E&M Laboratory 1106, Summer 2025

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https://www.physics.smu.edu/tneumann/110X_Summer2025/

Lab 6 — Ohm's law and DC circuits: Kirchhoff's laws Max. points: 58.5

Your preparation: Work through before coming to the lab

- Prepare for the lab by thoroughly reading and understanding the measurement and analysis procedures on this worksheet. Photos of the equipment and further introductory material will be made available on https://www.physics.smu.edu/tneumann/110X_Summer2025/schedule-em/.
- Collect all your questions and ask your instructor at the beginning of the lab.
- Work through chapter "Circuits" of Halliday, Resnick, and Walker [1]. Focus especially on these topics: Understand the concepts of electromotive force (EMF) and internal resistance of a battery. Understand series and parallel connections of resistors and how to calculate equivalent resistances. Study and understand Kirchhoff's Junction Rule (Current Law) and Loop Rule (Voltage Law) and their application to circuit analysis. Review Ohm's Law and its application in DC circuits.

Pre-lab: Upload to Canvas before coming to the lab

A reminder: Upload your answers as a text document (exported as PDF) to Canvas before the lab begins (Canvas uploads are no longer possible 30 minutes before the lab starts!).

Pre-lab 1 10 points

- (2 points) Explain in your own words why Kirchhoff's Current Law (KCL) is a fundamental principle in circuit analysis. Imagine you are explaining it to a classmate who is confused – what analogy or simple physical principle would you use to help them understand it?
- (2 points) Explain in your own words why Kirchhoff's Voltage Law (KVL) is a fundamental principle in circuit analysis. Think about what happens to energy as charges move around a closed loop in a circuit – how does KVL relate to this?
- (2 points) Consider you are building a series circuit in the lab using a battery and two resistors, R_1 and R_2.
  (a) Predict how the current measured at different points in a series circuit (e.g., before R_1, between R_1 and R_2, after R_2) will compare. Explain why you expect this based on the nature of series circuits.

(b) If you increase the value of R_2 in this series circuit, what would you expect to happen to the current through R_1? Explain your reasoning based on Ohm's Law and the concept of total series resistance.

- (2 points) Consider you are building a parallel circuit in the lab with a battery and two resistors, R_1 and R_2.

  (a) Predict how the voltage drop across R_1 will compare to the voltage drop across R_2 in a parallel circuit. Explain why based on the properties of parallel connections.

  (b) If you add another resistor R_3 in parallel to R_1 and R_2, what would you expect to happen to the total current drawn from the battery? Explain your prediction in terms of equivalent resistance and Ohm's Law.

- (1 point) In Measurement 1 of the lab, you will measure the open-circuit voltage and terminal voltage of a battery with a 100 Ohm load. Why is there a difference between these two voltages in a real battery?

- (1 point) Sketch a simple circuit diagram of Measurement 1 in the lab procedure (internal resistance measurement). Label the battery (EMF and internal resistance), the external resistor, and where you would connect a voltmeter to measure terminal voltage and an ammeter to measure current.

Lab measurements and report: submission by end of class

A reminder: All measurements must be fully documented . The final report must be uploaded to Canvas *by the end of the class* exported as PDF with plots and tables from Excel or Capstone embedded as images. Canvas will stop accepting uploads 10 minutes after the class ends. If you have not fully completed your report, you must upload the documents as far as you have completed them for grading.

In this lab, you will investigate Kirchhoff's rules in DC circuits using dry-cell batteries and resistors. You will perform four sets of measurements: first, to determine the internal resistance of a battery, second and third, to study series and parallel resistor combinations, and fourth, to verify Kirchhoff's rules in a two-loop circuit.

Measurement 1 *Measurement 1: Internal resistance of a dry-cell battery (range of resistors)* 6.5 points
In this first measurement, you will use a range of load resistors to experimentally determine the internal resistance of a dry-cell battery using linear regression analysis.

Equipment: Digital Multimeter (DMM), 1.5V or 3.0V battery, resistors (10 Ω, 20 Ω, 30 Ω, 100 Ω), broadboard, connecting cables, cable clips.

