E&M Laboratory 1106, Summer 2025

Prof. D. Balakishiyeva, Prof. R. Guarino

https://www.physics.smu.edu/tneumann/110X_Summer2025/schedule-em/

Lab 9 — Helmholtz coils and earth's magnetic field Max. points: 51

Your preparation: Work through before coming to the lab

- Prepare for the lab by thoroughly reading and understanding the measurement and analysis procedures on this worksheet. Photos of the equipment and further introductory material will be made available on https://www.physics.smu.edu/tneumann/110X_Summer2025/schedule-em/.
- Collect all your questions and ask your instructor at the beginning of the lab.
- Study the following topics in Halliday, Resnick, and Walker [1]: Understand the concept of a magnetic field and how it exerts a force on moving charges and magnetic dipoles. Study the concept of a magnetic dipole moment. Understand how a magnetic field exerts a torque on a magnetic dipole. Review the characteristics of SHM, including the relationship between period, frequency, and angular frequency. Pay attention to the equation of motion for SHM. Learn about the construction and properties of Helmholtz coils, specifically their ability to create a nearly uniform magnetic field. Review the concept of moment of inertia and how to calculate it for simple shapes (in this case, a cylinder).

Pre-lab: Upload to Canvas before coming to the lab

A reminder: Upload your answers as a text document (exported as PDF) to Canvas before the lab begins (Canvas uploads are no longer possible 30 minutes before the lab starts!).

Pre-lab 1 — 8 points

1. (1 point) What is a Helmholtz coil, and what is its primary advantage compared to a single coil for generating a magnetic field?
2. (1 point) Write down the formula for the magnitude of the magnetic field (B) at the center of a Helmholtz coil in terms of the current (I), the number of turns per coil (N), the radius of the coils (R), and the permeability of free space (μ_0). You do not need to derive it, but you need to explain the meaning of each symbol.
3. (1 point) A cylindrical magnet with magnetic dipole moment μ is placed in a uniform magnetic field B. Write down the equation for the *torque* (τ) on the magnet when it is at an angle θ relative to the magnetic field. What is the direction of this torque?

4. (2 points) In this experiment, you will measure the frequency of oscillation (f) of the magnet in the combined magnetic field of the Helmholtz coils (B) and the Earth (B_E). You will then plot f^2 versus the current (I) in the coils. According to the provided lab description, what type of graph (linear, quadratic, inverse, etc.) do you expect to obtain, and why? What physical quantities are represented by the slope and y-intercept of this graph?

5. (1 point) Explain, in your own words, how the small angle approximation is used in deriving the equation of motion for the oscillating magnet. Why is this approximation necessary?

6. (2 points) Identify *two* potential sources of *systematic* error in this experiment (different from those mentioned in the introduction), and explain how each would affect your results. Be specific about *which measured or calculated quantity* would be affected and *how* (higher or lower than the true value).

Lab measurements and report: submission by end of class

A reminder: All measurements must be fully documented . The final report must be uploaded to Canvas *by the end of the class* exported as PDF with plots and tables from Excel or Capstone embedded as images. Canvas will stop accepting uploads 10 minutes after the class ends. If you have not fully completed your report, you must upload the documents as far as you have completed them for grading.

Analysis 1 *Theoretical background: Derivation of the oscillation frequency* 3 points

In this experiment, a cylindrical magnet is suspended in the combined magnetic field of the Helmholtz coils ($\vec{B}$) and the Earth ($\vec{B}_E$). When the magnet is slightly displaced from its equilibrium position (aligned with the net magnetic field), it experiences torque and oscillates. We will derive the relationship between the oscillation frequency and the magnetic fields.

1. **Torque on the magnet:** The torque (τ) on a magnetic dipole with moment μ in a magnetic field $\vec{B}_{net}$ is given by:
$$\vec{\tau} = \vec{\mu} \times \vec{B}_{net}$$
When the magnet is at an angle θ with respect to the magnetic field, the magnitude of the torque is:
$$\tau = \mu B_{net} \sin\theta$$
where B_{net} is the magnitude of the net magnetic field, which is the sum of the coil's field and Earth's field since they are aligned: $B_{net} = B + B_E$.

2. **Rotational analog of Newton's second law:** The rotational analog of Newton's Second Law states:
$$\tau = I_{magnet}\alpha = I_{magnet}\frac{d^2\theta}{dt^2}$$
where I_{magnet} is the moment of inertia of the magnet and $\alpha = \frac{d^2\theta}{dt^2}$ is the angular acceleration.

