E&M Laboratory 1106, Summer 2025

Prof. D. Balakishiyeva, Prof. R. Guarino

https://www.physics.smu.edu/tneumann/110X_Summer2025/

Lab 8 — Time-varying RL circuits and high-pass filter Max. points: 56.5

Your preparation: Work through before coming to the lab

- Prepare for the lab by thoroughly reading and understanding the measurement and analysis procedures on this worksheet. Photos of the equipment and further introductory material will be made available on https://www.physics.smu.edu/tneumann/110X_Summer2025/schedule-em/.
- Collect all your questions and ask your instructor at the beginning of the lab.
- Review the chapter on "Inductance and RL Circuits" in Halliday, Resnick, and Walker [1]. Focus especially on these topics: Understand the concept of inductance and inductors. Understand behavior of inductors in DC circuits and with changing currents. Understand charging and discharging of current in RL circuits with DC voltage sources. Understand the definition and significance of the RL time constant ($\tau = L/R$). Understand exponential current rise and decay in RL circuits. Understand frequency response of RL circuits and the concept of a high-pass filter using an RL circuit.

Pre-lab: Upload to Canvas before coming to the lab

A reminder: Upload your answers as a text document (exported as PDF) to Canvas before the lab begins (Canvas uploads are no longer possible 30 minutes before the lab starts!).

Pre-lab 1 6.5 points

1. RL circuit time domain behavior:

   (a) (1 point) Sketch a simple circuit diagram for an RL circuit connected to a DC voltage source through a switch, to demonstrate current rise and decay in the inductor. Label Resistor (R), Inductor (L), Voltage Source (V), and switch.

   (b) (1.5 points) Imagine you build the Measurement 1 RL circuit with a specific resistor R_1 and inductor L_1. You observe the current rise waveform on the oscilloscope when driven by a square wave. Now, you replace the resistor R_1 with a larger resistor $R_2 > R_1$, while keeping the inductor L_1 and the function generator settings unchanged. Sketch how you expect the current rise waveform to change (if at all) compared to the original circuit with R_1. Focus on changes in the initial rate of current rise, the final steady-state current value, and the time constant of the circuit. Explain your reasoning for each of these expected changes based on RL circuit theory.

(c) (0.5 points) Indicate the time constant $\tau = L/R$ on your sketch of the current rise curve.

2. RL high-pass filter — frequency response:

   (a) (1.5 points) In Measurement 2, you'll investigate the RL circuit as a high-pass filter, observing the voltage across the resistor. Suppose you wanted to design this RL high-pass filter to more effectively block low-frequency sine wave signals from reaching the resistor, while still effectively passing high-frequency signals to the resistor. Based on your understanding of RL circuit resistance and frequency response, would you recommend increasing or decreasing the resistance R (while keeping the inductance L constant) to achieve this goal? Explain your reasoning in terms of how changing R affects the filter's frequency response characteristics.

   (b) (1 point) Qualitatively, what do you expect to happen to the voltage across the resistor as you increase the frequency of the sine wave input? Will it increase, decrease, or stay roughly the same? Explain your reasoning in terms of inductor resistance and how it changes with frequency.

   (c) (1 point) Consider the practical aspect of building the RL circuit. When connecting the oscilloscope probe to measure the voltage across the resistor in Measurement 2, should you connect the probe directly across the resistor terminals using short wires, or is it acceptable to use long oscilloscope probe cables? Think about the potential effect of cable inductance and capacitance, especially at higher frequencies, on your voltage measurements. Briefly explain your consideration and recommendation for probe connection.

Lab measurements and report: submission by end of class

A reminder: All measurements must be fully documented . The final report must be uploaded to Canvas *by the end of the class* exported as PDF with plots and tables from Excel or Capstone embedded as images. Canvas will stop accepting uploads 10 minutes after the class ends. If you have not fully completed your report, you must upload the documents as far as you have completed them for grading.

In this lab, you will investigate time-varying RL circuits and their frequency-dependent behavior, learn about high-pass filters and continue practice with function generators and oscilloscopes.

Measurement 1 *Measurement 1: Observing RL transient response with a square wave* 13 points

In this measurement, you will build a series RL circuit and drive it with a square wave from a function generator to observe the transient response using two different oscilloscope connection methods.

