Elementary Particles I, Spring 2025

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Final exam study guide: Key topics and outcomes based on lecture notes and "Elementary particle physics" by Andrew J. Larkoski (Cambridge University Press, 2019)

- This list summarizes key topics and learning outcomes from lectures 12-25 to help you prioritize and focus your study efforts for the final examination.
- While the list highlights and identifies core concepts, it is not a comprehensive inventory of all material covered in these lectures. You remain responsible for all content presented and discussed.
- Further, while these topics represent important learning outcomes, this list does not define the content of the final exam. Topics may appear on the exam whether or not they are explicitly listed here. Treat this purely as a study aid.
- The final exam may include elements from before the midterm, as they are a crucial background prerequisite for the second half of lectures covered here.

Lecture 12: The Parton Model

- Define and explain the physical significance of the Mandelstam variables (s, t, u) for $2 \to 2$ scattering.
- Understand and apply the concept of crossing symmetry to relate different scattering processes, e.g. $e^+e^- \to q\bar{q}$ and $e^-q \to e^-q$.
- Calculate the differential cross section for $e^-q \to e^-q$ scattering in terms of Mandelstam variables.
- Describe the concept of Deep Inelastic Scattering (DIS) $e^-p \to e^-X$ as a probe of proton structure.
- Explain the postulates of the Parton Model: protons composed of quasi-free, point-like partons (quarks, gluons) carrying fractions of the proton's momentum.
- Define the Bjorken scaling variable x and inelasticity y and relate them to experimental observables and parton kinematics.
- Relate the parton-level Mandelstam variables $(\hat{s}, \hat{t}, \hat{u})$ to the DIS variables (x, y, Q^2).
- Explain how the Parton Model predicts the form of F_2 in terms of Parton Distribution Functions (PDFs) $f_q(x)$.
- Define Bjorken scaling (independence of F_1, F_2 on Q^2 at fixed x) and describe the experimental evidence from SLAC that supported it and the Parton Model.

Lecture 13: The Gluon

- Explain the need for a new force carrier (gluon) for the strong interaction, distinct from the photon.

- Describe the hypothesized properties of the gluon (massless, spin-1) based on analogy with QED.
- Explain how gluon radiation can occur in e^+e^- annihilation ($e^+e^- \to q\bar{q}g$) and draw the corresponding Feynman diagrams.
- Understand the role of the gluon polarization vector ϵ_μ and the quark propagator in Feynman diagram calculations.
- Outline the structure of the amplitude calculation for $e^+e^- \to q\bar{q}g$, recognizing the need to sum diagrams.
- Understand the concept of spinor helicity formalism and its utility in simplifying calculations (no detailed calculations required).
- Describe the structure of the differential cross section $d\sigma(e^+e^- \to q\bar{q}g)/dx_1 dx_2$.
- Explain how the $e^+e^- \to q\bar{q}g$ process leads to 3-jet events and how their properties (rate, angular distributions) provide evidence for gluons, allow measurement of α_S, and confirm the gluon's spin-1 nature.
- Recognize the physical origin of soft and collinear divergences in the $q\bar{q}g$ cross section.

Lecture 14/15: Non-Abelian Gauge Theories & QCD

- Describe the concept of color charge and the representation of quarks as 3-component vectors in color space.
- Identify $SU(3)$ as the gauge group of color symmetry based on the requirement of unitary transformations preserving probability.
- Understand the concept of group generators (T^a) and structure constants (f^{abc}) for $SU(3)$.
- Define non-Abelian gauge theories and explain why $SU(3)$ is non-Abelian.
- Distinguish between global and local gauge invariance.
- Explain why the free Dirac Lagrangian is invariant under global but not local gauge transformations.
- Define the gauge covariant derivative $D_\mu = \partial_\mu - ig_s A_\mu^a T^a$ and explain its role in restoring local gauge invariance.
- State the transformation properties of the gauge field A_μ^a under a local gauge transformation.
- Define the gluon field strength tensor $F_{\mu\nu}^a$ via the commutator $[D_\mu, D_\nu]$ and identify the non-linear term arising from the non-Abelian nature.
- Explain how to construct a gauge-invariant kinetic term for the gluon field using $\mathrm{Tr}[F_{\mu\nu}F^{\mu\nu}]$.
- Write down the full Lagrangian for massless Quantum Chromodynamics (QCD) $\mathcal{L}_{QCD} = -\frac{1}{4}F_{\mu\nu}^a F^{\mu\nu a} + \bar{\psi} i\gamma^\mu D_\mu \psi$ and identify its components.

