Intro to Uncertainty Analysis

Contents

Important Note:

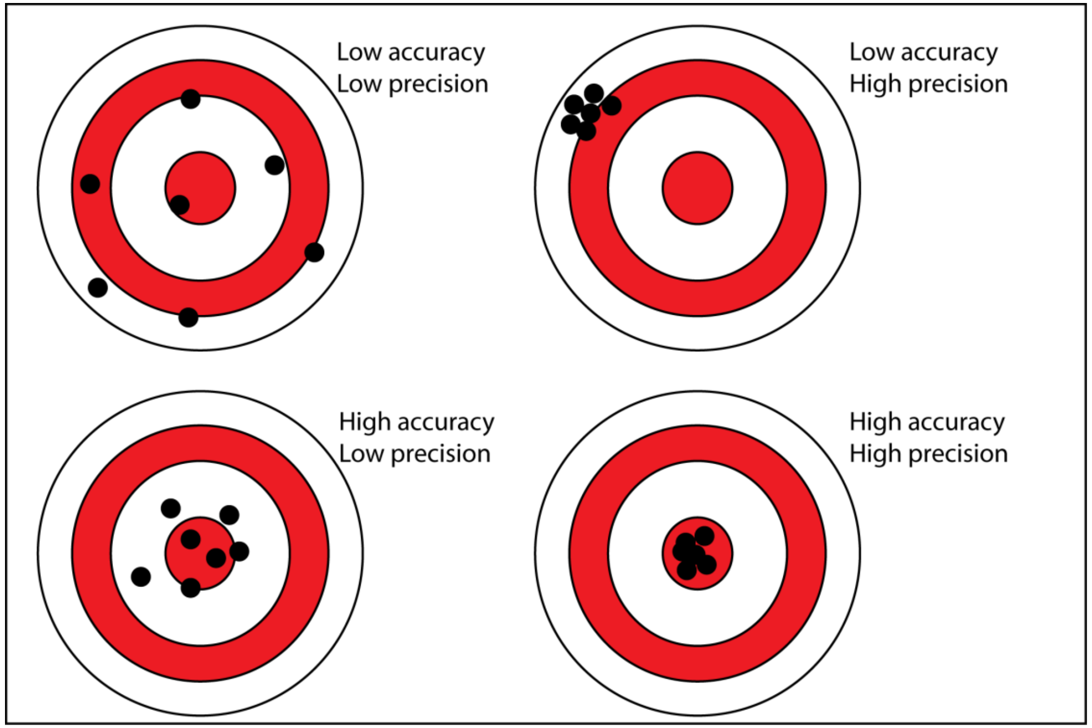
You may see different textbooks/sources use the words ‘uncertainty’ and ‘error’ interchangeably. While this is not ideal, for the purpose of this class, you may take them to mean the same thing. Nevertheless, we prefer "uncertainty" since it makes a better distinction to a definite mistake (error).

Why uncertainty analysis?

- In physics, experiments are often done to find a quantity.
- The purpose of uncertainty analysis:
  - To see if measurements are reproducible
  - Do measurements agree with theoretical prediction
  - Identify places where we can improve the experiment
- <u>An experiment is not complete until uncertainty analysis has been done!</u>

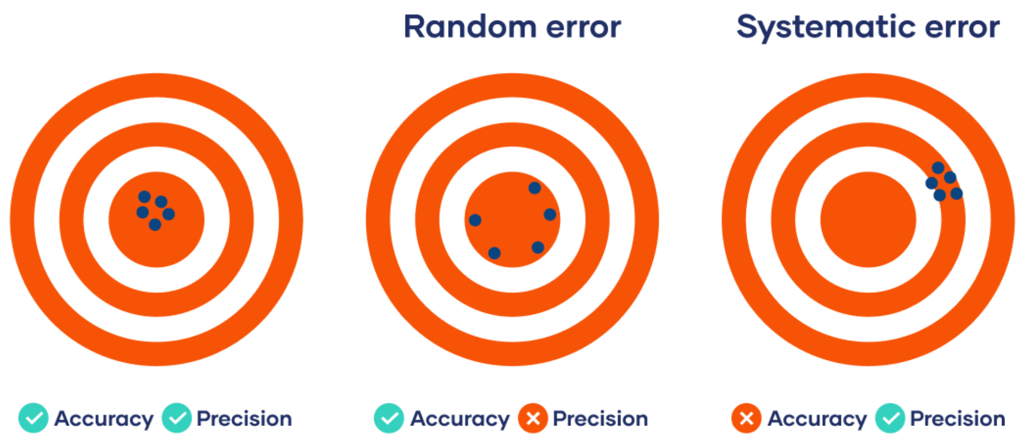
Accuracy and Precision

- **Accurate** measurement - agrees with 'accepted' value
- **Precise** measurement – one where the spread of results is 'small'