3. **Combining torque and Newton's law:** Combining the equations from steps 1 and 2, we get:

$$I_{magnet}\frac{d^2\theta}{dt^2} = -\mu(B + B_E)\sin\theta$$

   The negative sign indicates that the torque is a restoring torque, always acting to bring the magnet back to its equilibrium position.

4. **Small angle approximation:** For small angles ($\theta \leq 20°$, as specified in the lab), we can use the small angle approximation:

$$\sin\theta \approx \theta$$

   This simplifies the equation of motion to:

$$I_{magnet}\frac{d^2\theta}{dt^2} = -\mu(B + B_E)\theta$$

5. **Simple harmonic motion:** Rearranging the equation, we get:

$$\frac{d^2\theta}{dt^2} = -\frac{\mu(B + B_E)}{I_{magnet}}\theta$$

   This is the equation of motion for a simple harmonic oscillator, where the angular frequency squared is:

$$\omega^2 = \frac{\mu(B + B_E)}{I_{magnet}}$$

6. **Frequency and period:** The angular frequency (ω) is related to the frequency (f) by $\omega = 2\pi f$. Therefore:

$$f^2 = \frac{\mu(B + B_E)}{4\pi^2 I_{magnet}}$$

7. **Relating to coil current:** The magnetic field produced by the Helmholtz coils is proportional to the current (I): $B = C \cdot I$, where $C = \frac{8N\mu_0}{\sqrt{125}R}$. Substituting this into the equation for f^2, we get:

$$f^2 = \frac{\mu(C \cdot I + B_E)}{4\pi^2 I_{magnet}} = \frac{\mu C}{4\pi^2 I_{magnet}}I + \frac{\mu B_E}{4\pi^2 I_{magnet}}$$

8. (3 points) **Final result and linear relationship:** This final equation has the form of a straight line, $y = mx + b$, where:

   - $y = f^2$
   - $x = I$
   - slope $= m = \frac{\mu C}{4\pi^2 I_{magnet}}$
   - y-intercept $= b = \frac{\mu B_E}{4\pi^2 I_{magnet}}$

Thus, by plotting f^2 versus I and performing a linear regression, we can determine the slope and y-intercept, and from these, calculate the magnetic dipole moment (μ) of the magnet and the horizontal component of Earth's magnetic field (B_E). *Show in your report that you followed this derivation and understand the steps to get to the final result. Summarize the key assumptions and approximations used in this derivation.*

Measurement 1 *Helmholtz Coil and Magnet Measurements* 21 points

Equipment: Helmholtz coils, cylindrical magnet, thread, ruler, digital micrometer, triple beam balance, digital caliper, compass, DC power supply, spreadsheet software.

1. (4 points) **Coil radius measurements:**
   (a) Use a ruler to measure the *outer* radius of *each* coil. Make several measurements (at least 3) in different directions across each coil.
   (b) Record all your measurements in your spreadsheet.
   (c) Calculate and record the average outer radius for *each* coil separately.
   (d) Calculate the overall average outer radius by averaging the two coil averages. Report this value with an appropriate number of significant figures.
2. (5 points) **Inner radius calculation:**
   (a) Use the digital micrometer to measure the diameter of the copper wire used in the coils. Make several measurements (at least 3) at different points along the accessible wire. **Do not pull the wire out of the coils.**
   (b) Record all your wire diameter measurements in your spreadsheet.
   (c) Calculate the average wire diameter.
   (d) Calculate the thickness of the coil windings (5 layers times average wire diameter).
   (e) Calculate the *inner* radius of the coils by subtracting the coil winding thickness from the average *outer* radius. Report this value.
   (f) Calculate the average radius, R, by averaging the inner and outer radii and record this in your spreadsheet.
   (g) Explain, in your report, how the fact that the wires nestle between layers (rather than sitting directly on top of each other) affects your calculated value of the inner radius (and therefore, the average radius R). Does this systematic error cause your calculated R to be an overestimate or an underestimate of the true average radius?
3. (3 points) **Magnet measurements:**
   (a) Use the triple beam balance to measure the mass (m) of the cylindrical magnet.
   (b) Use the digital caliper to measure the length (L) of the magnet.
   (c) Record the mass and length in your spreadsheet.

