An application of nonperturbative Pauli – Villars regularization to QED

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Light Cone 2007 Relativistic Hadronic and Nuclear Physics
May 14-18 2007, Columbus

Electron's magnetic moment $\dfrac{ge\hbar}{4mc}$

g — gyromagnetic ratio

Theoretical prediction, computed perturbatively up to order α^4, is

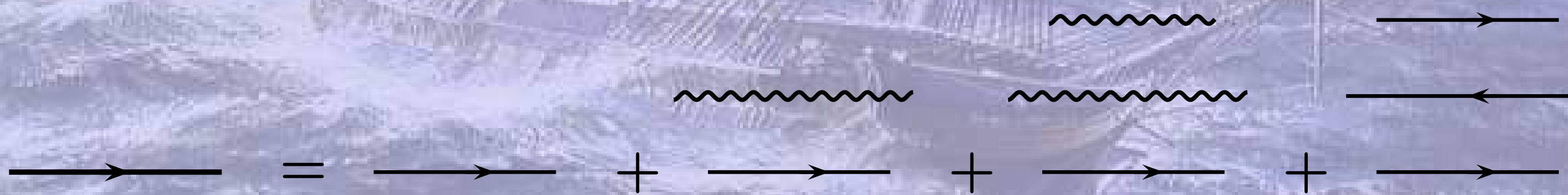
$$\frac{g-2}{2} = \frac{\alpha}{2\pi} - (0.328\,478\,965\ \ldots)\times\left(\frac{\alpha}{\pi}\right)^2 + (1.176\,11\ \ldots)\times\left(\frac{\alpha}{\pi}\right)^3 - (1.434\ \ldots)\times\left(\frac{\alpha}{\pi}\right)^4 = 0.001\,159\,652\,140\ \ldots$$

“A nonperturbative calculation of the electron’s magnetic moment”

S.J.Brodsky, V.A.Franke, J.R.Hiller, G.McCartor, S.A.Paston, E.V.Prokhvatilov

Light-cone quantization regulated with PV fields in $3+1$ *dimensions*

- *Light-cone gauge* $A^+ = 0$, 3 *PV electrons*
- *Feynman gauge,* 1 *PV electron* $+$ 1 *PV photon*
- *Again light-cone gauge,* 3 *PV electrons* $+$ 1 *PV photon, higher-order derivatives*