Lecture 16: Introduction to Quantum Chromodynamics (QCD)

- Identify the key components of the QCD Lagrangian: quark fields (ψ), gluon fields (A^a_μ), covariant derivative (D_μ), field strength tensor ($F^a_{\mu\nu}$).
- Explain why gluons are predicted to be massless in QCD (due to gauge invariance).
- Determine the number of physical propagating degrees of freedom for the gluon field (2 per color type).
- Explain the origin and significance of gluon self-interactions (3-gluon and 4-gluon vertices) arising from the F^2 term in the Lagrangian.
- Contrast gluon self-interaction with the lack thereof for photons in QED.
- Describe the concept of a running coupling constant, $\alpha(Q^2)$, using the QED example of vacuum polarization and screening.
- Define the beta function $\beta(\alpha) = Q d\alpha/dQ$.
- Explain the contributions of quark loops (screening) and gluon loops (anti-screening) to the QCD beta function $\beta(\alpha_S)$.
- State the result for the one-loop QCD beta function $\beta(\alpha_S) = \frac{\alpha_S^2}{2\pi}(-11 + \frac{2}{3}n_f)$ and explain why it is negative for $n_f < 16.5$.
- Define Asymptotic Freedom and explain how it arises from the negative beta function ($\alpha_S \to 0$ as $Q^2 \to \infty$).
- Explain how asymptotic freedom provides a justification for the success of the Parton Model at high energies.

Lecture 17: QCD at Hadron Colliders

- Apply crossing symmetry to relate $e^+e^- \to q\bar{q}$, $e^-q \to e^-q$, and $q\bar{q} \to \ell^+\ell^-$.
- Describe the Drell-Yan process at the parton level ($q\bar{q} \to \ell^+\ell^-$) and in hadron collisions ($pp \to \ell^+\ell^- X$).
- Calculate the partonic cross section $\sigma(q\bar{q} \to \ell^+\ell^-)$.
- Construct the hadronic cross section for Drell-Yan production by convoluting the partonic cross section with Parton Distribution Functions (PDFs) $f_q(x)$.
- Define rapidity y and relate the parton momentum fractions (x_1, x_2) to the experimentally measurable variables Q^2 and y of the lepton pair.
- Derive the expression for the differential cross section $d\sigma/(dQ^2dy)$ for Drell-Yan production in terms of PDFs.
- Explain why PDFs must depend on the energy scale Q^2 due to gluon radiation and splitting in QCD (violation of Bjorken scaling).

- Define the splitting function $P_{q\leftarrow q}(z)$ and understand its role in describing collinear parton radiation.
- Explain the physical origin and interpretation of collinear divergences in perturbative QCD.
- State the structure of the DGLAP evolution equations $Q^2 \frac{df_q(x,Q^2)}{dQ^2} = \frac{\alpha_S(Q^2)}{2\pi} \int_x^1 \frac{dz}{z} P_{q\leftarrow q}(z) f_q(x/z, Q^2)$ and understand qualitatively how they describe the Q^2 evolution of PDFs.

Lecture 18: Jets in QCD

- Define scale invariance (dilation symmetry) and test the action for a massless scalar field $\mathcal{S}[\phi] = \int d^4x \, \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi)$ for scale invariance.
- Explain how mass terms explicitly break scale invariance.
- Determine the scaling dimensions of the quark and gluon fields in QCD.
- Show that the classical massless QCD Lagrangian is scale invariant.
- Explain how the running of the coupling constant α_S leads to the breaking of scale invariance in quantum QCD (scale anomaly).
- Understand the concept of approximate scale invariance at high energies in QCD.
- Relate approximate scale invariance to fractal-like behavior and self-similarity.
- Explain how the DGLAP evolution and parton splitting lead to parton showers (cascades of soft/collinear emissions).
- Describe how parton showers hadronize into collimated sprays of particles called Jets.
- Explain the experimental significance of observing jets as evidence for quarks, gluons, and QCD.
- Understand the concept of event shape variables like Thrust τ.
- Describe the physical origin of Sudakov suppression (resummation of soft/collinear logarithms) and the Sudakov form factor.

Lecture 19: Parity Violation in the Weak Interaction

- Describe the historical puzzles in neutron beta decay (spin statistics, continuous electron energy spectrum) that hinted at a new particle.
- Explain Pauli's neutrino hypothesis and the properties required for the neutrino.
- Understand neutron decay as $n \to pe^-\bar{\nu}_e$.
- Define the discrete symmetry operations Parity (P), Time Reversal (T), and Charge Conjugation (C).