Procedure:

1. (0.5 points) Measure and record open-circuit voltage (V_{oc}): Use the DMM configured as a voltmeter to measure the open-circuit voltage of the battery. To do this, connect the voltmeter

directly across the battery terminals, ensuring that no external circuit is connected to the battery. Record this measured open-circuit voltage as V_{oc}.

2. (4 points) Perform varying load resistance measurements and record data: For each of the specified load resistors R_L (namely, $R_L = 10\,\Omega$, $20\,\Omega$, $30\,\Omega$, and $100\,\Omega$), perform the following sequence of measurements and record all measured values.
   (a) Connect load resistor: For a chosen load resistor R_L, connect it across the terminals of the battery using connecting cables and cable clips. This will create a simple closed circuit with the battery and the load resistor.
   (b) Measure terminal voltage: Use a DMM as a voltmeter to measure the terminal voltage (V_{t,R_L}) across the battery. Make sure to measure the voltage while the load resistor R_L is connected and the circuit is closed (i.e., under load). Record this terminal voltage.
   (c) Measure circuit current: Use a DMM configured as an ammeter to measure the current (I_{R_L}) flowing through the circuit. To do this, you will need to temporarily break the circuit, insert the ammeter in series at the break, and then close the circuit again so the current flows through the ammeter. Record the measured current. Also, carefully note down the nominal resistance value of the load resistor R_L you used for this set of measurements.

3. (2 points) Sketch and include circuit diagram in report: In your report, sketch the basic circuit diagram that you used for these internal resistance measurements. Your diagram should include: a symbol for the battery, clearly indicating its EMF ($\mathcal{E}$) and internal resistance (r); the load resistor R_L; the placement of the voltmeter used to measure the terminal voltage V_t; and the placement of the ammeter used to measure the current I_{R_L}.

Analysis 1 *Analysis 1: Calculation of internal resistance using linear regression* 11 points

In this analysis section, you will perform linear regression on your measurement data from Measurement 1 to determine the internal resistance (r) and electromotive force (EMF, $\mathcal{E}$) of the battery.

1. (2 points) Create a data table and scatter plot: Create a table in your lab report to organize the data you collected in Measurement 1. This table should have columns for: Load resistance (R_L [Ω]), measured current (I_{R_L} [A]), and measured terminal voltage (V_{t,R_L} [V]). Using the data from your table, create a scatter plot of the terminal voltage (V_{t,R_L}) on the vertical (y) axis versus the measured current (I_{R_L}) on the horizontal (x) axis. Include this data table and scatter plot in your lab report, ensuring that the axes of your plot are properly labeled, including units.

2. (4 points) Perform a linear regression, determine parameters, report uncertainties, and compare EMF to open-circuit voltage: Perform a linear regression (best-fit straight line) on the scatter plot of V_t vs. I. From the linear regression analysis, report the slope (m) and the y-intercept (c) of the best-fit straight line. Using these reported values, calculate the internal resistance $r = -m$ and the EMF $\mathcal{E} = c$ of the battery. Report the calculated values for r and $\mathcal{E}$, making sure to include appropriate units for both quantities.

   In your report, also include the following:

   - (1 point) Provide a brief explanation of how the equation $V_t = \mathcal{E} - Ir$ is represented graphically by your V_t vs. I plot, and specifically discuss how the slope of the best-fit line is related to the internal resistance of the battery.

- (1 point) If your linear regression analysis provides uncertainty values (standard errors) for the slope and y-intercept, report the internal resistance and EMF as $r \pm \Delta r$ and $\mathcal{E} \pm \Delta \mathcal{E}$, where Δr and $\Delta \mathcal{E}$ are the uncertainties. If uncertainty values are not directly available, qualitatively discuss the spread of your data points around the best-fit line as an indication of the uncertainty in your measurements.
- (1 point) Compare the EMF value ($\mathcal{E}$) that you obtained from the linear fit to the open-circuit voltage V_{oc} that you measured directly in Measurement 1. Discuss whether these two values are in reasonable agreement, and if there are differences, suggest possible reasons for these differences.

