Mechanics Laboratory 1105, Summer 2025

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https://www.physics.smu.edu/tneumann/110X_Summer2025/

Lab 2 — Kinematics: Time derivatives of position Max. points: 50

Your preparation: Work through before coming to the lab

- Your need to thoroughly prepare for the lab by reading and understanding the measurement and analysis procedures on the worksheet! Photos of the equipment and further introductory material are available on https://www.physics.smu.edu/tneumann/110X_Summer2025/schedule-mechanics/.
- Collect all your questions and ask your instructor at the beginning of the lab.
- Read through chapter 2 (Motion along a straight line) Halliday, Resnick, and Walker [1] to understand the topics of motion, position and displacement, velocity and speed and acceleration. Learn about higher-order time derivatives of the position [2]. Review chapter 3 Halliday, Resnick, and Walker [1] about vectors.
- Learn about the polar coordinate system (two dimensions) and the spherical coordinate system (three dimensions), e.g. ref. [3].

Pre-lab: Upload to Canvas before coming to the lab

A reminder: Upload your answers as a text document (exported as PDF) to Canvas before the lab begins (Canvas uploads are no longer possible 60 minutes before the lab starts!).

Pre-lab 1 6 points

1. (1 point) Given a point $P = (x, y, z) = (\sqrt{4}\,\text{cm}, \sqrt{4}\,\text{cm}, \sqrt{8}\,\text{cm}) \equiv (\sqrt{4}, \sqrt{4}, \sqrt{8})\,\text{cm}$ in a cartesian coordinate system, determine the distance of the point P to the origin $(0, 0, 0)$ in cm. Write down the exact, non-rounded answer.
2. (1 point) Look up the transformation equation for obtaining a coordinate triplet (r, θ, φ) in spherical coordinates from cartesian coordinates (x, y, z). Then determine the value of r for the point P.
3. (1 point) Determine the polar angle θ and azimuthal angle φ for the point P in a spherical coordinate system in *radians*. What are the angles expressed in degrees?
4. (1 point) Confirm analytically that for your obtained values of $r, \theta\,\varphi$ the following equations hold: $x = r \sin\theta \cos\varphi$, $y = r \sin\theta \sin\varphi$, and $z = r \cos\theta$. Show your steps and do not round any numbers. Hint: $\sin(\pi/4) = \cos(\pi/4) = \sqrt{2}/2$.

5. (1 point) In your own words, what is the difference between speed and velocity in physics?
6. (1 point) Jerk is the time derivative of acceleration. "Engineers who design a train are typically required to keep the jerk less than $2\,\mathrm{m/s^3}$ for passenger comfort. The aerospace industry even has a jerkmeter: an instrument for measuring jerk." [4, 2]. Assume that you accelerate your car with an acceleration of $a(t) = 4\,\mathrm{m/s^2}$. What is the value of the jerk?

Lab measurements and report: submission by end of class

A reminder: All measurements and steps must be followed in order and must be fully documented (using Excel spreadsheets for tables and plots and a text document for the text answers). The final report must be uploaded to Canvas *by the end of the class* (Canvas will stop accepting uploads 10 minutes after the class ends). If you have not fully completed your report, you must upload the documents as far as you have completed them for grading.

Measurement 1 *Measurement of position and acceleration of a moving cart* 20 points

We use a "smart" cart with builtin position and acceleration sensors to compare measured acceleration with acceleration computed using finite difference formulas. The smart cart can be connected wirelessly via Bluetooth to your computer using the Capstone software.

Raise the end of the cart's track to $10\,\mathrm{cm}$, and take position, velocity and acceleration data simultaneously using Capstone while releasing the cart from the top end. Export as CSV to import into Excel. Only work with the data that is taken until the cart hits the bumper at the end of the track for the first time. Remember to not push the cart to not influence the measurement, but to just let it go.

Follow your instructor and pick sampling frequency larger than $100\,\mathrm{Hz}$. The higher the frequency, the more measurement points are taken per second.

