Mechanics Laboratory 1105, Summer 2025

Prof. D. Balakishiyeva, Prof. R. Guarino

https://www.physics.smu.edu/tneumann/110X_Summer2025/

Lab 1 — Measurement uncertainties Max. points: 56.5

Your preparation: Work through before coming to the lab

- Your need to thoroughly prepare for the lab by reading and understanding the measurement and analysis procedures on the worksheet! Photos of the equipment and further introductory material are available on https://www.physics.smu.edu/tneumann/110X_Summer2025/schedule-mechanics/.
- Collect all your questions and ask your instructor at the beginning of the lab.
- Read chapters 1-1 through 1-7 in Halliday, Resnick, and Walker [1]: Measuring things, International System of Units, Changing Units, Length, Time, Mass.
- Read about uncertainty/error analysis, the difference between random (statistical) and systematic uncertainties. Recommended resources: Wikipedia [2, 3, 4], Hughes and Hase [5], Taylor [6].

Pre-lab: Upload to Canvas before coming to the lab

A reminder: Upload your answers as a text document (exported as PDF) to Canvas before the lab begins (Canvas uploads are no longer possible 60 minutes before the lab starts!).

Pre-lab 1 9 points

1. (1 point) In your own words, explain the difference between accuracy and precision.
2. (1 point) In your own words, explain the difference between random and systematic uncertainties.
3. (1 point) In your own words, explain the difference between a measurement uncertainty and a mistake/error caused by the experimenter.
4. (1 point) In your own words, explain the role of significant figures/digits in specifying a measurement value.
5. (1 point) We want to measure the density $\rho(m, l)$ of a cube based on its length $l = 10.4 \pm 0.1$ cm and mass $m = 1042 \pm 1$ g. What is the density in $\mathrm{kg/m^3}$ and its uncertainty? Pay attention to the units, and comment on the correct number of significant figures.
6. (2 points) In your own words, explain the difference between standard deviation of a single measurement and the standard deviation of the mean.

7. (1 point) Write down the formula to compute the volume of a cylinder in terms of height h and radius R.

8. (1 point) Consider again the cylinder. If you cut out a cylindrical hole of radius $r < R$, what is the volume of that object? In the lab we will use this formula for the volume of the metal washer, see below and https://www.physics.smu.edu/tneumann/110X_Summer2025/intro/1105/worksheet1/.

Lab measurements and report: submission by end of class

All measurements and steps must be followed in order and must be fully documented (using Excel spreadsheets for tables and plots and a text document for the text answers). The final report must be uploaded to Canvas *by the end of the class* (Canvas will stop accepting uploads 10 minutes after the class ends). If you have not fully completed your report, you must upload the documents as far as you have completed them for grading.

The experiments are performed in groups of three, and you should discuss the experiments and questions within your group. Nevertheless, the report submissions are individual, and your submitted answers should be in your own words and not exactly copied from your group members.

Measurement 1 *Measurements of dimensions and volume* 10 points

We begin by measuring the volume of some objects, learning about associated uncertainties and the propagation of uncertainties. Use Excel to keep track of the measurement data and uncertainties.

1. (3 points) Use the *transparent plastic ruler* and measure the three spatial dimensions of the provided rectangular block, and derive from that the volume. One source of uncertainty is due to the limited precision of the ruler. The Excel spreadsheet should contain columns for each spatial dimension measurement (x,y,z), and each dimension's uncertainty $(\Delta x, \Delta y, \Delta z)$, as well as derived volume V and derived uncertainty ΔV. Pay attention to the units and significant digits. The columns in Excel must have a header row that labels the measurement variable and the correct units.

2. (1 point) a) How precisely can you measure the length with the plastic ruler? This should match the uncertainty you have used in the Excel spreadsheet. b) How many measurements do you need to take to measure the length of something with it? If this is more than one, describe them and describe the possible impact on the measurement uncertainty.

3. (4 points) Perform the same volume measurement as in (1.) for the metal washer with a hole in the center, that is, set up a spreadsheet with columns for all the necessary measured input dimensions and their uncertainties, as well as a derived volume and derived volume uncertainty.

   Hint: The volume of a full cylinder is $V = \pi r^2 \cdot h$ for a cylinder with radius r and height h.

4. (2 points) Name two other sources of uncertainty in addition to your ability to read off the

length with a certain precision. For each of them argue why they are systematic or random/statistical and roughly estimate their size, argue if it can be neglected.

Measurement 2 *Calibration of the ruler and correction of volume measurement* 14 points

We will now check the calibration of the transparent plastic ruler. Ask your instructor for a *wooden ruler*.