4. (2 points) **Setup and alignment:**
   (a) Suspend the magnet by thread so that it hangs horizontally and level in the central region of the Helmholtz coils. Adjust the thread length as needed.
   (b) Use the compass (held far away from metal tables and the coils themselves) to determine the direction of the magnetic north.
   (c) Connect the coils to the DC power supply. *Briefly* turn on the power supply (adjusting the voltage to maximum and current to around 0.30 A) and use the compass, *placed between the coils*, to determine the direction of the magnetic field produced by the coils.
   (d) Rotate the Helmholtz coil apparatus so that the magnetic field produced by the coils (when energized) will be aligned *with* Earth's magnetic field (i.e., both point North). In your report, describe how you ensured this alignment.
5. (7 points) **Oscillation frequency measurements:**
   (a) Set the power supply current to $I = 0.20$ A. Wait about 30 seconds for the coils to reach thermal equilibrium. Readjust the current if necessary to maintain 0.20 A.
   (b) Gently displace the magnet by a small angle (no more than 20 degrees) from its equilibrium position and release it, causing it to oscillate.
   (c) Time 20 complete oscillations of the magnet. Record the total time in your spreadsheet.
   (d) Calculate the period (T) of one oscillation by dividing the total time by 20.
   (e) Calculate the frequency (f) of oscillation using the formula $f = 1/T$.
   (f) Repeat steps (a)-(e) for coil currents of $I = 0.30$ A, 0.40 A, 0.50 A, 0.60 A, 0.70 A, 0.80 A, 0.90 A, 1.00 A, 1.10 A, and 1.20 A. Record all data in your spreadsheet. Ensure you maintain the current at the set value for each measurement.

Analysis 2 *Analysis of Helmholtz coil and Earth's magnetic field* 19 points

1. (4 points) **Data plotting and linear regression:**
   (a) In your spreadsheet, create a new column and calculate f^2 (frequency squared) for each value of the current, I.
   (b) Create a plot of f^2 (on the y-axis) versus I (on the x-axis).
   (c) Perform a linear regression (fit a straight line) to your data. Display the equation of the best-fit line and the R-squared value on the plot. Include a screenshot of this plot in your report.
2. (2 points) **Moment of inertia calculation:**
   a) Calculate the moment of inertia (I_{magnet}) of the cylindrical magnet using the formula $I_{magnet} = \frac{1}{12}mL^2$, where m is the mass and L is the length of the magnet.
3. (3 points) **Helmholtz coil field constant calculation:**

a) The magnetic field at the center of the Helmholtz coils is given by $B = C \cdot I$, where I is the coil current. Calculate the value of the proportionality constant C using the formula $C = \frac{8N\mu_0}{\sqrt{125}R}$, where $N = 60$ (turns per coil), $\mu_0 = 4\pi \times 10^{-7}$ H/m, and R is the average radius of the coils (from Measurement step 2). Show your calculation with units, including your determined value for R.

4. (3 points) **Magnetic dipole moment calculation:**
   (a) From the equation of the best-fit line on your f^2 vs. I plot, obtain the numerical value of the *slope*.
   (b) Calculate the magnetic dipole moment (μ) of the magnet using the formula derived in the introduction: $\mu =$ slope $\cdot\, 4\pi^2 I_{magnet}\sqrt{125}R/(8N\mu_0) =$ slope $\cdot\, 4\pi^2 I_{magnet}/C$. Use your calculated values for I_{magnet} and C. Show your calculation.

5. (3 points) **Earth's magnetic field calculation:**
   (a) From the equation of the best-fit line on your f^2 vs. I plot, obtain the numerical value of the *y-intercept*.
   (b) Calculate the horizontal component of Earth's magnetic field (B_E) using the formula derived in the introduction: $B_E =$ y-intercept $\cdot\, 4\pi^2 I_{magnet}/\mu$. Use your calculated values for I_{magnet} and μ. Show your calculation.

6. (4 points) **Discussion:**
   (a) Why was it desirable to limit the coil current to 1.2 A? Suggest at least *two* reasons.
   (b) Identify *two* sources of *random* uncertainty in this experiment (different from those in the prelab).
   (c) How would your plot of f^2 vs. I have differed if Earth's magnetic field ($\vec{B}_E$) and the applied magnetic field ($\vec{B}$) had been in *opposite* directions, rather than aligned? Would the slope have changed? Would the y-intercept have changed? Explain your reasoning.

References

[1] D. Halliday, R. Resnick, and J. Walker. *Fundamentals of Physics*. Fundamentals of Physics. John Wiley & Sons.