Mechanics Laboratory 1105, Spring 2025

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https://www.physics.smu.edu/tneumann/110X_Spring2025/

Lab 7 — Uniform circular motion Max. points: 58

Your preparation: Work through before coming to the lab

- Prepare for the lab by thoroughly reading and understanding the measurement and analysis procedures on this worksheet. Photos of the equipment and further introductory material will be made available on
https://www.physics.smu.edu/tneumann/110X_Spring2025/schedule-mechanics/.
- Collect all your questions and ask your instructor at the beginning of the lab.
- Work through the chapter on "Uniform Circular Motion" [1]. Focus especially on these topics: Understand the concept of centripetal force and centripetal acceleration. Understand the relationship between centripetal force, mass, radius, linear velocity and angular velocity in uniform circular motion. Review how to relate angular velocity ω to linear velocity v for circular motion.

Pre-lab: Upload to Canvas before coming to the lab

A reminder: Upload your answers as a text document (exported as PDF) to Canvas before the lab begins (Canvas uploads are no longer possible 30 minutes before the lab starts!).

Pre-lab 1 — 8 points

1. (1 point) Explain in your own words what centripetal force is and what its role is in uniform circular motion. Provide an example.
2. (1 point) Write down the formula for centripetal force in terms of mass m, radius r, and linear velocity v. Also write down the formula in terms of mass m, radius r, and angular velocity ω. Explain the symbols and units.
3. (1 point) You will be plotting centripetal force F vs. angular velocity squared ω^2, and centripetal force F vs. linear velocity squared v^2. Based on the formulas for centripetal force, what type of graph (linear, quadratic, inverse, etc.) do you expect to see in each case? Explain why, referring to the relevant formula in each case.
4. (1 point) In your own words, explain the physical meaning of angular velocity ω and linear velocity v in the context of this rotating experiment. How are these two quantities related for the steel ball in uniform circular motion?

5. (1 point) Imagine you are performing the experiment and you increase the voltage to the motor, causing the rotor to spin faster. Describe in qualitative terms how you expect to see the readings on the force sensor and the photogate change as the speed increases. Explain your reasoning based on the principles of uniform circular motion.
6. (1 point) Consider potential sources of error in this experiment. Describe one specific factor that could cause your experimental measurements of centripetal force to deviate from the theoretical prediction, and explain how this factor might influence your results. (Think about the equipment and setup described in the worksheet.)
7. (2 points) Linear Regression and R-squared: In the Analysis section, you will perform linear regression and look at the R-squared value to assess how well your data fits a linear relationship. Explain in your own words what linear regression does. What does the R-squared value tell you about how well a linear model fits the data? Imagine you get an R-squared value of 0.98 for your F vs. ω^2 plot, and 0.75 for your F vs. v^2 plot. Which plot indicates a better linear fit? Explain.

Lab measurements and report: submission by end of class

A reminder: All measurements must be fully documented . The final report must be uploaded to Canvas *by the end of the class* exported as PDF with plots and tables from Excel embedded as images. Canvas will stop accepting uploads 10 minutes after the class ends. If you have not fully completed your report, you must upload the documents as far as you have completed them for grading.

In this lab, you will experimentally verify the relationship between centripetal force and angular/linear velocity in uniform circular motion.

Measurement 1 *Uniform Circular Motion: Centripetal Force vs. Velocity* 21 points
In this measurement, you will use a rotating setup with a force sensor and photogate to measure centripetal force, angular velocity, and linear velocity, and verify their relationship.

Equipment: Force sensor with angular velocity measurement capability, dual-beam photogate, rotor with motor and reduction gear, steel ball, string, DC Power Supply (Max 12V), weighing scale, meter stick or ruler, Capstone software for data acquisition and plotting

Measurement procedure:

1. (2 points) Prepare the setup:
   (1) Weigh the steel ball using the weighing scale and record its mass m.
   (2) Attach the steel ball to the force sensor using the string.
   (3) Measure the radius of rotation r, which is the distance from the center of the steel ball to the center of the rotor. Record this radius.
   (4) Mount the force sensor onto the rotor such that the force sensor is at the center of rotation.

(5) Position the dual-beam photogate to measure the linear velocity of the rotor as it passes through the photogate beams.

(6) Measure the radius R at which the dual-beam photogate measures the linear velocity.

(7) Before turning on the motor, zero the force sensor using the Capstone software to eliminate any initial readings.

2. (8 points) Vary motor voltage and record data for different speeds:

   (1) Connect the DC power supply to the motor. Set the initial voltage to $1\,\mathrm{V}$. *Ensure the voltage does not exceed 12V at any point.*

   (2) Turn on the power supply output and set the rotor in motion. Wait until the rotation is stable at a constant speed.

   (3) Using Capstone, record the centripetal force F measured by the force sensor, and the angular velocity ω measured by the force sensor.

   (4) Simultaneously, measure the linear velocity $v(R)$ using the dual-beam photogate and record the value.

   Important: Note that the dual-beam photogate does not measure the velocity at the radius of the steel ball ($v(r)$), but at the edge of the rotor blade ($v(R)$). Knowing that the angular velocity, which is the rate of change in the angle, is the same at both positions on the blade, you can obtain $v(r) = v(R) \cdot r/R$, where r is the radius of rotation of the steel ball and R is the radius of rotation where the photogate measures the velocity.

   (5) Increase the voltage of the power supply in steps of $0.5\,\mathrm{V}$, up to a maximum of 12V, and repeat steps (3)-(4) for each voltage setting. Record all values of F, ω, and $v(R)$ for each voltage setting.

3. (5 points) Repeat measurements for a shorter radius:

   (1) Shorten the string length (e.g., by using a loop-back to approximately halve its length).

   (2) Measure the new, shorter radius r_{short}. Record this value.