1. (3 points) Using the wooden ruler, measure the distances between each of the following marks on the transparent plastic ruler: $0\,\mathrm{cm}$, $5\,\mathrm{cm}$, $10\,\mathrm{cm}$, $15\,\mathrm{cm}$, $20\,\mathrm{cm}$. That is, measure the distance between the $0\,\mathrm{cm}$ mark and the $5\,\mathrm{cm}$ mark, the distance between the $5\,\mathrm{cm}$ mark and the $10\,\mathrm{cm}$ mark, etc. Fill out an Excel spreadsheet with columns for the length on the plastic ruler (5 cm), another column for its uncertainty, as well as columns for your wooden ruler measurement and its uncertainty. Do the numbers agree within uncertainties? If not, explain why.
2. (0.5 points) If you find a discrepancy between wooden and plastic ruler beyond their uncertainties, argue which ruler, if any of them, gives the correct measurement.
3. (1.5 points) For each length measurement between $0\,\mathrm{cm}$, $5\,\mathrm{cm}$, $10\,\mathrm{cm}$, $15\,\mathrm{cm}$, $20\,\mathrm{cm}$, compute the difference between plastic and wooden ruler, and the propagated uncertainty of the difference. Obtain one systematic calibration length by averaging over these five measurements. Assume an uncertainty of that calibration length equal to the systematic uncertainty of one measurement.
4. (2.5 points) Now, assuming that the wooden ruler gives the correct measurement, correct your volume measurements of just the rectangular block in the first measurement for the systematic calibration length you have found and its uncertainty.

   Do this as follows. Copy your spreadsheet from measurement 1.1 and add two more columns for the systematic correction (the averaged difference between plastic ruler and wooden ruler) and its uncertainty. Add new columns for the *corrected* length measurements (x_{corr}, y_{corr}, z_{corr} and their uncertainties (Δx_{corr}, Δy_{corr}, Δz_{corr}). Compute these corrected lengths from the original measurements and adding/subtracting the systematic correction. Remember to do this in in a separate (copied) spreadsheet so that the first measurement can still be followed.
5. (2 points) Compute the corrected volume V_{corr} and its uncertainty ΔV_{corr}.
6. (2.5 points) Repeat the correction procedure for the metal washer with the hole, obtaining corrected radius and height.
7. (2 points) Compute the corrected volume and its uncertainty of the metal washer with the hole.

Measurement 3 *Measurements of mass and density* 11.5 points

We now determine the density of the rectangular block and the washer with hole based on the calibrated/corrected volume measurement.

1. (3 points) Use both the triple-beam balance and the digital scale to weigh each of the objects from the first measurement. Add columns to your Excel spreadsheet for the mass and its

uncertainty measured by the triple-beam balance and the digital scale. Briefly, in one or two sentences, justify the measurement uncertainty for balance and digital scale.

2. (3 points) Using your most precise mass and volume measurements, derive the density of each of the objects in Excel, and propagate the mass and volume uncertainties to the density uncertainty.

3. (2 points) Research a table of common metal densities and estimate each block's material. Justify this based on your measurement, its uncertainty and other properties, for example color.

4. (3 points) For both the rectangular block and the washer with hole: Use a combination of multiple of the calibration weights to create a weight close to the mass of your measured objects. For example, if you measured your object's mass to be around $213\,\mathrm{g}$, add the calibration weights of $200\,\mathrm{g}$ and $10\,\mathrm{g}$ to create a calibration weight of $210\,\mathrm{g}$.

   Use the combined calibration weights to measure their (combined!) mass with the triple-beam balance and the digital scale. Record the (combined!) calibration weight's stated nominal pre-calibrated mass, your mass measurement of the calibration weight and its measurement uncertainty. Do your measurements agree with the pre-calibrated mass within uncertainties?

5. (0.5 points) Whether or not your measurements agree with the pre-calibrated mass within uncertainties, let's imagine that it does not. In that case, what would that mean for your measurement?

Measurement 4 *Random uncertainties in length measurement* 12 points

So far we were only had to deal with systematic uncertainties from reading off a length from a ruler. We will now experience a significant source of random uncertainties.

1. (6 points) Using a piece of cord, determine the length of the outline of the tree in fig. 1. Each person in your group must repeat the measurement independently at least one time and estimate the *systematic uncertainty* from measuring the cord's length. Take note of the measurements and their uncertainties in an Excel spreadsheet. Each of you should copy the data from the others, so that all of you have all measurement data and the uncertainties.

2. (3 points) While reading off the cord's length is a systematic uncertainty, the length will differ depending on who aligned it with the outlined fir in a certain way. This is an additional random/statistical uncertainty that you determine using the *standard deviation of the mean.* For that compute the mean, and the standard deviation of the mean as its uncertainty (additional columns in your Excel spreadsheet).