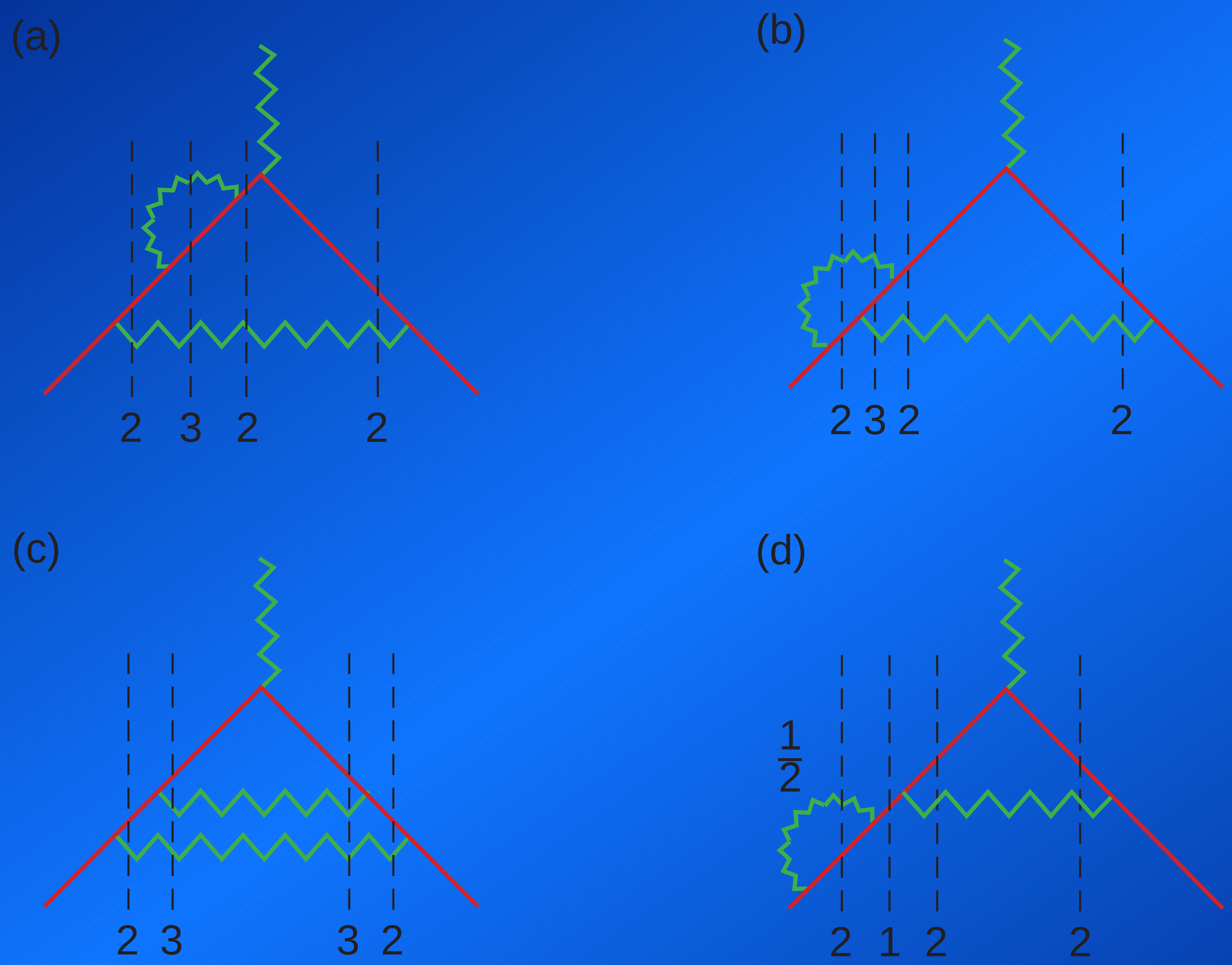


Three problems must be solved to produce useful calculation:

- *Problem of maintaining gauge invariance (exact solution exists and has all symmetries and a close approximation can safely break symmetries)*

- *New singularities (when the bare mass is less than the physical mass, as is the case in QED, then can be zero in energy denominator) — principal value prescription*

- *Uncancelled divergencies (missing of corrections due to truncation of Fock state) — keep at least on PV mass finite*


(a)
2 3 2 2
(b)
2 3 2 2
(c)
2 3 3 2
(d)
1/2
2 1 2 2

$$\sum_{i=0}^{1}\left(-\frac{1}{4}(-1)^i F_i^{\mu\nu}\,F_{i,\mu\nu}+(-1)^i\overline{\psi}_i(i\gamma^\mu\partial_\mu-m_i)\psi_i+B_i\partial_\mu A_i^\mu+\frac{1}{2}B_iB_i\right)-e\,\overline{\psi}\,\gamma^\mu\,\psi\,A_\mu,$$