Equipment: Resistor ($1\,\mathrm{k}\Omega$), Inductor ($6.8\,\mathrm{mH}$), Function Generator, Oscilloscope (preferably dual-channel), (small sensing resistor ($10\,\Omega$)), Connecting wires and breadboard.

Setup:

1. Construct a series RL circuit with the resistor and inductor. Connect the function generator output as the input voltage to your circuit.

   Via placing a small sensing resistor in series in your circuit you would measure the current indirectly via Ohm's law. In this lab you do not need to include the small sensing resistor, see the expert note below.

2. Set the function generator to output a square wave. Set the frequency to e.g., $100\,\mathrm{Hz}$. Set the Amplitude to $5\,\mathrm{V}$ peak-to-peak.

3. (3 points) In your report, include a clear hand-drawn (or digitally created) circuit schematic of your complete setup, including the function generator, resistor, inductor, sensing resistor, and oscilloscope connections (showing which channels are connected where).

Measuring inductor voltage (1a):

4. Connect channel 1 of the oscilloscope to measure the voltage across the inductor. Connect channel 2 to measure the input voltage from the function generator. Turn on the function generator and the oscilloscope. Adjust the Volts/division and Time/division knobs for a clear waveform display showing inductor voltage transient. Adjust the trigger settings for a stable waveform.

5. (6 points) Save and include a screenshot showing the inductor voltage transient waveform. Clearly record and report the Time/division and Volts/division settings used for each channel. Export your data as a CSV file to process on your computer.

Expert note on measuring current indirectly. In an earlier version of this lab (method 1b), we explored measuring the current indirectly by using the voltage across the sensing resistor. While this approach is conceptually sound in an *ideal* scenario, real-world components introduce complications.

Specifically, real inductors possess an internal Ohmic resistance, often referred to as DC resistance (DCR). The inductors used in this lab have a non-negligible DCR. To observe the *ideal* exponential rise and fall of current (and therefore, the resistor voltage), the external resistance, R, would need to be significantly *larger* than the inductor's internal resistance.

However, this creates a new problem: To make R much larger than the inductor's DCR, we'd need to choose a very large value for R. This, in turn, leads to very small currents flowing through the circuit.

With these extremely small currents, the circuit becomes highly sensitive to other non-ideal characteristics that are usually negligible. A primary concern is *parasitic capacitance*. Every component and wire in the circuit, including the oscilloscope probe, has a small amount of capacitance. At high resistances and low currents, these tiny capacitances can significantly impact the circuit's behavior.

You might observe this on the oscilloscope. If you connect channel 1 to measure the voltage across the resistor, you might see a very small voltage signal (representing the small current) and potentially oscillatory behavior. These oscillations are a consequence of the interaction between the inductance, the large resistance, and the unavoidable parasitic capacitances, forming an underdamped RLC circuit.

Expert note: why use a separate sensing resistor? You might be wondering: Why would we use a separate, small-value "sensing resistor" to measure current? Since the sensing resistor is in series with the main resistor R, wouldn't their resistances just add up? Couldn't we simply measure the voltage across the existing resistor R using Ohm's Law ($V = I \cdot R$) and calculate the current that way?

In our lab setup, with the specific resistor values we're using, directly measuring across resistor R *might* give a reasonably close result. However, in general, and for accurate measurements in most circuits, this is *not* the best approach. Here's why:

Every measurement device (like a voltmeter or oscilloscope) has its own internal resistance (called "input impedance"). When you connect the device across a component to measure its voltage, you're actually placing that internal resistance in *parallel* with the component. This parallel combination *changes* the overall resistance of that part of the circuit. You're trying to observe the circuit's natural behavior, but your measurement tool is *becoming part of the circuit* and altering it.

- A sensing resistor is intentionally chosen to have a *very small* resistance (often just a few ohms). This is *much smaller* than the typical resistance values in the rest of the circuit.
- By inserting this tiny resistance, the voltage drop across it (which is proportional to the current) can be measured *without significantly changing the total resistance* of the circuit, and therefore, without significantly affecting the current we're trying to measure.
- Because the sensing resistor's value is precisely known and its impact is minimal, we get a much more accurate measurement of the *actual* current flowing in the circuit *before* we connected our measurement device.

