Gauged Axions and Anomaly-Mediated Interactions

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Gauged axions are extension of the traditional (invisible) Peccei-Quinn axions where the "shift parameter" is gauged.

The gauging, as we are going to show, has consequences at cosmological level, as in the PQ case.
We will try to highlight some of the main feature of this light particle, under the assumption that a mechanism of "sequential misalignment" is induced Both at the electroweak and at the QCD phase transition.

These types of axions, as we are going to argue, are typical of several types of high energy "completion theories" such as

INTERSECTING BRANE models
GAUGED SUPERGRAVITIES.

The role played by these axions are quite specific, since they could Take the role both of dark matter and of dark energy, under some conditions That we are going to discuss.

Organization of the talk

1) ABC of chiral gauge anomalies and the structure of the correlators

2) Stuckelberg fields and the Stuckelberg Mechanism

3) Extracting the physical component of the Stuckelberg field: the Gauged Axion (Higgs-Axion Mixing)

4) Realistic Models: MLSOM (minimal low-scale orientifold model) and the USSM-A (supersymmetric version).

5) Relic densities of axions (MLSOM) and of axions/neutralinos in the USSM-A

6) Implications at the LHC for anomalous extra Z prime's

One important feature of all anomalies

Anomalies are associated to massless (scalar or pseudoscalar degrees of freedom) which are affect both the infrared (light cone region) of a certain theory and the UV)

(R. Armillis, Delle Rose, C.C.)

They can be "captured" in perturbation theory, as I am going to show next.

This feature appears both in all the types of anomalies that we have studied so far. Axions are a manifestation of this mechanism.

Anomalies are related to some pitfalls of a given quantum field theory in the presence of a chiral fermion spectrum.
The Standard Model is a chiral theory.
The resolution of chirality may have enormous consequences at phenomenological level but we may have overlooked at the way it appears.
Some of the implications of the anomalies at low energy, even within the "most traditional" phenomenology are still only partially understood, and some finer details may soon or later going to appear in some experiment.

The models that we analyze have **gauge anomalies** and these have to cancel.
There are various possibilities to enforce this cancellation.
One of them requires an axion.

Anomalies in the Standard Model cancel by charge assignment.

However, this is not the only way in which one can cancel the anomaly
Once we allow strings/branes, new mechanism may appear

For gauge anomalies

A Green-Schwarz mechanism involving an axion
Anomaly inflow

Other anomalies (global)

Anomalies related to global currents (the "U(1)" anomaly)
Conformal anomaly (breaking of scale invariance)
Are not subject to cancellation in a 4 D theory. They are genuine predictions of the theory.

Goal: to study the effective field theory of
a class of models containing a gauge structure of the form

SM x U(1) x U(1) x U(1)

$SU(3) \times SU(2) \times U(1)_Y \times U(1)$.....

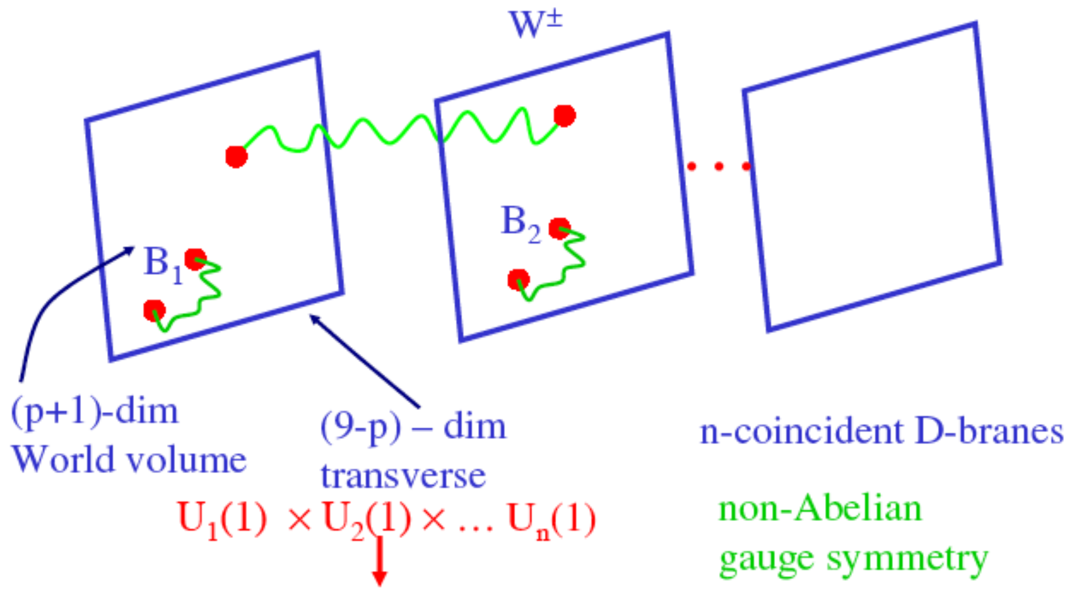
from which the hypercharge is assigned in a given string construction, corresponding to a certain class of vacua in string theory (Minimal Low Scale orientifold Models).