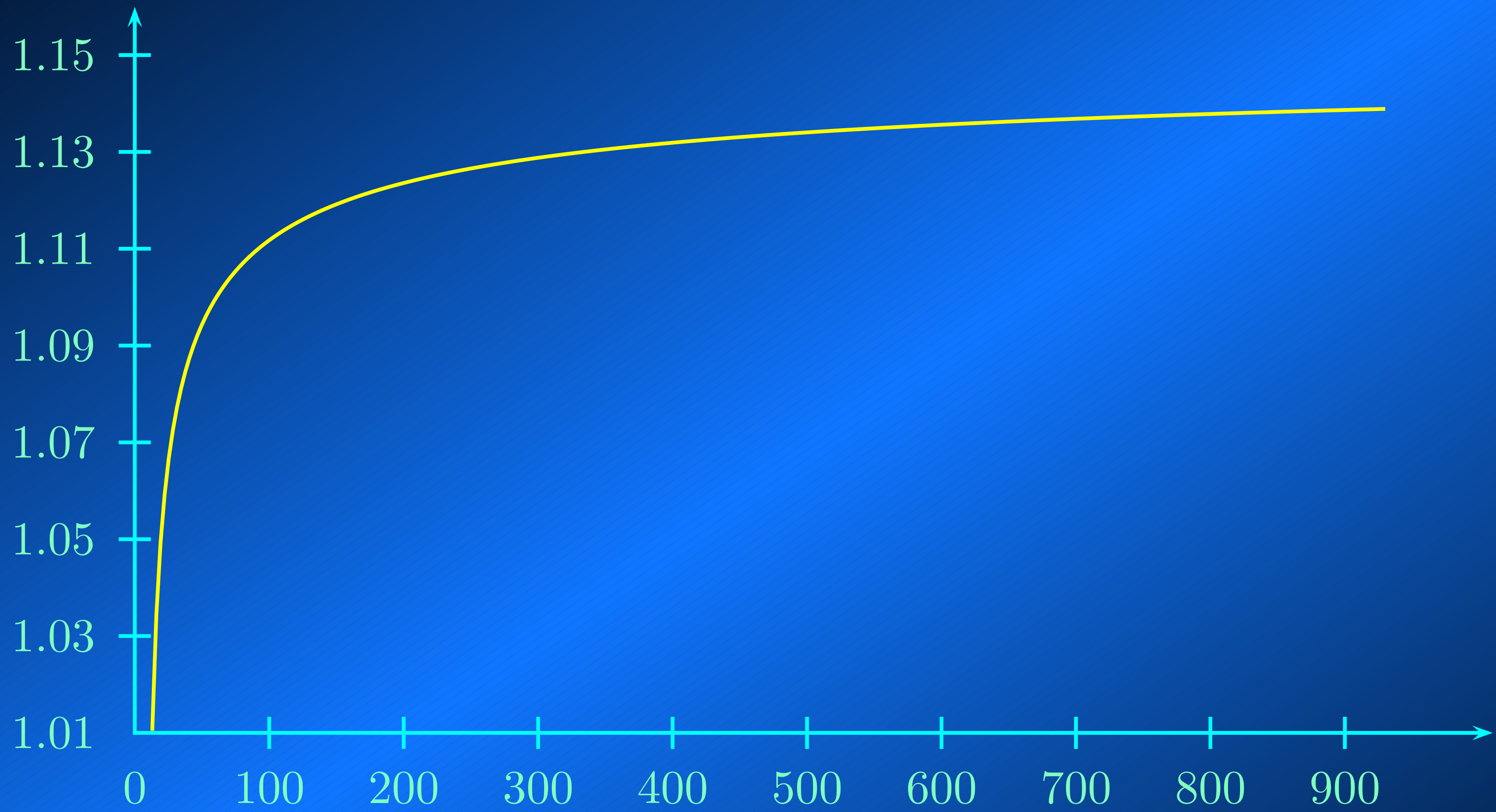
where $A^\mu=\sum_{i=0}^{1}A_i^\mu,\qquad \psi=\sum_{i=0}^{1}\psi_i,\qquad F_i^{\mu\nu}=\partial^\mu A_i^\nu-\partial_\nu A_i^\mu$

$$P^-=\sum_{i,s}\int d\underline{p}\frac{m_i^2+p_\perp^2}{p^+}\,(-1)^i\,b_{i,s}^\dagger(\underline{p})\,b_{i,s}(\underline{p})+\sum_{l,\mu}\int d\underline{k}\frac{m_l^2+k_\perp^2}{k^+}\,(-1)^l\epsilon^\mu a_l^{\mu\dagger}(\underline{k})\,a_l^\mu(\underline{k})+$$

$$+\sum_{i,j,l,s,\mu}\int d\underline{p}\,d\underline{q}\left\{b_{i,s}^\dagger(\underline{p})\left[b_{j,s}(\underline{q})Q_{ij,2s}^\mu(\underline{p},\underline{q})+b_{j,-s}(\underline{q})R_{ij,-2s}^\mu(\underline{p},\underline{q})\right]a_{l\mu}^\dagger(\underline{q}-\underline{p})+h.c.\right\}$$