While it might seem simpler to just measure the voltage across an existing resistor, using a separate, small sensing resistor is a crucial technique for accurate current measurement because it minimizes the disturbance to the circuit being measured. This is a fundamental principle in electronics: good measurement practices aim to observe a system without changing it.

(4 points) From your transient waveforms (inductor voltage), estimate the experimental time constant τ_{exp}. You can do this based on the screenshot, or, better, based on the exported CSV file on your computer. Determine and report the time for the waveform to reach approximately 63% of its maximum (rise) or fall to 37% (decay). Record your estimated τ_{exp} value(s) and clearly describe your estimation method in your report, indicating which waveform (inductor voltage or current) you used for each estimation.

Measurement 2 *Measurement 2: Exploring sine wave response of an RL circuit (high-pass behavior)* 12 points

You will investigate the frequency-dependent behavior of the RL circuit with a sine wave input, and explore its high-pass filter characteristics by observing how the voltage across the resistor changes with frequency.

Note: Measurement 2 should be conducted after you have fully completed Measurement 1. You will reuse the same RL circuit components from Measurement 1.

Measurement Procedure:

1. Function generator setup for sine wave input:
   1. Change the waveform output of the function generator from square wave to sine wave.
2. Varying frequency and observing resistor and inductor voltages:
   1. Start at low frequency: Set the function generator frequency to a low value: 10 Hz.
   2. Frequency stepping: Gradually increase the frequency of the sine wave in steps. Suggested frequency values for observation are: 20 Hz, 40 Hz, 100 Hz, 200 Hz, 500 Hz, 1 kHz, 2 kHz, 5 kHz, 10 kHz, 20 kHz, 50 kHz, 100 kHz and higher frequencies if changes are still observable.
   3. For each frequency step measure and observe the voltage waveform across the inductor. Take note of the amplitude of the inductor voltage waveform.

(4 points) Qualitative description: Carefully observe and describe in your report how the inductor voltage amplitude changes with frequency. Focus on qualitative descriptions such as ”increases,” ”decreases,” ”remains approximately constant,” etc.

For your report, under the section ”Measurement 2”, include the following:

- (4 points) Screenshots of the frequency response: Include two oscilloscope screenshots that effectively demonstrate the frequency response of the RL circuit as a high-pass filter:
  - One screenshot taken at a lower frequency from the range you tested.
  - One screenshot taken at a higher frequency from your tested range. Choose a higher frequency that clearly shows a noticeable change in the resistor voltage amplitude compared to the lower frequency screenshot, illustrating the high-pass filter behavior.
  - Clearly label each screenshot with the exact frequency value at which it was recorded. For each screenshot, ensure that both the sine wave input (channel 2) and the inductor voltage waveform (channel 1) are visible.
  - For each of the two screenshots you included, clearly report the oscilloscope settings used for both channel 1 and channel 2 (if used): Time/division setting, Volts/division setting
- (4 points) Create a table in your report that lists the measured peak-to-peak amplitude voltage across the inductor. Your table should have columns for: Frequency (in Hz or kHz), Peak-to-peak voltage across the inductor.

Analysis 1 *RL transient response* 12 points

This analysis section focuses on verifying the theoretical behavior of an RL circuit under transient conditions. You will calculate the theoretical time constant, sketch the expected waveforms, and compare these predictions to your experimental measurements from Measurement 1.

1. (2 points) Calculate the theoretical time constant (τ_{theory}): For the RL circuit you constructed in Measurement 1 (with $R = 1\,\mathrm{k\Omega}$ and $L = 6.8\,\mathrm{mH}$), calculate the theoretical time constant (τ_{theory}) using the formula $\tau_{theory} = L/R$. Report the value of τ_{theory} including units in μs.

