the uncertainty in g a significant contributor to the uncertainty in F_g compared to the mass uncertainty? Show your calculation's result and comment.
2. (3 points) Analyze your acceleration vs. time plots from experiment 2. Identify regions on the plots where the acceleration is approximately constant during downhill motion (after release and before stopping). For each elevation ($5\,\mathrm{cm}$ and $10\,\mathrm{cm}$), estimate the average acceleration during the downhill motion $|a_{\text{down},5}|$ and $|a_{\text{down},10}|$. Estimate the uncertainties in these acceleration values based on the spread or fluctuations of the acceleration data in the chosen regions. Briefly explain in one or two sentences how you estimated the accelerations and their uncertainties from the plots.
3. (4 points) Assuming friction is negligible ($F_f \approx 0$), the downhill net force is approximately equal to the gravitational force component along the incline: $F_{\text{down}} \approx |F_g| = mg\sin\alpha$. Using $F_{\text{down}} = m|a_{\text{down}}| \approx mg\sin\alpha$, solve for the incline angle $\alpha = \arcsin(|a_{\text{down}}|/g)$. Determine the incline angle $\alpha_{\text{derived},5}$ for the $5\,\mathrm{cm}$ elevation and $\alpha_{\text{derived},10}$ for the $10\,\mathrm{cm}$ elevation using your measured downhill accelerations $|a_{\text{down},5}|$ and $|a_{\text{down},10}|$, including uncertainties by propagating uncertainties in the accelerations and g.

4. (3 points) For both elevations ($5\,\mathrm{cm}$ and $10\,\mathrm{cm}$), calculate the difference between the measured incline angle (α_{measured}) from track dimensions and the derived incline angle (α_{derived}) from motion. Include uncertainties in these differences. Are the differences consistent with zero, considering the uncertainties? Comment on the consistency and discuss whether the assumption of negligible friction is reasonable based on your results.

5. (2 points) If friction were *not* negligible, how would friction affect the *downhill* acceleration of the cart compared to the case with negligible friction (i.e., would the acceleration be larger, smaller, or the same)? Explain your reasoning based on equation (1).

6. (2 points) Discuss *one* factor, besides friction, that could cause a discrepancy between your measured incline angle (from track dimensions) and the derived incline angle (from motion). Classify this factor as contributing to systematic or random error, and explain your reasoning.

References

[1] D. Halliday, R. Resnick, and J. Walker. *Fundamentals of Physics*. Fundamentals of Physics. John Wiley & Sons.