3. (2 points) Advantages of a linear regression analysis: In your report, briefly discuss the advantages of using the linear regression method with measurements from multiple different load resistors to determine the internal resistance of a battery, compared to a method that might rely on measurements with only a single load resistor.

Measurement 2 *Measurement 3: Investigation of resistors in parallel (using four resistors)* 7 points

In this measurement, you will experimentally investigate the relationships between voltage and current in a parallel circuit with four resistors (100 Ω, 220 Ω, 330 Ω, and 470 Ω).

Equipment: Digital Multimeter (DMM), 1.5V or 3.0V battery, 100 Ω, 220 Ω, 330 Ω, 470 Ω resistors, breadboard, connecting cables, cable clips.

Procedure:

1. (2 points) Set up parallel circuit and sketch diagram: Connect the four resistors (100 Ω, 220 Ω, 330 Ω, and 470 Ω) in parallel with each other. Then, connect this parallel combination to the battery to form a closed parallel circuit. In your lab report, sketch a clear circuit diagram of your parallel circuit. Label each resistor in your diagram as R_1, R_2, R_3, and R_4, and indicate their respective nominal resistance values.
2. (1 point) Measure individual parallel resistor voltages: Use the DMM as a voltmeter to measure the voltage drop across each resistor in the parallel combination. Record these voltage values in your report as $V_{R1,parallel}$, $V_{R2,parallel}$, $V_{R3,parallel}$, and $V_{R4,parallel}$.
3. (1 point) Measure total parallel voltage: Measure the voltage across the entire parallel combination of the four resistors. To do this, place the voltmeter probes across the points where the parallel combination connects to the battery. Record this voltage as $V_{parallel,total}$.
4. (1 point) Measure individual branch currents: Use the DMM as an ammeter to measure the current flowing through each branch of the parallel circuit. For each resistor branch (containing R_1, R_2, R_3, and R_4 respectively), temporarily disconnect that branch from the parallel combination and insert the ammeter in series within the branch to measure the current flowing through it. Record these measured branch currents as $I_{R1,parallel}$, $I_{R2,parallel}$, $I_{R3,parallel}$, and $I_{R4,parallel}$.

5. (1 point) Measure total parallel current: Measure the total current ($I_{parallel,total}$) being drawn from the battery into the parallel combination. To do this, insert the ammeter in series in the main branch of the circuit, between the battery and the point where the parallel combination begins. Record this total current.

6. (1 point) Measure equivalent parallel resistance: Now, disconnect the parallel combination of resistors from the battery, but do not disconnect the resistors from each other. Use the DMM in resistance measurement mode to measure the equivalent resistance of this parallel combination. To do this, connect the DMM probes across the points where the parallel resistor combination would connect to the battery. Record this measured equivalent parallel resistance as $R_{parallel,measured}$.

Analysis 2 *Analysis 3: Analysis of resistors in parallel (four resistors)* 5 points

In this analysis section, you will analyze the data you collected in Measurement 3 to verify the principles of current division in parallel circuits and to determine the equivalent resistance of parallel combinations.

1. (2 points) Verify current division and current summation: Verify the current division principle for parallel resistors. To do this, choose at least two different pairs of resistor branches in your parallel circuit (for example, the branches containing R_1 and R_3, and another pair like R_2 and R_4). For each chosen pair, calculate the ratio of their measured branch currents (e.g., $I_{R1,parallel}/I_{R3,parallel}$) and compare this ratio to the inverse ratio of their nominal resistances (R_3/R_1). Discuss in your report whether your measurements are consistent with the expected current division rule for parallel resistors. Also, verify whether the sum of the individual branch currents ($I_{R1,parallel} + I_{R2,parallel} + I_{R3,parallel} + I_{R4,parallel}$) is approximately equal to the total current entering the parallel combination ($I_{parallel,total}$). Calculate the percentage difference between the sum of branch currents and $I_{parallel,total}$.

