Mechanics Laboratory 1105, Summer 2025

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https://www.physics.smu.edu/tneumann/110X_Summer2025/

Lab 3 — Free-body force and the equilibrium condition Max. points: 47

Your preparation: Work through before coming to the lab

- Your need to thoroughly prepare for the lab by reading and understanding the measurement and analysis procedures on the worksheet! Photos of the equipment and further introductory material are available on https://www.physics.smu.edu/tneumann/110X_Summer2025/schedule-mechanics/.
- Collect all your questions and ask your instructor at the beginning of the lab.
- Review the transformation of cartesian into spherical coordinates and the opposite transformation. Review chapter 5 "Force and motion" in Halliday, Resnick, and Walker [1] to understand Newton's second law, the topics of free-body diagrams, vectorial forces, condition of equilibrium.

Pre-lab: Upload to Canvas before coming to the lab

A reminder: Upload your answers as a text document (exported as PDF) to Canvas before the lab begins (Canvas uploads are no longer possible 60 minutes before the lab starts!).

Pre-lab 1 — 6 points

1. (2 points) Newton's Third Law is often summarized as "For every action, there is an equal and opposite reaction." Explain what this means in your own words. Give a simple, real-world example.
2. (1 point) If you push on a wall with a force of 20 Newtons, what is the force the wall exerts on you? In what direction does the wall's force act?
3. (1 point) A book rests on a table. Identify the action-reaction pair of forces involved. What object is exerting each force, and in what direction is each force acting?
4. (2 points) You are walking forward. What is the force that propels you forward? Identify the action-reaction pair involved in this motion.

Lab measurements and report: submission by end of class

A reminder: All measurements and steps must be fully documented (using Excel spreadsheets for tables and plots and a text document for the text answers). The final report must be uploaded as a PDF document to Canvas *by the end of the class* (Canvas will stop accepting uploads 10 minutes after the class ends). If you have not fully completed your report, you must upload the documents as far as you have completed them for grading.

Measurement 1 *Spherical coordinate system* 16 points

We use the eCube coordinate learning instrument and right-angle ruler to compare measurements in (r, θ, φ) coordinates with measurements in (x, y, z) coordinates.

1. (6 points) *Measure* $(r_i, \theta_i, \varphi_i)$ for the following cartesian reference coordinate points and include uncertainties. For simplicity we assume that there is only an uncertainty in reading off $(r_i, \theta_i, \varphi_i)$.
   a) $(x_1, y_1, z_1) = (10.0, 0.0, 0.0)$ cm
   b) $(x_2, y_2, z_2) = (5.0, 5.0, 0.0)$ cm
   c) $(x_3, y_3, z_3) = (8.0, 6.0, 0.0)$ cm
   d) $(x_4, y_4, z_4) = (7.1, 7.1, 0.0)$ cm
   e) $(x_5, y_5, z_5) = (3.0, 3.0, 5.0)$ cm
   f) $(x_6, y_6, z_6) = (-4.0, 3.0, 6.0)$ cm
2. (6 points) For each of these six points, calculate the $(r_i, \theta_i, \varphi_i)$ from the (x_i, y_i, z_i) using the cartesian-to-spherical coordinate conversion formulas. Comment if your measured coordinates agree with the theoretical predictions.
3. (2 points) Measure (x_i, y_i, z_i) for the following spherical reference coordinate points, include uncertainties.
   a) $(r_1, \theta_1, \varphi_1) = (10.0\,\text{cm}, 1.6\,\text{rad}, 0.8\,\text{rad})$
   b) $(r_2, \theta_2, \varphi_2) = (6.6\,\text{cm}, 0.7\,\text{rad}, 0.8\,\text{rad})$
4. (2 points) For each of these two points, calculate the (x_i, y_i, z_i) from the $(r_i, \theta_i, \varphi_i)$ using the spherical-to-cartesian coordinate conversion formulas. Comment if your measured coordinates agree with the theoretical predictions.