- Classify physical quantities (vectors, pseudovectors, scalars, pseudoscalars) according to their transformation properties under Parity.
- State that QED, QCD, and Gravity conserve P, C, and T individually.
- State the CPT theorem and its implications.
- Describe the motivation (Lee & Yang) and experimental setup of the Wu experiment testing parity conservation in ^{60}Co beta decay.
- Explain how the Wu experiment tests parity by measuring the correlation $\langle \vec{J} \cdot \vec{p}_e \rangle$.
- State the result of the Wu experiment and its conclusion: the weak force violates parity conservation.
- Explain the concept of helicity ($h = \vec{s} \cdot \hat{p}$) and the experimental observation that weak interactions preferentially couple to left-handed particles ($h \approx -1/2$) and right-handed anti-particles ($h \approx +1/2$).
- Understand the structure of the V-A (Vector minus Axial-vector) theory Lagrangian for four-fermion interactions, incorporating maximal parity violation, e.g., $\mathcal{L}_{int} \propto G_F(\bar{u}_L \gamma^\mu d_L)(\bar{e}_L \gamma_\mu \nu_{eL})$.

Lecture 20: The V-A Theory

- Apply the four-fermion interaction theory to calculate the amplitude for muon decay $\mu^- \to e^- \bar{\nu}_e \nu_\mu$.
- Outline the steps in calculating the spin-averaged squared amplitude $\overline{|\mathcal{M}|^2}$ for muon decay, recognizing the key result $\overline{|\mathcal{M}|^2} \propto G_F^2 (p_e \cdot p_{\nu_\mu})(p_\mu \cdot p_{\bar{\nu}_e})$.
- State the result for the differential decay rate $d\Gamma/dE_e$ and the total decay rate $\Gamma = G_F^2 m_\mu^5/(192\pi^3)$.
- Perform the numerical calculation of the muon lifetime $\tau = \hbar/\Gamma$ using the known values of G_F and m_μ.
- Appreciate the excellent agreement between the V-A prediction and the experimental measurement of the muon lifetime.
- Identify the theoretical shortcomings of the V-A theory (point interaction, divergent at high energy) that motivate a gauge theory description with intermediate vector bosons.

Lecture 21 Spontaneous Symmetry Breaking

- Explain the theoretical problems with the V-A theory of weak interactions (dimensionality of G_F, high-energy behavior).
- Argue why a mediated theory of the weak force implies the mediators (W, Z bosons) must be massive, and estimate the relevant mass scale from G_F.
- Explain the conflict between explicit mass terms for gauge bosons and the principle of local gauge invariance.

- Define Spontaneous Symmetry Breaking (SSB) and illustrate the concept using quantum mechanical potentials (e.g., double well, Mexican hat).
- Distinguish between the symmetry of the Lagrangian/Hamiltonian and the symmetry of the ground state (vacuum).
- Explain how expanding fields around a symmetry-breaking minimum makes the original symmetry non-manifest.
- State Goldstone's Theorem and explain the connection between spontaneously broken *continuous* global symmetries and the existence of massless Goldstone bosons.
- Analyze SSB for a complex scalar field with a "Mexican hat" potential, identifying the Vacuum Expectation Value (VEV), the unstable symmetric point ($\phi = 0$), and the stable minimum ($|\phi| = v$).
- Parameterize the complex scalar field in terms of magnitude (h) and phase (θ) fluctuations around the VEV.
- Derive the particle spectrum after SSB for the global symmetry case, identifying the massive scalar (h) and the massless Goldstone boson (θ).
- Explain the Higgs Mechanism: Combine SSB with a *local* (gauge) symmetry (using the U(1) toy model as an example).
- Understand the role of the covariant derivative in coupling the scalar field to the gauge field.
- Explain the concept of the Unitary Gauge and how gauge freedom can be used to eliminate the Goldstone boson field from the scalar sector description.
- Derive the mass acquired by the gauge boson ($m_A \propto ev$) and the Higgs boson ($m_h \propto \sqrt{\lambda}v$) in the Higgs mechanism.
- Explain the interpretation of the gauge boson "eating" the Goldstone boson to acquire mass and its longitudinal degree of freedom.
- Understand that the Higgs mechanism allows gauge bosons to be massive while preserving the underlying gauge invariance of the theory.