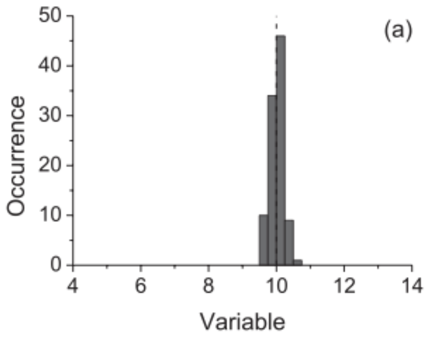


Types of Uncertainties

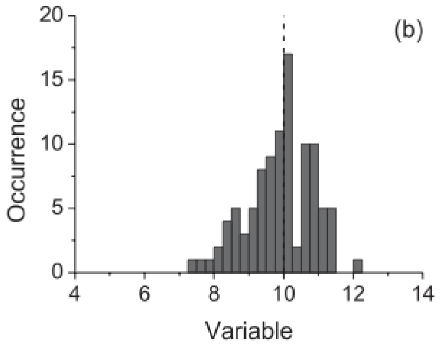
1. **Random (Statistical)** – affect the precision, but remain accurate
   - Signature of random errors is that repeated measurements are scattered over a range
2. **Systematic** – affect the accuracy but remain precise
   - Causes measurements to shift away from the accepted, or predicted, value
3. **Mistakes** – bad data points (outliers)
   - Ex: misreading a scale



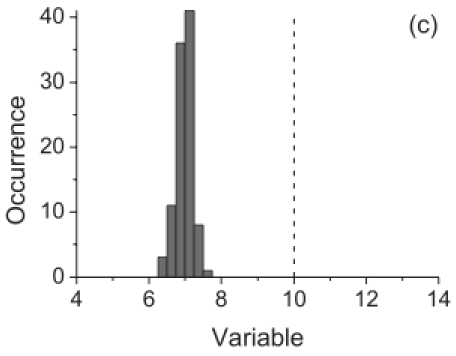
Let’s look at data…



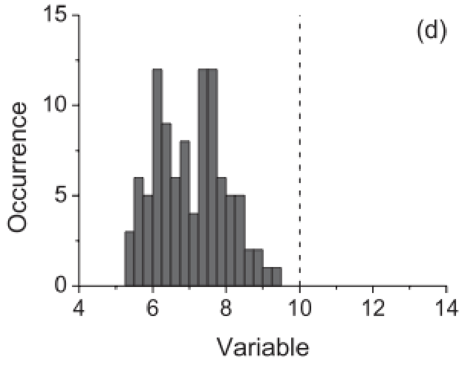
Precise and Accurate



Imprecise and Accurate



Precise and Inaccurate



Imprecise and Inaccurate

- Let’s say these histograms show the results of 100 measurements of students recording the time it took for a ball to drop from a certain height. (Dashed line shows correct value)

1. Describe each histogram as precise/imprecise and accurate/inaccurate.
2. What kind of errors are most obviously present in each histogram (random or systematic)?

Statistical Quantities: Mean & Standard Deviation

- **Mean** – the average of a set of data. Consider N measurements, then the mean is calculated by…

$$\tilde{x} = \frac{1}{N}(x_1 + x_2 + \cdots + x_N) = \frac{1}{N}\sum_{i=1}^{N} x_i$$

- **Standard deviation** – a measure of the amount of variation of the values of a measurement variable about its mean