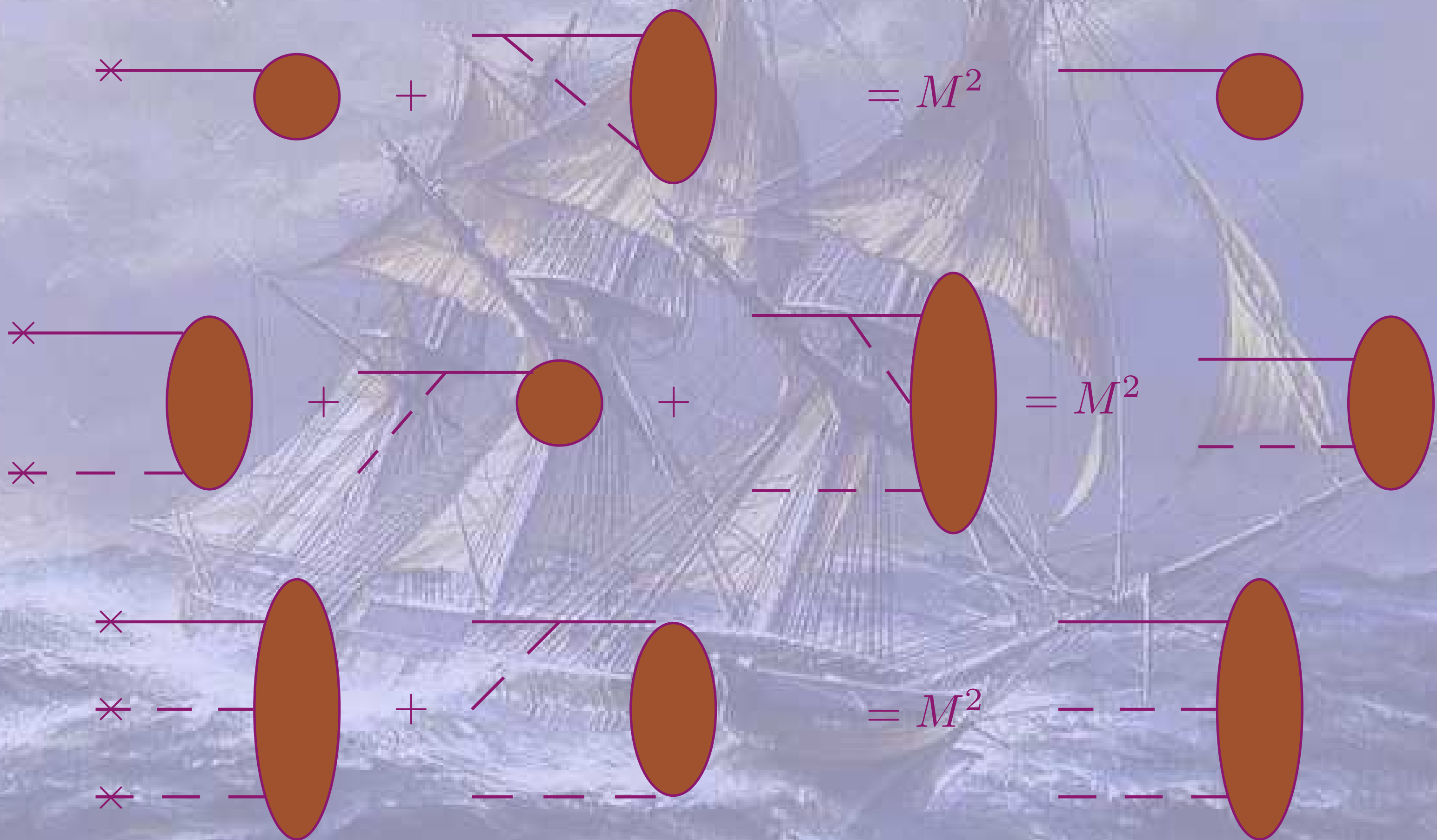
$$\Phi_+(\underline{P})=\sum_i z_i b_{i+}^\dagger(\underline{P})|0\rangle+\sum_{ijs}\int d\underline{q}\,f_{ijs}(\underline{q})\,b_{is}^\dagger(\underline{P}-\underline{q})\,a_j^\dagger(\underline{q})|0\rangle+$$

$$+\sum_{ijks}\int d\underline{q_1}d\underline{q_2}\,f_{ijks}(\underline{q_1},\underline{q_2})\frac{1}{\sqrt{1+\delta_{jk}}}\,b_{is}^\dagger(\underline{P}-\underline{q_1}-\underline{q_2})\,a_j^\dagger(\underline{q_1})\,a_k^\dagger(\underline{q_2})|0\rangle+\dots$$



The anomalous moment of the electron in units of the Schwinger term $\left(\frac{\alpha}{2\pi}\right)$ plotted versus the PV photon mass, μ_1.

$$P^+P^-|\Phi_+\rangle = M^2|\Phi_+\rangle$$



THE EQUATION FOR TWO-PARTICLE AMPLITUDES ONLY:

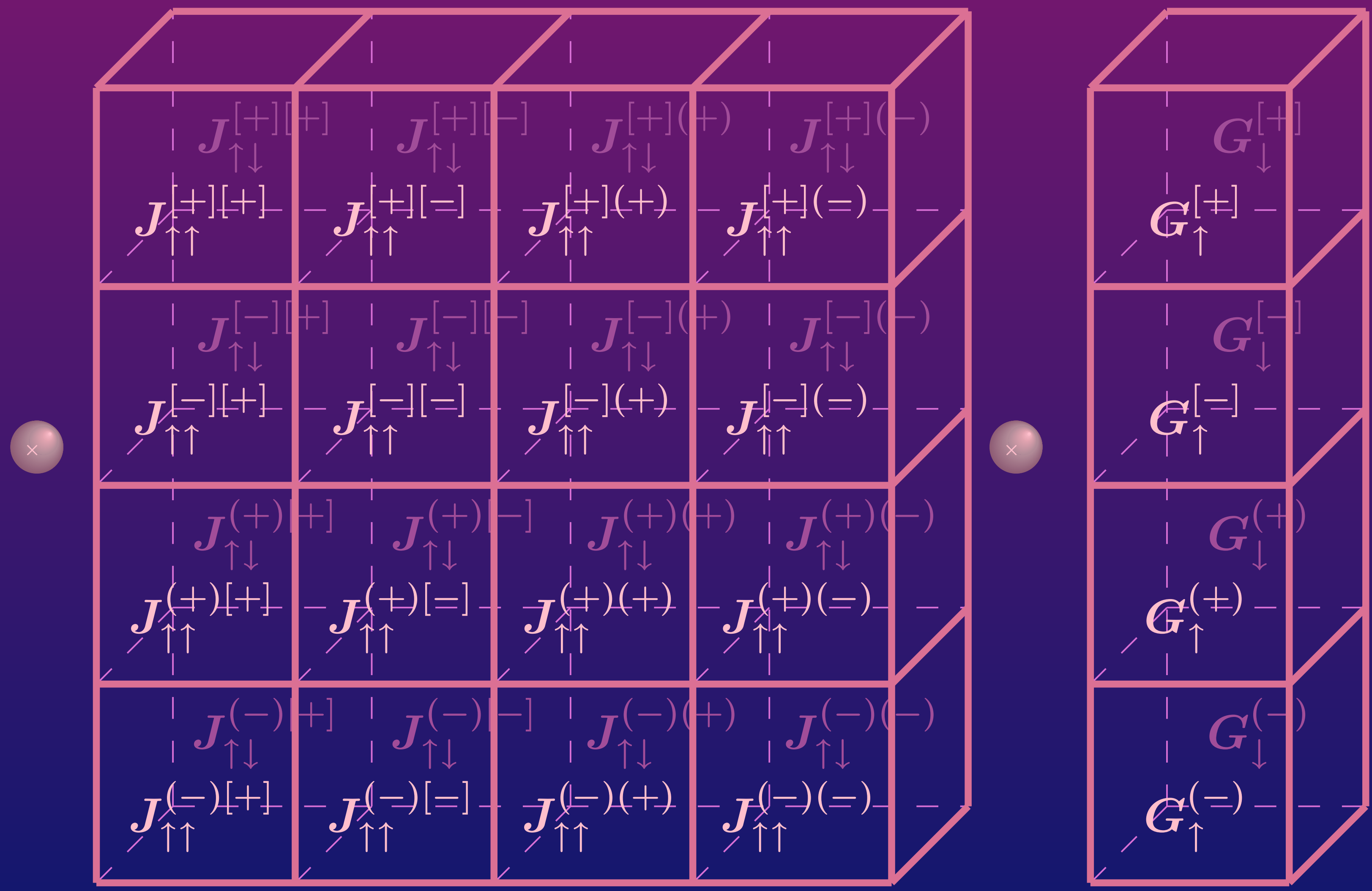
$$\left[M^2-\frac{m_i^2+q_\perp^2}{1-y}-\frac{\mu_j^2+q_\perp^2}{y}\right]G^{\lambda}_{ijs}(y,q_\perp)=$$