Lecture 22: Higgs Mechanism; giving the photon a mass

- Recall the concept of spontaneous symmetry breaking (SSB) in scalar field theory with a "Mexican Hat" potential $V(\phi) = \lambda(|\phi|^2 - v^2)^2$.
- Remember Goldstone's Theorem for global symmetries: SSB leads to massless Goldstone bosons corresponding to broken generators.
- Understand the goal of using SSB in a gauge theory to give mass to gauge bosons.
- Couple a complex scalar field ϕ to a $U(1)$ gauge field A_μ using the covariant derivative $D_\mu = \partial_\mu - ieA_\mu$.

- Write down the Lagrangian for the $U(1)$ gauge theory with the scalar field and Mexican Hat potential, and identify its local gauge symmetry.
- Parameterize the scalar field around its VEV $|\phi| = v$ using radial (h) and phase (θ) modes.
- Explain the concept of the Unitary Gauge and how it can be used to eliminate the phase field $\theta(x)$ from the scalar field parameterization ($\phi \to v + h/\sqrt{2}$).
- Analyze the scalar kinetic term $(D_\mu \phi)(D^\mu \phi^*)$ in the unitary gauge after SSB.
- Identify the term $\propto e^2 v^2 A_\mu A^\mu$ that emerges from the kinetic term and recognize it as a mass term for the gauge boson A_μ.
- Determine the mass of the gauge boson $m_A = \sqrt{2} ev$.
- Determine the mass of the remaining physical scalar (Higgs) particle h from the potential term $m_h = 2\sqrt{\lambda} v$.
- Explain the Higgs Mechanism: the process by which a gauge boson acquires mass by 'eating' the Goldstone boson corresponding to the spontaneously broken gauge symmetry, with the Goldstone becoming the longitudinal mode of the massive boson.

Lecture 23: The W & Z Bosons and Electroweak Unification

- Explain why the dimensionality of the Fermi constant G_F implies the weak force must be mediated by massive particles.
- Deduce the properties of the charged current mediator ($W^\pm$) from processes like beta decay (charged, color-neutral, massive, spin-1).
- Describe the experimental evidence for a neutral weak mediator (Z^0) from the resonance observed in $e^+e^- \to \mu^+\mu^-$ around 91 GeV.
- Explain how the Z^0 resonance in $e^+e^- \to \nu\bar{\nu}$ links the neutral current to neutrino interactions.
- Argue for the need for a unified Electroweak Theory incorporating $\gamma, W^\pm, Z^0$.
- Justify the choice of the $SU(2)_W \otimes U(1)_Y$ gauge group based on the requirement of 4 gauge bosons.
- Define Weak Isospin ($SU(2)_W$) and Hypercharge ($U(1)_Y$) and the associated gauge bosons (A^a_μ, B_μ).
- Define the Higgs field as an $SU(2)_W$ doublet $\phi = (\phi^+, \phi^0)^T$ with hypercharge $Y = 1/2$.
- Explain the importance of the Higgs VEV residing in the neutral component $\langle \phi \rangle = (0, v/\sqrt{2})^T$ to preserve massless QED.
- Write down the covariant derivative D_μ for the Higgs doublet interacting with A^a_μ and B_μ.
- Describe how the Higgs kinetic term $(D_\mu \phi)^\dagger (D^\mu \phi)$, evaluated at the VEV, generates mass terms for combinations of A^a_μ and B_μ.
- Define the physical $W^\pm, Z^0, A$ fields as linear combinations of $A^1_\mu, A^2_\mu, A^3_\mu, B_\mu$.

- State the predicted masses $m_W = g_W v/2$ and $m_Z = m_W/\cos\theta_W$, and explain why the photon A remains massless.
- Define the weak mixing angle θ_W and its relation to the couplings g_W, g_Y.
- Derive the relationship between electric charge Q, weak isospin I_3, and hypercharge Y: $Q = I_3 + Y$.
- State the relationship between the fundamental couplings $e = g_W \sin\theta_W = g_Y \cos\theta_W$.
- Understand the relationships between the measurable quantities ($m_W, m_Z, G_F, e, \theta_W$) and the fundamental parameters (g_W, g_Y, v), and appreciate the predictive power of the theory.