3. (2 points) State your final best length estimate and briefly describe how you obtained it. Quote both the systematic uncertainty and the statistical/random uncertainty. The statistical uncertainty is related to the randomness in different people's measurement process of aligning the cord with the tree. Is this error larger or smaller than the systematic uncertainty in measuring the length of the cord?

4. (1 point) To improve the measurement, do you need to reduce the systematic or statistical uncertainty? Suggest a way to do this.

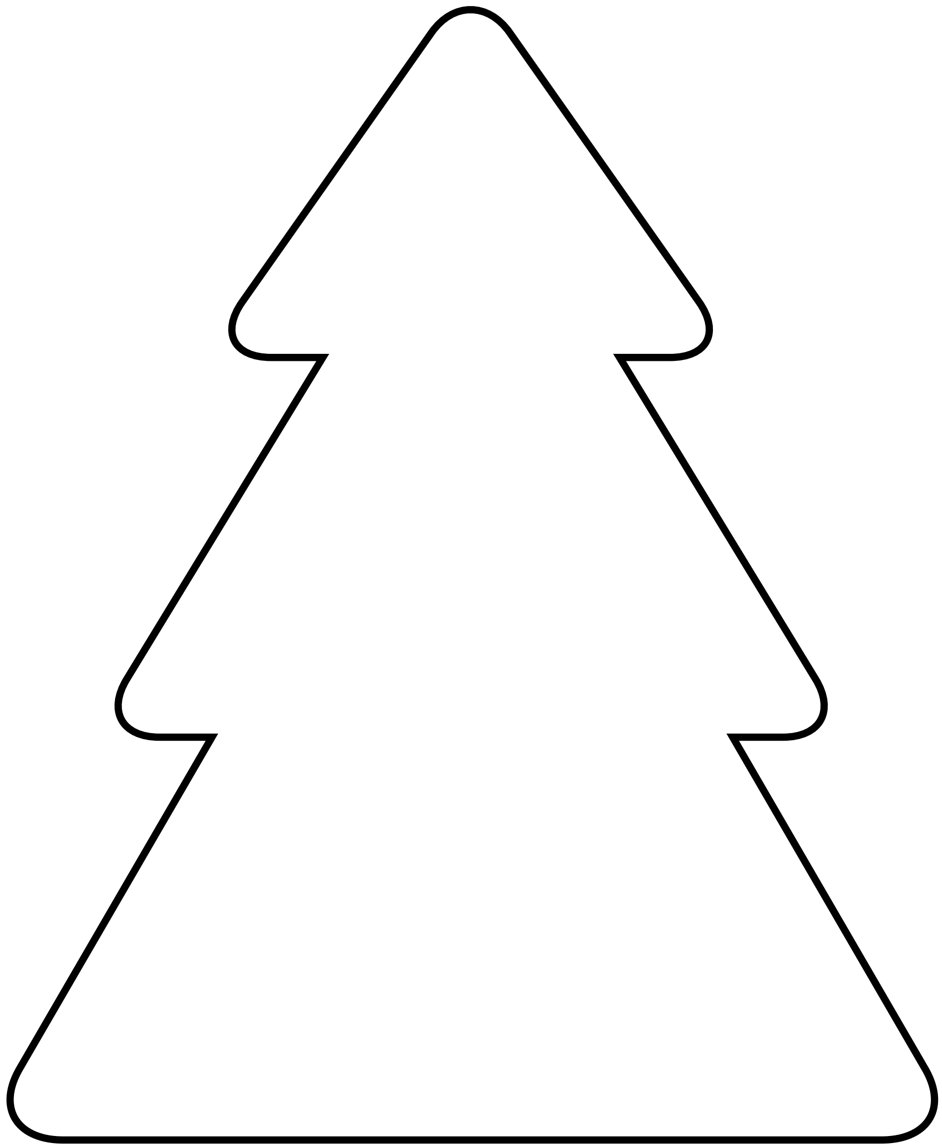

Figure 1: Outline of a stylized fir.

References

[1] D. Halliday, R. Resnick, and J. Walker. *Fundamentals of Physics*. Fundamentals of Physics. John Wiley & Sons.

[2] URL: https://en.wikipedia.org/wiki/Observational_error.

[3] URL: https://en.wikipedia.org/wiki/Accuracy_and_precision.

[4] URL: https://en.wikipedia.org/wiki/Significant_figures.

[5] Ifan Hughes and Thomas Hase. *Measurements and their Uncertainties. A practical guide to modern error analysis*. Oxford University Press, 2010.

[6] John R. Taylor. *An Introduction to Error Analysis*. University Science Books, 1996.