$$\frac{e^2}{8\pi^2}\sum_l I_{ijl}(y,q_\perp)\,G^{\lambda}_{ljs}(y,q_\perp)+$$

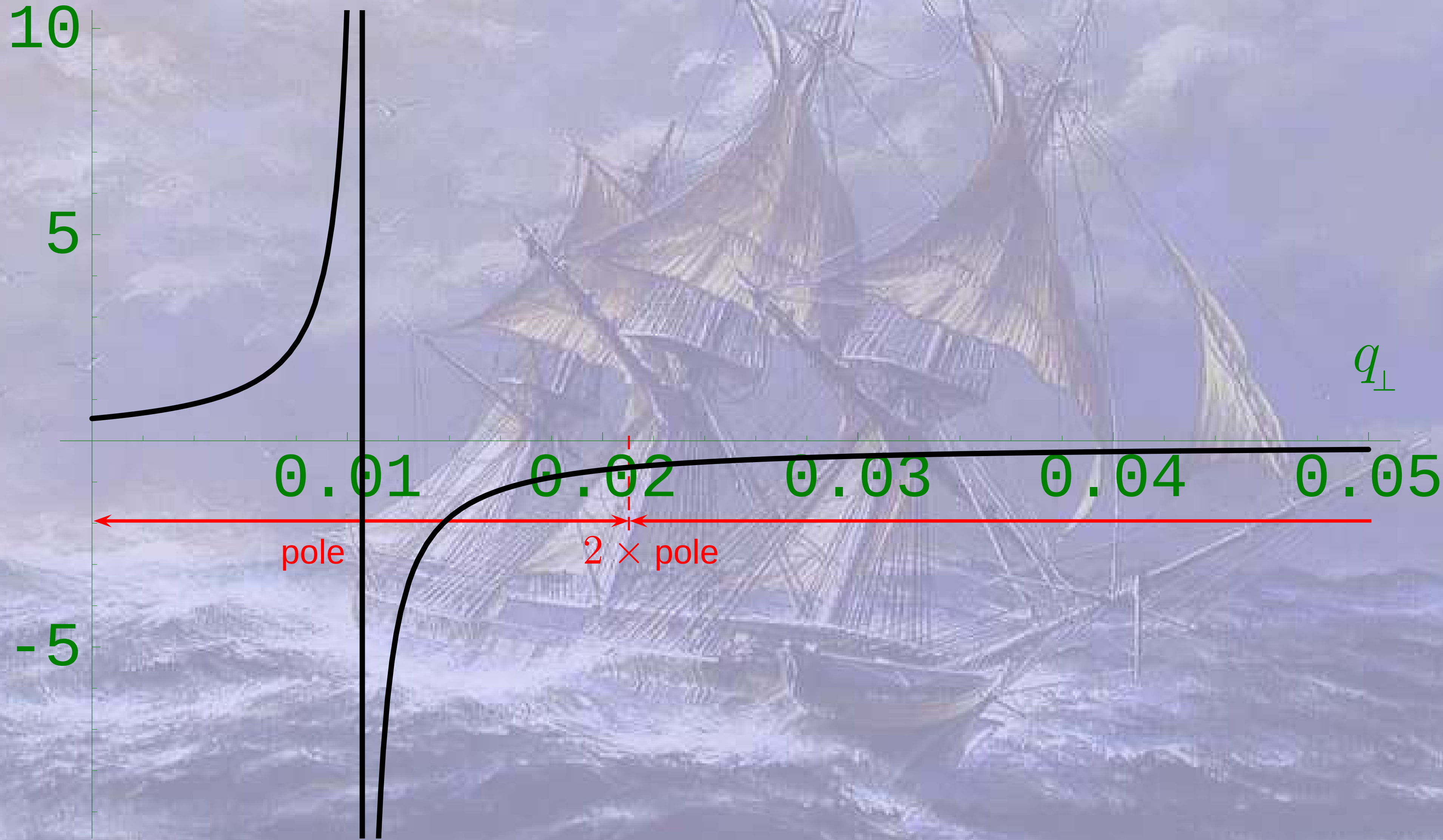
$$+\frac{e^2}{8\pi^2}\sum_{n,k,s',\lambda'}\int_0^1 dy'\int_0^{+\infty} q'_\perp dq'_\perp\, J^{(0)\lambda\lambda'}_{ijs,nks'}(y,q_\perp;y',q'_\perp)\,G^{\lambda'}_{nks'}(y',q'_\perp)+$$

$$+\frac{e^2}{8\pi^2}\sum_{n,k,s',\lambda'}\int_0^{1-y} dy'\int_0^{+\infty} q'_\perp dq'_\perp\, J^{(2)\lambda\lambda'}_{ijs,nks'}(y,q_\perp;y',q'_\perp)\,G^{\lambda'}_{nks'}(y',q'_\perp)$$

