Mechanics Laboratory 1105, Summer 2025

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https://www.physics.smu.edu/tneumann/110X_Summer2025/

Lab 6 — Projectile Motion Max. points: 64

Your preparation: Work through before coming to the lab

- Prepare for the lab by thoroughly reading and understanding the measurement and analysis procedures on this worksheet. Photos of the equipment and further introductory material are available on https://www.physics.smu.edu/tneumann/110X_Summer2025/schedule-mechanics/.
- Collect all your questions and ask your instructor at the beginning of the lab.
- Work through the chapter on "Motion in two and three dimensions" and "Projectile Motion" in Halliday, Resnick, and Walker [1]. Focus especially on these sections: "Projectile Motion": Understand the basic concepts of projectile motion, including independent treatment of horizontal and vertical motion, acceleration due to gravity, and the trajectory of a projectile. "Kinematic Equations": Review the kinematic equations for constant acceleration in two dimensions. Understand how to apply these equations to projectile motion, considering initial velocity, launch angle, and gravity. "Range and Maximum Height": Understand the derivation and application of formulas for the horizontal range and maximum height of a projectile, especially for launches on level ground and from a height. "Velocity and Acceleration Vectors": Understand how to decompose the velocity vector into horizontal and vertical components and how acceleration acts only in the vertical direction (neglecting air resistance).

Pre-lab: Upload to Canvas before coming to the lab

A reminder: Upload your answers as a text document (exported as PDF) to Canvas before the lab begins (Canvas uploads are no longer possible 60 minutes before the lab starts!).

Pre-lab 1 10 points

1. (2 points) In your own words, describe projectile motion. What are the key assumptions we make when analyzing projectile motion theoretically (in the absence of air resistance)?
2. (2 points) Write down the kinematic equations that describe the horizontal and vertical motion of a projectile launched with an initial velocity v_0 at an angle θ above the horizontal, from an initial height y_0. Define all variables and state which component of motion each equation describes.

3. (2 points) For a projectile launched horizontally from a height y_0 above the ground with an initial horizontal velocity v_0, derive the formula for the theoretical horizontal range (distance traveled before hitting the ground). Show your steps.

4. (2 points) For a projectile launched at an angle θ above the horizontal from a height $y_0 = 0$ (level ground), what launch angle theoretically maximizes the horizontal range? Briefly explain why. What happens to the optimal angle if the projectile is launched from a height $y_0 > 0$ above the ground?

5. (1 point) Explain how a dual beam photogate can be used to measure the initial velocity of a projectile. What quantity does the photogate directly measure, and how is velocity calculated from it?

6. (1 point) In this experiment, we will use a protractor on the projectile launcher to set the launch angle. What is a potential source of systematic error with a protractor, and how can we calibrate it to improve accuracy?

Lab measurements and report: submission by end of class

A reminder: All measurements must be fully documented . The final report must be uploaded to Canvas *by the end of the class* exported as PDF with plots and tables from Excel embedded as images. Canvas will stop accepting uploads 10 minutes after the class ends. If you have not fully completed your report, you must upload the documents as far as you have completed them for grading.

In this lab, you will experimentally investigate projectile motion by launching steel balls horizontally and at angles, measuring their initial velocity and range, and comparing your measurements to theoretical predictions.

Measurement 1 *Horizontal Launch: Range and Initial Velocity* In this first measurement, you will launch steel balls horizontally to determine their initial muzzle velocity and horizontal range. 11 points

Equipment: Projectile launcher with C-clamp and protractor, Steel balls, Dual beam photogate, Level, Plumb bob, Measuring tape or meter stick, Paper tape or multiple sheets of paper, Carbon paper, Masking tape, (Safety glasses)

1. (1 point point) Launcher setup and calibration for horizontal launch: Securely mount the projectile launcher to the table using the C-clamp and position it to launch across the floor.

   Calibrate the protractor using the level to find the true horizontal setting. Record the protractor angle reading for the horizontal launch. Keep the launcher at this calibrated (true) horizontal setting. Record the difference between nominal (protractor) angle and calibrated true horizontal 0 degrees angle as your calibration offset angle to correct any other nominal protractor angles.

2. Use the plumb bob to find the vertical point directly beneath the launcher muzzle on the floor. Mark this point on the floor paper as your reference origin.