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \tilde{x})^2}$$

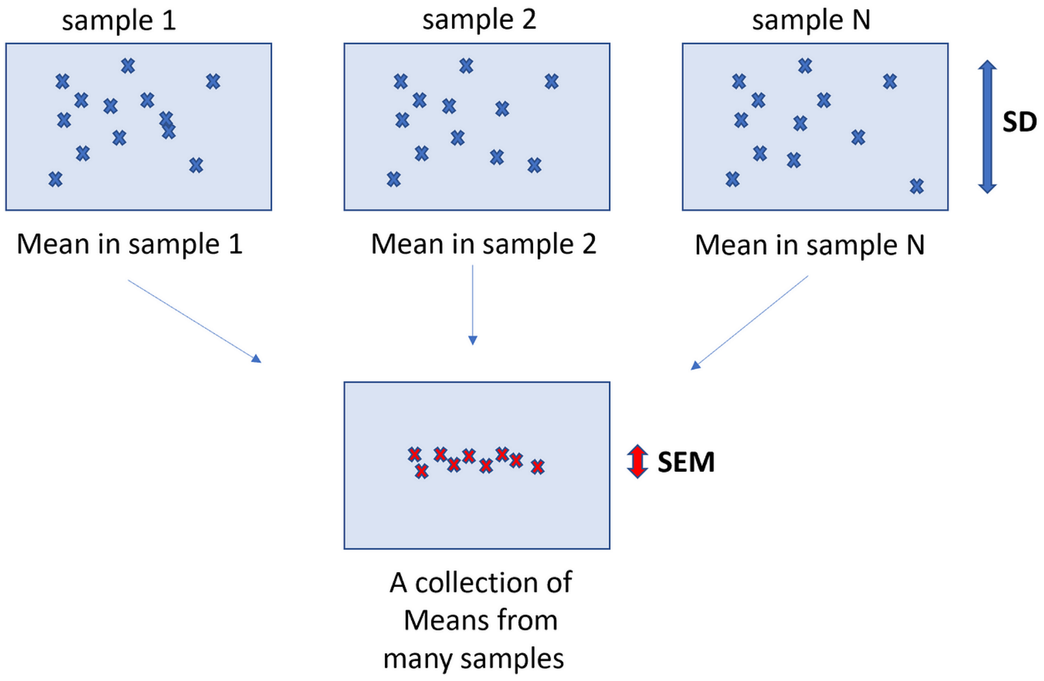
Statistical Quantities: Standard Deviation of the Mean

- **Standard deviation of the mean** (SDOM) - Uncertainty of mean between different smaller sets of measurements