   (3) Repeat step 3 (Vary motor voltage and record data for different speeds) for this shorter radius r_{short}, recording all values of F, ω, and v.

4. (6 points) Data recording:

   (1) Create an Excel spreadsheet to record all your measurements, including: Voltage setting [V], Radius r (or r_{short}), Force F [N], Angular velocity ω [rad/s], Linear velocity $v(r)$ [m/s], Linear velocity $v(R)$ [m/s]. Organize your data clearly, possibly using separate tables for radius r and r_{short} measurements.

   (2) In Excel, calculate ω^2 and $v^2(r)$ for each data point. Note that you will have to translate $v(R)$ into $v(r)$.

   (3) Create two scatter plots in Excel: (1) Centripetal force F vs. Angular velocity squared ω^2 (for both radii), and (2) Centripetal force F vs. Linear velocity squared $v^2(r)$ (for both radii). Ensure plots have clear axis labels with units, titles, and legends.

(4) Take a screenshot of your Excel spreadsheet including all data and calculations.

(5) Take screenshots of your Excel plots (Force vs. ω^2 and Force vs. $v^2(r)$).

(6) Include all screenshots in your lab report PDF.

Analysis 1 *Analysis of Uniform Circular Motion Data* 29 points

In this analysis, you will analyze the data you collected to verify the relationship between centripetal force and velocity in uniform circular motion. Refer to your Excel spreadsheet and plots from Measurement 1.

1. Linearity verification from plots:

    (1) (2 points) Examine your four scatter plots of Centripetal force F vs. Angular velocity squared ω^2 and Centripetal force F vs. Linear velocity squared $v^2(r)$.

    For each plot (Force vs. ω^2 and Force vs. $v^2(r)$), visually assess whether the data points approximately follow a linear trend. Describe the observed trend in each plot (e.g., approximately linear, curved, etc.).

    (2) (6 points) For each plot, perform a linear regression (best-fit straight line) in Excel. Display the equation of the best-fit line and the R-squared value on each plot. Include the plots with trendlines, equations, and R-squared values in your report (as screenshots from Excel).

    (3) (2 points) Based on the R-squared values and visual inspection of the plots, discuss to what extent your data supports a linear relationship between F and ω^2, and between F and $v^2(r)$. What do the R-squared values suggest about the goodness of fit for a linear model in each case?

2. Comparison of experimental slope with theoretical prediction:

    (1) (2 points) For the force vs. ω^2 plot, the theoretical prediction is $F = mr\omega^2$, which is a linear relationship with slope $S_{\omega^2} = mr$. From your linear regression in step 1, record the experimental slope S_{exp,ω^2} for both radii (r and r_{short}). Report the uncertainty in the experimental slope $\Delta S_{exp,\omega^2}$ (typically provided by the linear regression function).

    (2) (2 points) Calculate the theoretical slope $S_{theory,\omega^2} = mr$ using the measured mass m of the steel ball and the radius r (and r_{short}) you used. Using estimated uncertainties in r and m, estimate the uncertainty in the theoretical slope $\Delta S_{theory,\omega^2}$. Show your error propagation calculation, including formulas and values with units. Calculate this theoretical slope and its uncertainty for both radii.

    (3) (2 points) Calculate the percentage difference between the experimental slope S_{exp,ω^2} and the theoretical slope S_{theory,ω^2} for both radii.

    (4) (2 points) Compare the experimental slope $S_{exp,\omega^2} \pm \Delta S_{exp,\omega^2}$ with the theoretical slope $S_{theory,\omega^2} \pm \Delta S_{theory,\omega^2}$ for the Force vs. ω^2 relationship, for both radii. Do they agree within their uncertainties? To check for agreement within uncertainties, see if the difference between the central values $|S_{exp,\omega^2} - S_{theory,\omega^2}|$ is less than or equal to the sum

of their uncertainties $\Delta S_{exp,\omega^2} + \Delta S_{theory,\omega^2}$. State clearly whether they agree within uncertainties or not for both radii. Discuss the level of agreement.

(5) (8 points) Repeat steps (1)-(4) for the Force vs. v^2 plot. The theoretical prediction is $F = \frac{m}{r}v^2$, so the slope is $S_{v^2} = \frac{m}{r}$. Record the experimental slope S_{exp,v^2} and its uncertainty $\Delta S_{exp,v^2}$. Calculate the theoretical slope $S_{theory,v^2} = \frac{m}{r}$. Using error propagation, estimate the uncertainty in the theoretical slope $\Delta S_{theory,v^2}$. Show your error propagation calculation. Calculate the percentage difference, and discuss the agreement (within uncertainties) for both radii.

3. (3 points) Discussion of Measurement Errors and Limitations: In your report, in a few paragraphs, discuss potential sources of experimental errors and limitations in this measurement. Consider and elaborate on factors such as the following, explain why they might be relevant or negligible compared to others:

   - Accuracy in measuring the radius r.
   - Precision and accuracy of the force sensor and velocity measurements.
   - Stability of the rotation and consistency of the motor speed.
   - Effect of string mass (if any).
   - Friction in the rotating system.
   - Alignment of the photogate.
   - Any other factors that could have affected your results.

Learning outcomes

- Experimentally verify the relationship between centripetal force, mass, radius, and velocity in uniform circular motion.
- Gain hands-on experience with measuring centripetal force, angular velocity, and linear velocity using sensors.
- Learn to use software (e.g., Capstone and Excel) for data acquisition, plotting, and linear regression analysis.
- Understand and analyze experimental data to verify physical laws.
- Discuss and evaluate potential sources of error in experimental measurements.

References

[1] D. Halliday, R. Resnick, and J. Walker. *Fundamentals of Physics*. Fundamentals of Physics. John Wiley & Sons.