3. Prepare landing area: Tape a long sheet of paper tape or paper to the floor, starting at the vertical reference point and extending outwards in the expected landing direction. Ensure the paper is flat and securely taped down. Place carbon paper (carbon side down) on top of the paper tape in the expected landing zone, but do *not* tape it in place.
4. (1 point) Measure and record the muzzle height (y_0), the vertical distance from the center of the launcher muzzle to the floor. Record the value in your Excel spreadsheet.
5. Position the dual beam photogate in front of the muzzle to measure the initial velocity.
6. (2 points) Muzzle velocity measurement (horizontal launch): Perform at least 5 horizontal successful test launches through the photogate that give a velocity reading. Record the initial velocity for each successful launch. Record each value in your Excel spreadsheet.
7. (1 point) Calculate the average initial muzzle velocity (v_0) from these measurements and the standard deviation of the mean (SDOM) in Excel and record these values. The SDOM is your uncertainty for the initial velocity $u(v_0)$.
8. (5 points) Horizontal range measurement: Perform 10 horizontal launches (using the same calibrated horizontal setting and launcher spring setting as for velocity measurement). Ensure the ball hits the carbon paper to mark the landing. Create a table in your Excel spreadsheet to record your measurements. For each landing point marked by the carbon paper, measure and record the horizontal range. Measure this distance on the floor, starting from your vertical reference origin point, along the *intended horizontal direction of launch* (which is the direction the launcher is aimed, extended along the floor), to the landing mark. Ensure you are measuring a straight, horizontal distance on the floor, aligned with the *launcher's intended direction*, not a slanted or angled distance. Record each range value in your Excel table.
9. (1 point) Calculate and record the average experimental horizontal range ($R_{exp,horizontal}$) and the SDOM in Excel and record these. The SDOM is your uncertainty for the horizontal range .

Analysis 1 *Analysis of Horizontal Launch* 5 points

In this analysis section, you will analyze the data from the horizontal launch experiment and compare it to theory. Refer to the data you recorded in your Excel table for Measurement 1, and the screenshot you included in your report.

1. (2 points) Theoretical horizontal range calculation and uncertainty: Using the average initial velocity ($v_0 \pm \Delta v_0$) and muzzle height (y_0), calculate the theoretical horizontal range ($R_{theory,horizontal}$) using the formula derived in the pre-lab: $R_{theory,horizontal} = v_0\sqrt{\frac{2y_0}{g}}$, where $g = 9.81\,\mathrm{m\,s^{-2}}$.

   Using error propagation, estimate the uncertainty in the theoretical horizontal range $\Delta R_{theory,horizontal}$ based on the uncertainty in the initial velocity Δv_0. Assume the uncertainty in muzzle height y_0 and g are negligible compared to Δv_0. Show your error propagation calculation. Record $\Delta R_{theory,horizontal}$.
2. (1 point points) Compare the experimental horizontal range $R_{exp,horizontal} \pm \Delta R_{exp,horizontal}$ (from Measurement 1) with the theoretical horizontal range $R_{theory,horizontal} \pm \Delta R_{theory,horizontal}$.

Do the experimental and theoretical ranges agree within their uncertainties? To check for agreement within uncertainties, see if the difference between the central values $|R_{exp,horizontal} - R_{theory,horizontal}|$ is less than or equal to the sum of their uncertainties $\Delta R_{exp,horizontal} + \Delta R_{theory,horizontal}$. State clearly whether they agree within uncertainties or not.

3. (2 points) Discuss the agreement (or disagreement) in the context of your calculated uncertainties and potential sources of systematic and random errors. Consider factors such as air resistance, measurement uncertainties, launcher inconsistencies, limitations of the theoretical model, and potential small vertical variations in the projectile launch trajectory. In your discussion, explicitly address whether the agreement (or disagreement) suggests that the theoretical model is a good representation of your horizontal launch experiment.

Measurement 2 *Angled Launches: Range vs. Launch Angle* In this second measurement, you will investigate how the horizontal range varies with the launch angle. 21 points

Choose calibrated launch angles to test: 35°, 40°, 45°, 50°, and 55° by taking into account the calibration offset.

1. Prepare angled launch: Follow the instructions of your intructor to turn the launcher towards the middle of the table. Ask your fellow students to remove any laptops or smartphones from the table as a precaution. Do not use the high launcher setting, but at most the medium launcher setting. Based on this, test where the ball will land, and tape paper to the target area and place carbon paper on it.

2. (1 point) For each target angle, set the launcher protractor to that value while taking into account the calibration offset. Record the true/calibrated angles and their uncertainties (protractor accuracy, e.g., $0.5°$ or $1°$ depending on the protractor resolution) in your Excel spreadsheet. For example, record $\theta = 35° \pm 0.5°$.

3. (10 points) Range and velocity measurement for each angle: For each calibrated launch angle (start with the lowest angle, e.g. 35°):

   Perform 5-10 launches at each angle, allowing the ball to land on the paper. Ensure the ball hits the carbon paper to mark landing points. Extend your Excel spreadsheet to include tables for angled launch measurements. For each landing point marked by the carbon paper, measure and record the horizontal range. Measure this distance on the table, starting from the muzzle origin point, along the *intended horizontal direction of launch* (which is the direction the launcher is aimed, extended along the table), to the landing mark. Ensure you are measuring a straight, horizontal distance on the table, aligned with the *launcher's intended direction*, not a slanted or angled distance. Record each range value in your Excel table for the corresponding angle.

   Calculate and record the average experimental horizontal range ($R_{exp,\theta}$). Calculate the average range and the SDOM in Excel and record these in your Excel table. The SDOM is your uncertainty for the horizontal range.