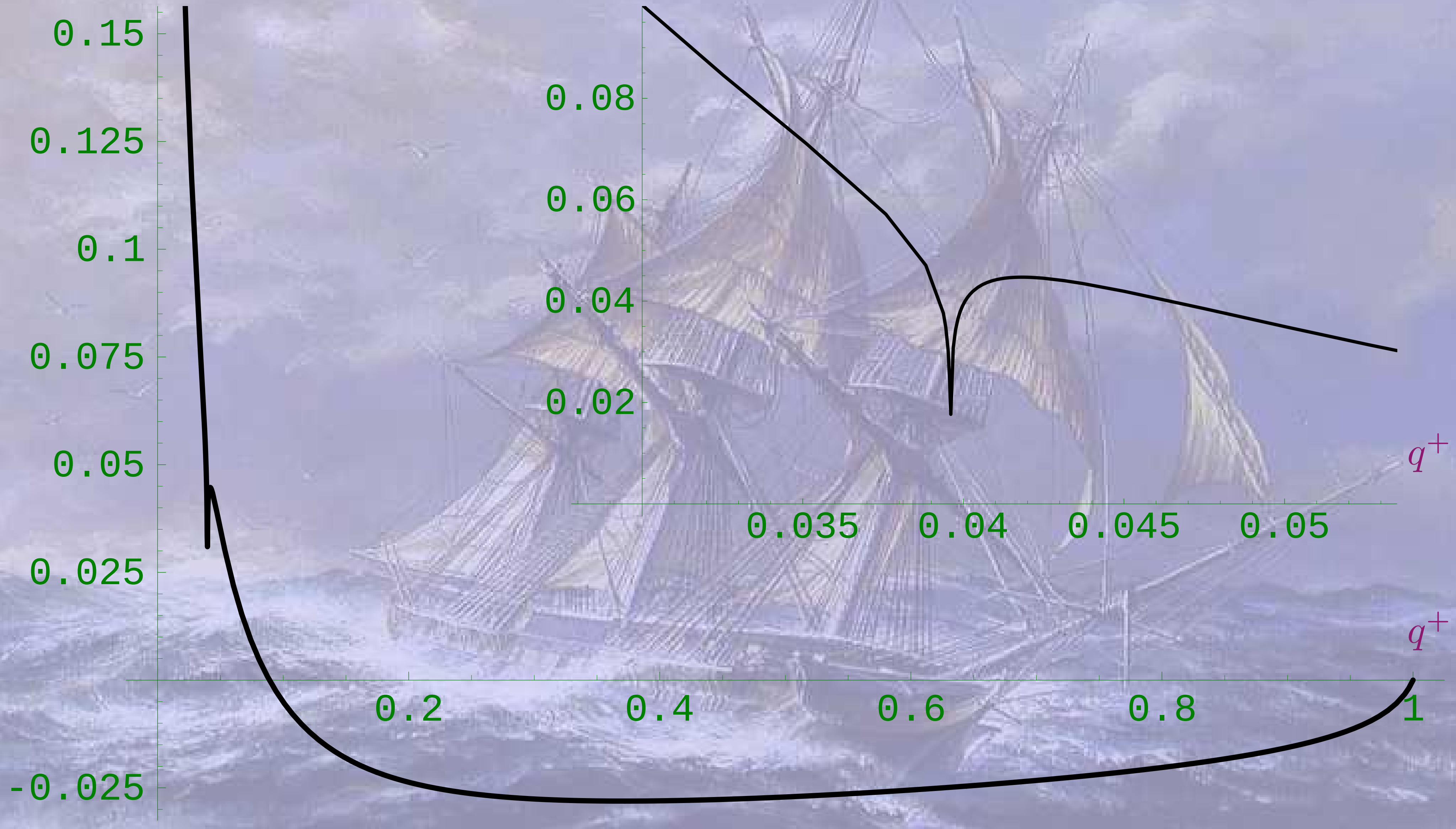
$$\sim \frac{e^2}{8\pi^2}\sum_{n,k}\int\limits_0^{1-y} dy' \int\limits_0^{+\infty} q'_\perp dq'_\perp$$