2. (2 points) Compare theoretical and measured equivalent parallel resistance: Calculate the theoretical equivalent resistance ($R_{parallel,theoretical}$) of your parallel combination using the formula for resistors in parallel: $\frac{1}{R_{parallel,theoretical}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$. Use the nominal resistance values of R_1, R_2, R_3, and R_4 for this calculation. Compare the calculated theoretical equivalent resistance $R_{parallel,theoretical}$ to the equivalent resistance that you directly measured using the DMM, $R_{parallel,measured}$, in Measurement 3. Calculate the percentage difference between $R_{parallel,theoretical}$ and $R_{parallel,measured}$.

3. (1 point) Explain voltage consistency in parallel circuits: Based on your measurements in Measurement 3, explain in your report how your experimental results demonstrate that the voltage is the same across all components (resistors) in a parallel circuit.

Measurement 3 *Measurement 4: Verification of Kirchhoff's rules in a two-loop circuit (mixed circuit configuration)* 9 points

In this measurement, you will construct a two-loop circuit with a mixed configuration of components and then experimentally verify Kirchhoff's Rules by measuring currents and voltages in the circuit.

Equipment: Digital Multimeter (DMM), 1.5V battery (label as $\mathcal{E}_1$), 3.0V battery (label as $\mathcal{E}_2$), 100 Ω resistor (label as R_1), 220 Ω resistor (label as R_2), 330 Ω resistor (label as R_{shared}), connecting cables, breadboard, cable clips.

Procedure:

1. (4 points) Construct the two-loop circuit and sketch diagram: Construct the two-loop circuit as described below. Loop 1 should consist of the 1.5V battery ($\mathcal{E}_1$), resistor R_1 (100 Ω), and resistor R_{shared} (330 Ω) connected in series. Loop 2 should consist of the 3.0V battery ($\mathcal{E}_2$), resistor R_2 (220 Ω), and the same resistor R_{shared} (330 Ω) connected in series. The resistor R_{shared} is thus shared between loop 1 and loop 2. When connecting the batteries, ensure you are consistent with the battery polarities. In your lab report, sketch a detailed diagram of this two-loop circuit. Clearly label all components in your diagram: label the resistors as $R_1 = 100\ \Omega$, $R_2 = 220\ \Omega$, and $R_{shared} = 330\ \Omega$; and label the batteries as $\mathcal{E}_1$ (1.5V) and $\mathcal{E}_2$ (3.0V). In your diagram, also assume and indicate directions for the three branch currents: I_1 (current through branch with $\mathcal{E}_1$ and R_1), I_2 (current through branch with $\mathcal{E}_2$ and R_2), and I_{shared} (current through the shared resistor R_{shared}). Finally, indicate the positive and negative terminals of each battery in your circuit diagram.

2. (2.5 points) Measure the branch currents: Use the DMM as an ammeter to carefully measure the current in each of the three branches of the two-loop circuit that you constructed. Specifically, measure and record the following:

   a) I_1: The current flowing through the branch containing resistor R_1 and battery $\mathcal{E}_1$. To measure this, you'll need to temporarily break this branch and insert the ammeter in series.

   b) I_2: The current flowing through the branch containing resistor R_2 and battery $\mathcal{E}_2$. Similarly, break this branch to insert the ammeter in series.

   c) I_{shared}: The current flowing through the shared resistor R_{shared}. Break the branch containing R_{shared} to measure this current.

   For each current measurement, ensure you are paying attention to the direction of current flow and the polarity of the ammeter connections. Record the sign of the current as well, and check if the measured current signs are consistent with the current directions you assumed and indicated in your circuit diagram. Record all measured branch currents (I_1, I_2, I_{shared}).

3. (2.5 points) Measure the voltage drops: Use the DMM as a voltmeter to measure the voltage drop across each resistor and across each battery in the two-loop circuit. Measure and record the following voltage drops:

   - V_{R1}: The voltage drop across resistor R_1.
   - V_{R2}: The voltage drop across resistor R_2.
   - $V_{R_{shared}}$: The voltage drop across the shared resistor R_{shared}.
   - $V_{\mathcal{E}1}$: The terminal voltage across battery $\mathcal{E}_1$.
   - $V_{\mathcal{E}2}$: The terminal voltage across battery $\mathcal{E}_2$.