- (6 points) Using Excel, generate a plot for the position as a function of time. Remember to select the relevant data section and label axes.
- (4 points) Similarly, using Excel, generate a plot for the velocity as a function of time using the velocity data provided by the smart cart.
- (4 points) Finally, using Excel, generate a plot for the acceleration as a function of time using the acceleration data as measured by the smart cart.
- (2 points) Give a list of uncertainties of the position measurement, and estimate the size of those. (Hint: Check the smart cart specification sheet [5].)
- (2 points) The smart cart will travel a certain distance in a certain amount of time (velcocity). Based on your set sampling frequency, compute how much distance it travels between two data points. The sampling frequency is determines how often the sensor transmits a measurement per second. Ask your instructor about this.
- (2 points) In your measurement, how does the traveled distance between two data points compare with the position measurement uncertainties? Does your result motivate to use a smaller

or larger sampling frequency?

Analysis 1 *Comparison with theoretical predictions for velocity and acceleration* 24 points

We now compare the measured velocity and acceleration data with theoretical predictions. The Capstone smart cart has an optical sensor to keep track of the position with a resolution of $0.2\,\mathrm{mm}$, as well as an accelerometer to measure the acceleration with an accuracy of about $\pm 0.2\,\mathrm{m/s^2}$. Their maximum sampling frequencies are $500\,\mathrm{Hz}$.

As there is no direct measurement of the velocity, the smart cart will internally compute the velocity also using the derivative just as we will do.

- (6 points) Now, calculate the velocity from the position data using the finite difference formula below in eq. (1) and show the computed velocity in the same plot as the measured data.
- (6 points) For the velocity, additionally generate a plot that compares the measured velocity with the calculated velocity using the higher-order finite difference formulas in eq. (2) and eq. (3). This additional plot should have a curve for the measured velocity, and the result from the calculated velocity based on all three finite difference approximations.
- (4 points) Calculate the difference between the calculated velocity using eq. (3) and eq. (1). How large, in percent, is this difference in comparison to the calculated velocity using eq. (1)? Generate a plot of this difference relative to the calculated result using eq. (1) as a function of time. Discuss what kind of uncertainty this is. Explain why there is a difference.
- (4 points) Generate a plot of the measured acceleration over time, and include the calculated acceleration using the difference between the measured acceleration and the calculated acceleration using one of the finite difference approximations in eqs. (4) to (6).
- (4 points) Generate a plot of the cart's jolt over time. For that, use any of the finite difference formulas, either first or second order, on either the acceleration or the velocity, respectively. Argue why the cart's jolt should or shouldn't be consistent with zero. Explain if your plot confirms your hypothesis. Explain if and how the finite-difference equation chosen and its order might have an impact on the reliability of your statement.

Finite difference formulas (forward) for approximating the derivative from two points separated by a distance h [6] are given as follows, ordered by increasing accuracy. In our case we need to consider the time derivatives, so $x = t$, and h corresponds to the time difference between two measurements (the inverse sampling frequency). The formulas below therefore operate on measurements taken at successive time steps.

$$\frac{\partial f}{\partial x} \simeq \frac{f(x+h) - f(x)}{h} \tag{1}$$

$$\frac{\partial f}{\partial x} \simeq \frac{4f(x+h) - f(x+2h) - 3f(x)}{2h} \tag{2}$$

$$\frac{\partial f}{\partial x} \simeq \frac{-11f(x) + 18f(x+h) - 9f(x+2h) + 2f(x+3h)}{6h} \tag{3}$$

$$\frac{\partial^2 f}{\partial x^2} \simeq \frac{f(x) - 2f(x+h) + f(x+2h)}{h^2} \tag{4}$$

$$\frac{\partial^2 f}{\partial x^2} \simeq \frac{2f(x) - 5f(x+h) + 4f(x+2h) - f(x+3h)}{h^2} \tag{5}$$

$$\frac{\partial^2 f}{\partial x^2} \simeq \frac{35f(x) - 104f(x+h) + 114f(x+2h) - 56f(x+3h) + 11f(x+4h)}{12h^2} \tag{6}$$

References

[1] D. Halliday, R. Resnick, and J. Walker. *Fundamentals of Physics*. Fundamentals of Physics. John Wiley & Sons.

[2] URL: https://en.wikipedia.org/wiki/Fourth,_fifth,_and_sixth_derivatives_of_position.

[3] URL: https://en.wikipedia.org/wiki/Spherical_coordinate_system.

[4] URL: https://math.ucr.edu/home/baez/physics/General/jerk.html.

[5] URL: https://www.pasco.com/products/lab-apparatus/mechanics/carts-and-tracks/me-1240#specs-panel.

[6] URL: https://en.wikipedia.org/wiki/Finite_difference_coefficient.