2. For a square wave input voltage $V_{in}(t)$ applied to the series RL circuit, the theoretical expressions for the current $I(t)$ and inductor voltage $V_L(t)$ during the rising and falling portions of the square wave are given by:

   During the rising part of the square wave (0 to $T/2$, assuming period T):

   - Current: $I(t) = \frac{V_0}{R}(1 - e^{-t/\tau})$
   - Inductor Voltage: $V_L(t) = V_0 \cdot e^{-t/\tau}$

   During the falling part of the square wave ($T/2$ to T): Assuming the square wave goes from V_0 to 0 V)

   - Current: $I(t) = I_{max} \cdot e^{-(t-T/2)/\tau}$, where $I_{max} = \frac{V_0}{R}$ is the steady-state current reached during charging.
   - Inductor voltage: $V_L(t) = -V_0 \cdot e^{-(t-T/2)/\tau}$

   Here, V_0 is the amplitude of the square wave (5 V), $R = 1\,\mathrm{k\Omega}$, $L = 6.8\,\mathrm{mH}$, and $\tau = L/R$.

   Using these equations and the values of R, L, and V_0 from your Measurement 1 setup:

   (a) (3 points) Sketch the theoretical current waveform for one period of the square wave. Clearly label the axes (time and current) and indicate key values such as the maximum current, initial rate of rise, and time constant on your sketch.

   (b) (3 points) Sketch the theoretical inductor voltage waveform for one period of the square wave. Clearly label the axes (time and voltage) and indicate key values such as the peak voltages (positive and negative) and time constant on your sketch.

3. (4 points) Compare theoretical and experimental time constants: In the Measurement 1, you were asked to estimate the experimental time constant (τ_{exp}) from your oscilloscope waveforms. Compare your theoretically calculated time constant (τ_{theory}) with your estimated experimental time constants (τ_{exp}). Calculate the percentage difference between τ_{theory} and your two values of τ_{exp}. Discuss possible reasons for any discrepancies observed between the theoretical and experimental values.

Analysis 2 *Sine wave frequency response* 13 points

This analysis focuses on verifying the high-pass filter characteristics of the RL circuit. You will calculate the theoretical cutoff frequency, analyze the voltage transfer function, and compare these theoretical predictions to your experimental observations of the circuit's behavior at different frequencies.

1. (1 point) For the RL circuit used in Measurement 2, calculate the cutoff frequency (f_c) of the high-pass filter using the formula:

$$f_c = \frac{R}{2\pi L}$$

Include units, and report the cutoff frequency f_c in kilo Hertz (kHz). The cutoff frequency is the frequency where the output voltage is $1/\sqrt{2}$ the input voltage.

2. (6 points) Theoretical frequency response — voltage transfer function: The voltage transfer function for the RL high-pass filter, defined as the ratio of the output voltage across the inductor (V_I) to the input voltage (V_{in}) as a function of frequency (f), is given by:

$$|H(f)| = \left|\frac{V_I(f)}{V_{in}(f)}\right| = \left|\frac{L \cdot 2\pi i f}{R + L \cdot 2\pi i f}\right| = \frac{1}{\sqrt{1 + \left(\frac{f_c}{f}\right)^2}} .$$

   Using this voltage transfer function and the cutoff frequency f_c you calculated in step 1: Plot the theoretical frequency response curve (Bode magnitude plot) for this RL high-pass filter, that is, plot the frequency versus the magnitude of the transfer function $|H(f)|$ on a logarithmic scale for both axes. Indicate the cutoff frequency f_c and comment on the approximate behavior of the filter at frequencies much lower than f_c and much higher than f_c on your plot.

3. (6 points) In Measurement 2, you were asked to record the frequency dependence of the inductor voltage amplitude. Compare your experimental observations of how the voltage amplitude changed with frequency to the theoretical frequency response predicted by the transfer function in step 2 by plotting your measurement together with the theoretical predictions.

   Do your experimental findings agree with the theoretical high-pass filter behavior? Discuss any differences between your experimental results and the theoretical predictions. Consider factors that might contribute to any deviations between theory and experiment.

Learning outcomes

- Learn basic operation of function generators and oscilloscopes for circuit analysis.
- Generate square and sine wave signals using a function generator.
- Visualize and qualitatively measure time-varying voltages and transient responses using an oscilloscope.
- Observe RL circuit transient response with square wave input and relate to the RL time constant.
- Investigate the qualitative frequency response of an RL circuit to sine wave inputs and identify high-pass filter behavior.
- Compare and contrast the behavior of RC and RL circuits in both time and frequency domains.

References

[1] D. Halliday, R. Resnick, and J. Walker. *Fundamentals of Physics*. Fundamentals of Physics. John Wiley & Sons.