When measuring voltage drops, ensure that you are consistently orienting your voltmeter probes to measure the voltage drop in a direction that is consistent with your assumed current directions (e.g., if you've assumed current flows from left to right through a resistor, measure the voltage with the positive voltmeter probe on the left side and the negative probe on the right side of the resistor). Record all measured voltage drops ($V_{R1}, V_{R2}, V_{R_{shared}}, V_{\mathcal{E}1}, V_{\mathcal{E}2}$).

Analysis 3 *Analysis 4: Verification of Kirchhoff's rules (mixed circuit)* 10 points

In this analysis section, you will analyze the current and voltage data you collected in Measurement 4 to experimentally verify Kirchhoff's Current Law (KCL) and Kirchhoff's Voltage Law (KVL) for the two-loop circuit.

1. (3 points) Set up Kirchhoff's Current Law (KCL) equations: First, identify and clearly label all junctions (nodes) in your two-loop circuit diagram. For each junction that you identify, write down the Kirchhoff's Current Law (KCL) equation. Remember that KCL states that the sum of currents entering any junction must equal the sum of currents leaving that junction. Use the assumed current directions you indicated in your circuit diagram to write down the KCL equation for each junction.

2. (2 points) Numerically verify KCL: Substitute the values of the measured currents (I_1, I_2, I_{shared}) from Measurement 4 into the KCL equations that you wrote down for each junction. Calculate the sum of currents at each junction. Ideally, for each junction, the sum should be very close to zero (or the currents entering should equal currents leaving). Calculate the percentage difference or fractional difference for the current sums at each junction to quantify how well your measurements satisfy KCL. In your report, discuss whether your measurements provide experimental support for Kirchhoff's Current Law.

3. (3 points) Set up Kirchhoff's Voltage Law (KVL) equations: Identify the independent loops in your two-loop circuit. For the circuit in Measurement 4, you can identify three loops: Loop 1 (containing $\mathcal{E}_1$, R_1, R_{shared}), Loop 2 (containing $\mathcal{E}_2$, R_2, R_{shared}), and the outer loop (containing $\mathcal{E}_1$, R_1, R_2, $\mathcal{E}_2$). Clearly indicate these loops and choose a direction of traversal (e.g., clockwise or counterclockwise) for each loop on your circuit diagram. For each loop, write down the Kirchhoff's Voltage Law (KVL) equation. Remember that KVL states that the sum of potential differences (voltages) around any closed loop in a circuit must be equal to zero. When writing the KVL equations, carefully consider whether each component in the loop represents a voltage rise or a voltage drop, depending on your chosen loop traversal direction and the assumed current directions.

4. (2 points) Numerically verify KVL: Substitute the measured voltage values ($V_{R1}, V_{R2}, V_{R_{shared}}, V_{\mathcal{E}1}, V_{\mathcal{E}2}$) from Measurement 4 into the KVL equations that you wrote down for each loop. Calculate the algebraic sum of the potential differences for each loop. Ideally, for each loop, this sum should be very close to zero. Discuss in your report how close your calculated sums are to zero for each loop, and whether your measurements provide experimental support for Kirchhoff's Voltage Law.

Learning outcomes

- Understand the concept of internal resistance of a battery and be able to measure it experimentally.
- Understand and experimentally verify the rules for equivalent resistance, voltage, and current in series and parallel resistor circuits.
- Understand and apply Kirchhoff's Current Law (KCL) and Kirchhoff's Voltage Law (KVL) to analyze DC circuits.
- Experimentally verify Kirchhoff's laws in a two-loop DC circuit using measurements of currents and voltages.
- Develop skills in using digital multimeters for voltage, current, and resistance measurements in DC circuits.
- Gain experience in circuit construction, data acquisition, analysis, and comparison of experimental results with theoretical predictions.

References

[1] D. Halliday, R. Resnick, and J. Walker. *Fundamentals of Physics*. Fundamentals of Physics. John Wiley & Sons.