Lecture 24: CP Violation

- Distinguish between particle states defined by mass (mass eigenstates) and states defined by their interactions (flavor eigenstates).
- Explain why mass and flavor eigenstates coincide for QED and QCD interactions (flavor-diagonal).
- Understand that the weak charged current interaction inherently changes flavor ($d \leftrightarrow u$, $e \leftrightarrow \nu_e$, etc.).
- Describe how fermion masses arise from Yukawa couplings to the Higgs field ($\mathcal{L}_{Yuk} = -y\bar{\psi}_L\phi\psi_R +$ h.c.) after SSB.
- Explain why neutrinos are massless in the minimal Standard Model (no right-handed neutrino ν_R coupling).
- Understand that the charged lepton mass matrix M_e can be diagonalized by unitary matrices U_{eL}, U_{eR}.
- Show that in the minimal SM with massless neutrinos, the freedom to redefine neutrino fields allows the charged current interaction in the lepton sector to remain flavor-diagonal.
- Write down the Yukawa couplings for quarks, involving matrices y_u and y_d. Let Q_L be the left-handed quark doublet, u_R, d_R the right-handed singlets, ϕ the Higgs doublet: $\mathcal{L}_{Yuk,q} = -y_d^{ij}\bar{Q}_{Li}\phi d_{Rj} - y_u^{ij}\bar{Q}_{Li}\epsilon\phi^* u_{Rj} +$ h.c..
- Explain that both up-type and down-type quark mass matrices (M_u, M_d) must be diagonalized independently using four unitary matrices ($U_{uL}, U_{uR}, U_{dL}, U_{dR}$).
- Show how transforming the charged current interaction ($\propto W_\mu^+\bar{u}_{Li}\gamma^\mu d_{Li}$) to the quark mass eigenbasis introduces the Cabibbo-Kobayashi-Maskawa (CKM) matrix $V_{CKM} = U_{uL}U_{dL}^\dagger$.
- Explain that the CKM matrix describes flavor mixing between quark generations in weak interactions.
- Perform parameter counting for an $N \times N$ unitary matrix, considering absorbable phases from field redefinitions.
- Explain why the 2×2 CKM matrix has only one real parameter (Cabibbo angle θ_C) and no physical phase.

- Explain why the 3×3 CKM matrix has 3 real angles and 1 irreducible complex phase (δ_{CP}).
- Understand that the complex phase in V_{CKM} is the source of CP violation in the Standard Model quark sector, leading to different interaction strengths for matter vs. anti-matter.
- Relate CP violation to T violation via the CPT theorem.

Lecture 25: Neutrino Oscillations

- Recall the Standard Model prediction of massless neutrinos and no lepton flavor mixing.
- Describe how neutrinos are produced and detected via weak interactions, identifying them by their flavor eigenstate ($|\nu_e\rangle, |\nu_\mu\rangle$, etc.).
- State the hypothesis underlying neutrino oscillations: flavor eigenstates differ from mass eigenstates ($|\nu_1\rangle, |\nu_2\rangle$, etc.), implying neutrinos have mass.
- Write down the mixing relation between flavor and mass eigenstates for two neutrino generations using a mixing angle θ: $|\nu_\alpha\rangle = \sum_i U_{\alpha i}|\nu_i\rangle$ where U is a rotation matrix involving θ.
- Describe the time evolution of a neutrino state initially produced in a flavor eigenstate, recognizing that the mass eigenstate components acquire different phases $e^{-ip_i \cdot x}$ during propagation.
- Apply the ultra-relativistic approximation ($p_i \approx E - m_i^2/2E$) to find the phase difference $\Delta\phi \approx \Delta m^2 L/2E$ after traveling distance L.
- Calculate the probability amplitude $\langle \nu_\alpha | \nu(L) \rangle$ and the probability $P(\nu_\alpha \to \nu_\beta)$ for neutrino oscillation.
- State the two-flavor neutrino oscillation probability formula: $P(\nu_\alpha \to \nu_\beta) = \sin^2(2\theta)\sin^2(\Delta m^2 L/4E)$ (for $\alpha \neq \beta$).
- Identify the conditions necessary for oscillations to occur ($\theta \neq 0$ and $\Delta m^2 \neq 0$).
- Define the oscillation length $\mathcal{L}_{osc} = 4\pi E/|\Delta m^2|$.
- Understand that oscillation experiments measure mass-squared differences (Δm^2) but not absolute masses.
- Describe the extension to three generations: the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix (U_{PMNS}), 3 mixing angles ($\theta_{12}, \theta_{23}, \theta_{13}$), and 1 CP-violating phase (δ_{CP}) relevant for oscillations.
- Explain that the observation of neutrino oscillations provides definitive evidence for physics Beyond the Standard Model (non-zero neutrino masses).
- Briefly describe the significance of neutrino detection from Supernova 1987A.