4. (10 points) Perform 5-10 velocity measurements at each launch angle using the photogate. Calculate and record the average initial velocity ($v_{0,\theta}$). Record each velocity value in your Excel spreadsheet. Calculate the average velocity and the SDOM in Excel and record these in

your Excel table. The SDOM is your uncertainty for the initial velocity at the corresponding angle.

Analysis 2 *Analysis of Angled Launches: Range vs. Angle* 17 points

In this analysis section, you will analyze the data from the angled launch experiment to investigate the relationship between launch angle and horizontal range and compare with theoretical predictions. Refer to the data you recorded for Measurement 2. For the theoretical calculations below, use the muzzle height y_0 you measured in Measurement 1, assuming it remains approximately constant for angled launches as well.

1. (4 points) Theoretical horizontal range calculation and uncertainty: For each true/calibrated launch angle $\theta \pm \Delta\theta$ tested (from your Excel table for Measurement 2), calculate the theoretical horizontal range ($R_{theory,\theta}$) using the formula for projectile motion from an initial height y_0 (measured in Measurement 1): $R_{theory,\theta} = \frac{v_{0,\theta}\cos\theta}{g}\left(v_{0,\theta}\sin\theta + \sqrt{(v_{0,\theta}\sin\theta)^2 + 2gy_0}\right)$.

   Use the average initial velocity $v_{0,\theta} \pm \Delta v_{0,\theta}$ measured at that angle (from your Excel table for Measurement 2). Use the muzzle height y_0 from Measurement 1.

2. (2 points) Error propagation for theoretical range uncertainty: For each angle, estimate the uncertainty in the theoretical horizontal range $\Delta R_{theory,\theta}$ using error propagation, based on the uncertainties in the initial velocity $\Delta v_{0,\theta}$ and launch angle $\Delta\theta$ and now also considering the uncertainty in the initial height Δy_0 (estimate based on your measurement precision). Assume the uncertainty in g is negligible. Record $R_{theory,\theta}$ and $\Delta R_{theory,\theta}$.

3. (2 points) Present a table showing the calibrated launch angles with uncertainties, measured average initial velocities with uncertainties, and the corresponding calculated theoretical horizontal ranges with propagated uncertainties. This table can be created in Excel and included as a screenshot in your report.

4. (2 points) For each launch angle, compare the experimental range $R_{exp,\theta} \pm \Delta R_{exp,\theta}$ (from your Excel table for Measurement 2) with the theoretical range $R_{theory,\theta} \pm \Delta R_{theory,\theta}$.

   For each angle, do the experimental and theoretical ranges agree within their uncertainties? Check for agreement within uncertainties as described in Analysis 1. State clearly for each angle whether they agree within uncertainties or not.

5. (4 points) Create a graph plotting both experimental average range ($R_{exp,\theta}$) and theoretical range ($R_{theory,\theta}$) as a function of the calibrated launch angle θ (all values with central values and uncertainties). For each data point, include error bars representing the calculated uncertainties. Plot both experimental and theoretical datasets on the same graph, with clear labels, axes, a legend, and appropriate data markers and lines with error bars. This graph should be created in Excel and included as a screenshot in your report.

6. (1 point) Describe the overall shape of both the experimental and theoretical curves and how the error bars affect your visual comparison. Do they show a similar trend, considering the uncertainties?

7. (1 point) At approximately what angle do you observe the maximum experimental range? Considering the uncertainties in your measurements, is this consistent with the theoretically pre-

dicted angle for maximum range (which will now be slightly less than 45° due to the launch height $y_0 > 0$)?

8. (1 point) Discuss the level of agreement and any discrepancies between the experimental and theoretical range vs. angle curves, considering the error bars and agreement within uncertainties for each angle. Are the differences, if any, systematic or random, and are they explained by your estimated uncertainties?

   Discuss possible reasons for any remaining systematic or random differences, and limitations of your error analysis. Reconsider factors like air resistance, measurement errors, launcher inconsistencies, protractor calibration accuracy, and limitations of the theoretical model. Also consider potential small vertical variations in the projectile launch trajectory and how their impact might be reflected in your range uncertainties or systematic deviations.

Learning outcomes

- Experimentally investigate and verify the principles of projectile motion.
- Measure initial velocity of a projectile using a dual beam photogate.
- Measure the horizontal range of projectiles launched horizontally and at various angles.
- Calibrate a protractor to minimize systematic errors in angle measurements.
- Apply kinematic equations to calculate the theoretical range of projectiles for horizontal and angled launches.
- Compare experimental measurements with theoretical predictions and analyze any discrepancies.
- Investigate the relationship between launch angle and horizontal range and determine the angle for maximum range experimentally and theoretically.
- Understand the limitations of simplified projectile motion models and discuss the influence of factors like air resistance and measurement uncertainties.

References

[1] D. Halliday, R. Resnick, and J. Walker. *Fundamentals of Physics*. Fundamentals of Physics. John Wiley & Sons.