Measurement 2 *Force table* 25 points

We use the force table to study forces as vectors and the forces' equilibrium. The ring at the center of the table is pulled in different directions by tension forces through the strings. The forces in the strings are generated by weights that pull them down via pulleys clamped to the edge of the force table.

Before starting this experiment, use the level and adjusting screws to ensure that the top of the force table is fully horizontal.

1. (4 points) We first attach two forces to the ring, that is we use two strings. Place the tension forces 180 degrees apart from each other, that is opposite to each other. Take two weights of equal mass and confirm that the ring aligns with the table center.

   Draw a free-body diagram of the forces acting on the ring which we model as a point. Pay attention to the length of the two forces in the diagram and their direction, and label lengths (in N) and directions corresponding to the vectorial forces applied.

2. (2 points) With two forces applied (i.e. two strings attached to the ring), what other angular differences between the two strings will lead to an equilibrium of forces (apart from 180 degrees)?

3. (3 points) Now attach a third string (force) by draping a string attached to the ring over another pulley and attach a weight equal to each of the other two. State and argue about a possible set of angular differences between the three strings to reach a force equilibrium.

4. (4 points) Draw the free-body diagram including those three forces. Clearly label lengths of the force vectors and their directions as relative angles between each of the vectors.

   For example, the full circle would be divided by 360 degrees. Force 1 might have an angle with respect to force 2 of 45 degrees, while the angle with respect to force 3 might be 90 degrees. Consequently the angle between force 2 and 3 would be $360 - 90 - 45 = 225$ degrees. Add those three angles in your diagram using your values.

5. (2 points) Using polar coordinates and in the situation of three equal forces (masses), write down the set of equations to obtain the set of angular differences between string 1 and string 2 $\Delta\phi_1$, between string 2 and string 3 $\Delta\phi_2$, and $\Delta\phi_2$ between string 3 and string 1 in the situation of three equal forces (masses).

   Without loss of generality, assume that the first force is at a polar angle of 0 degrees, the second force at $\Delta\phi_1$, and the second force at $\Delta\phi_1 + \Delta\phi_2$. This will allow you to write down two equations for the two unknowns $\Delta\phi_1$ and $\Delta\phi_2$. You do not have to fully solve this system of equations.

6. (4 points) Attach masses of $m_1 = m_2 = 150\,\mathrm{g}$ to two of the strings, and $m_3 = 250\,\mathrm{g}$ to the third string. Note that the masses hang on metal hangers, and you will have to include the mass of the hanger in your overall masses m_1, m_2 and m_3. Position the strings (by changing the string/clamp angles) such that there is a equilibrium of forces that centers the ring. Record your angles and draw the free-body force diagram including vectorial lengths and relative angles.

7. (4 points) Denote the absolute angles between 0 and 360 degrees shown on the force table as ϕ_1, ϕ_2, ϕ_3, associated with the masses m_1, m_2 and m_3.

   Estimate the uncertainties in your angle measurements and record them. Estimate the uncertainties in the attached masses (use a scale to determine the masses and their uncertainties) and record then.

   Then calculate the following vectorial function f with your measured values and compute the uncertainty Δf via uncertainty propagation. Is $f \pm \Delta f$ consistent with zero?

$$f(m_1, m_2, m_3, \phi_1, \phi_2, \phi_3) = m_1 \begin{pmatrix} \cos \phi_1 \\ \sin \phi_1 \end{pmatrix} + m_2 \begin{pmatrix} \cos \phi_2 \\ \sin \phi_2 \end{pmatrix} + m_3 \begin{pmatrix} \cos \phi_2 \\ \sin \phi_2 \end{pmatrix} \tag{1}$$

8. (2 points) To test the equilibrium of forces with specific masses m_1, m_2 and m_3 attached to the strings, you propagated uncertainties from the masses of the weights and measured angles. List at least two additional sources of uncertainties and argue why they are systematic or random.

References

[1] D. Halliday, R. Resnick, and J. Walker. *Fundamentals of Physics*. Fundamentals of Physics. John Wiley & Sons.