$$\alpha = \frac{\sigma}{\sqrt{N}}$$

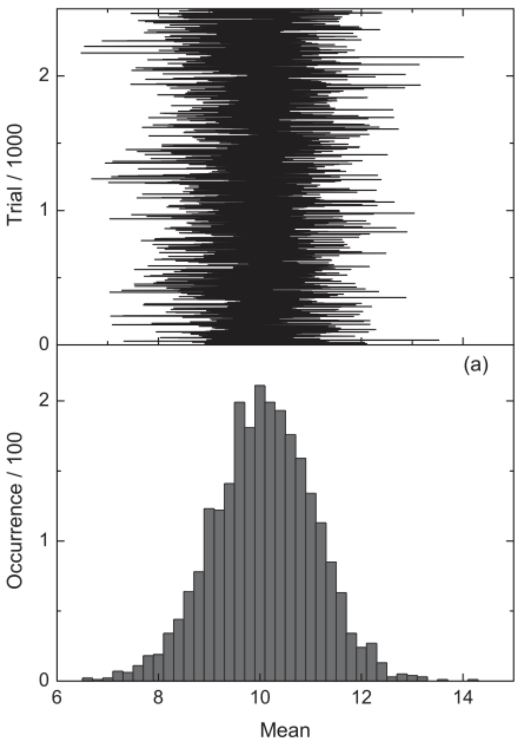
↑

This is the uncertainty that we quote in results

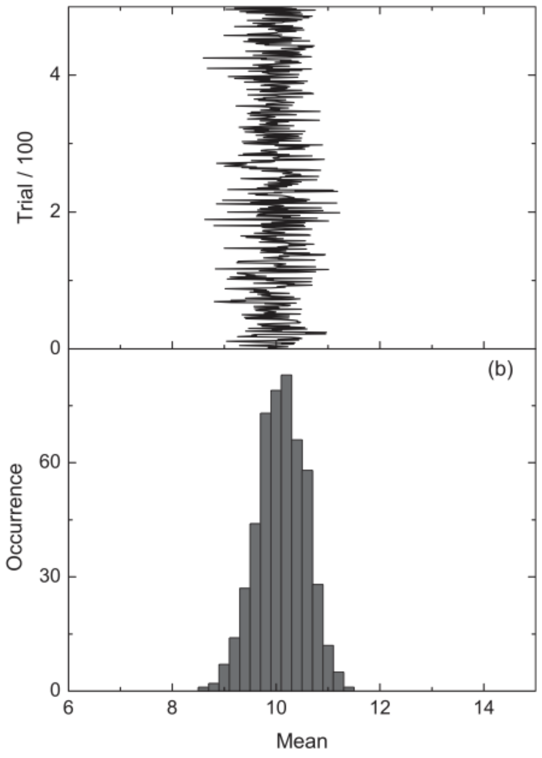


Statistical Quantities: Standard Deviation of the Mean - Example

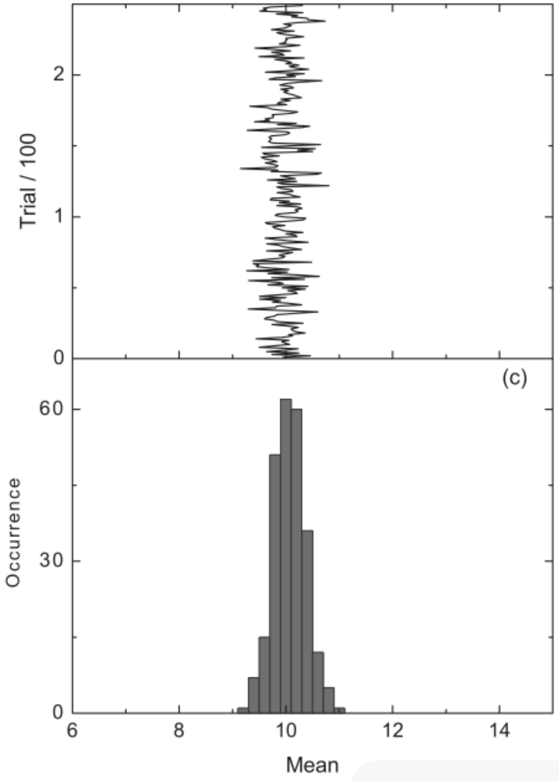
Raw data: 2500 data points



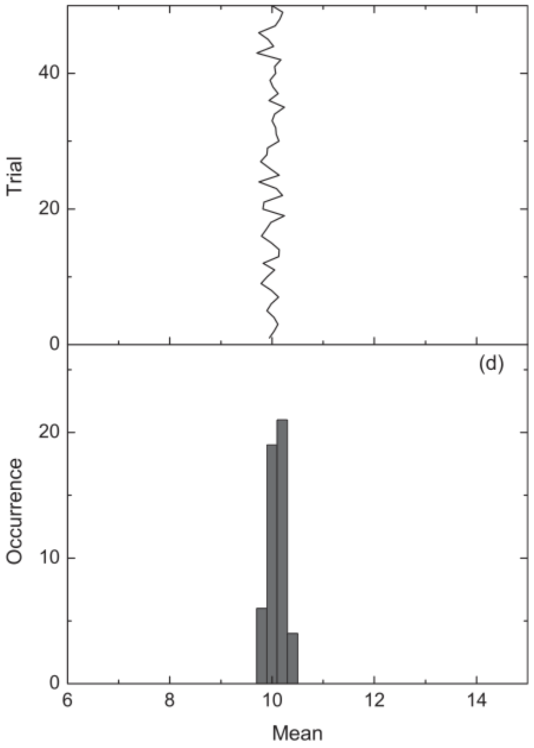
Mean of every 5 data points (500 means)



Mean of every 10 data points (250 means)



Mean of every 50 data points (50 means)



Significant Figures & Scientific Notation

- **Significant Figures:** in the absence of an error being quoted, we assume a number has 'significance' equal to a single unit in the last figure quoted i.e.
  - **A resistor with a resistance of 97Ω has 'absolute uncertainty' of 1 Ω**
  - **A resistor with a resistance of 100.04 Ω has 'absolute uncertainty' of 0.01 Ω**
  - **97 Ω** has **two** significant figures; **100.04 Ω** has **five** significant figures
- **Scientific Notation:** Shorthand for longer numbers
  - 2,700,000 Ω (2 significant figures, or 7 if there was a decimal at the end) can be written as..
  - 2.7 MΩ (2 significant figures)
  - 2.7×10^6 Ω (2 significant figures)

Uncertainty Propagation

- If the uncertainty in a measurement is known, and the measurement must be used in some equation to obtain a new, calculated number, the uncertainty can be *'propagated'*...

- $\Delta f(x) = |f(x) - f(x + \Delta x)|$
  - $\Delta \mathrm{f}(\mathrm{x})$ is the **uncertainty of our calculated number** $\mathrm{f}(\mathrm{x})$, and $\Delta \mathrm{x}$ is the **uncertainty of our measurement** x
  - This is done when the **calculated number** only depends one **measurement** and its associated **uncertainty**
- If our **calculated number** depends on multiple **measurements** and associated **uncertainties...**
  - $\Delta f(x, y, \ldots) = \sqrt{|f(x, y, \ldots) - f(x + \Delta x, y, \ldots)|^2 + |f(x, y, \ldots) - f(x, y + \Delta y, \ldots)|^2 + \ldots}$

Five Golden Rules for Uncertainty Analysis

1. The best estimate of a parameter is the mean.
2. The ‘error’ is the standard error in the mean.
3. Round up the error to the appropriate number of significant figures (should be similar to precision of measurements taken)
4. Match the number of decimal places in the mean to the standard error.
5. **Include units.**