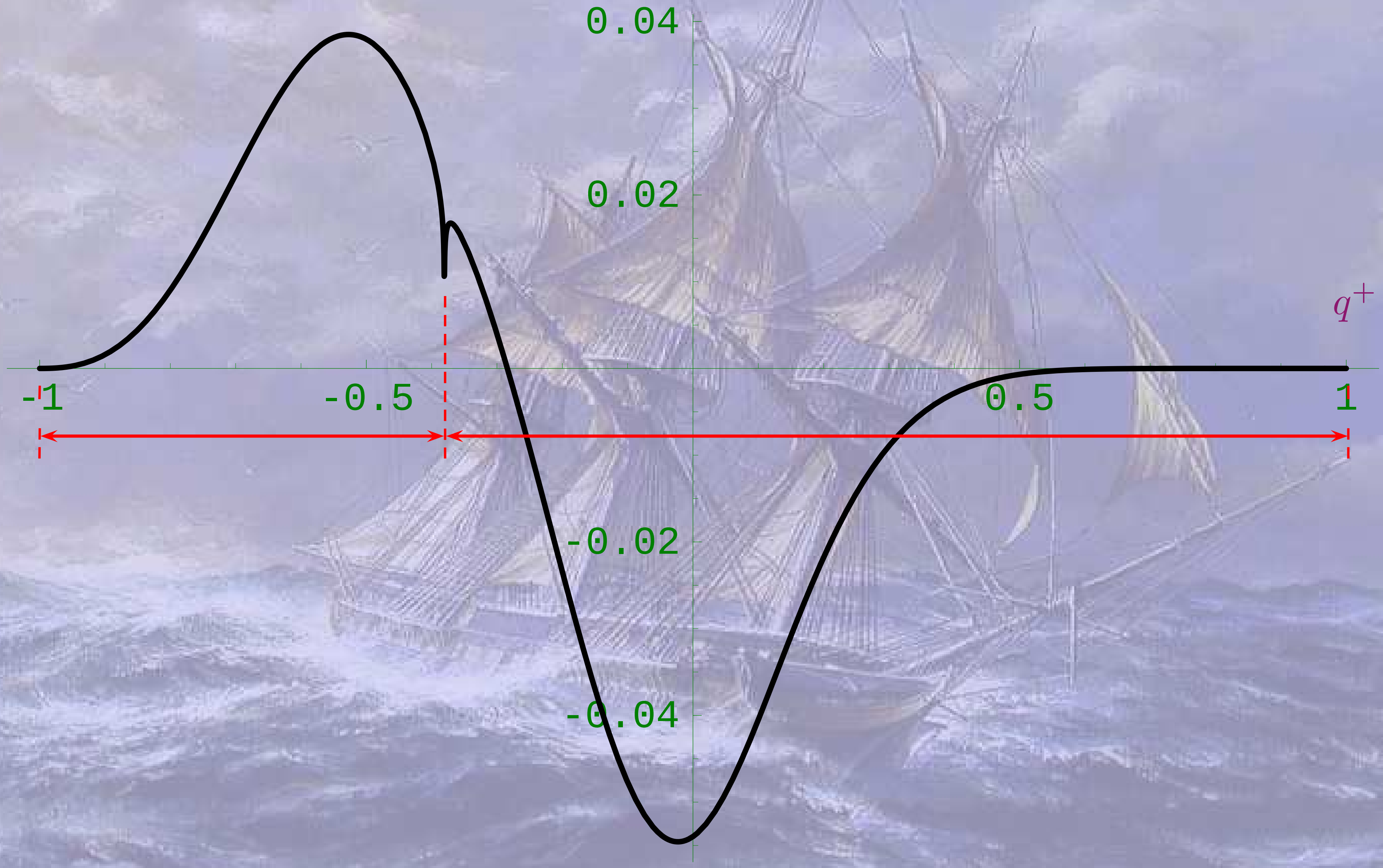
$$-M^2 y(1-y) + m^2 y + \mu^2(1-y) + q^2$$



Logarithmic singularity of longitudinal momentum



Changing of variables



One-photon truncated wave function was obtained analytically and anomalous electron's magnetic moment is within 14% accuracy of Schwinger term.

For two-photon truncated state anomalous magnetic moment is expected to get close to Sommerfield-Peterman term, but this case demands huge numerical calculation. Currently computer code is being checked for consistency with analytical solution derived from one-photon truncated state.