These models are the object of an intense scrutiny by many groups working on intersecting branes, from where they originally emerged

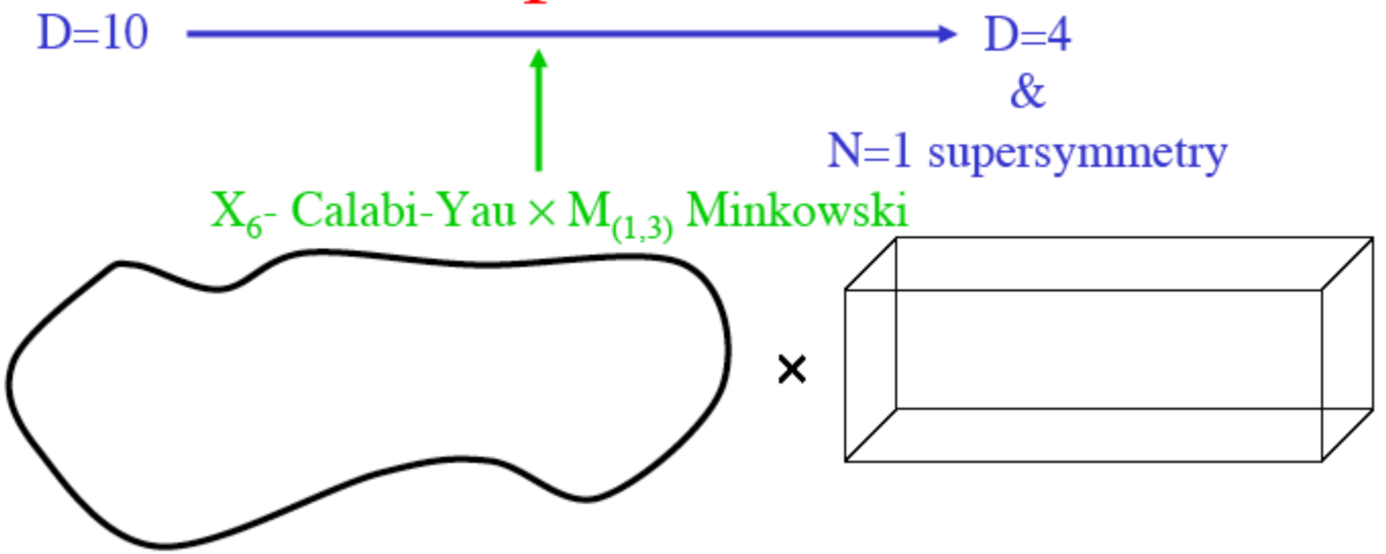
Antoniadis, Kiritsis, Rizos, Tomaras
Antoniadis, Leontaris, Rizos
Ibanez, Marchesano, Rabadan,
Ghilencea, Ibanez, Irges, Quevedo

first studied by us in 2006 (Irges, Kiritsis, CC). Since then we have realized the these extensions can be Justified also within anomaly free model when a part of the fermion spectrum is decoupled from the low energy theory (Armillis, Guzzi, C.C.)

D p-branes

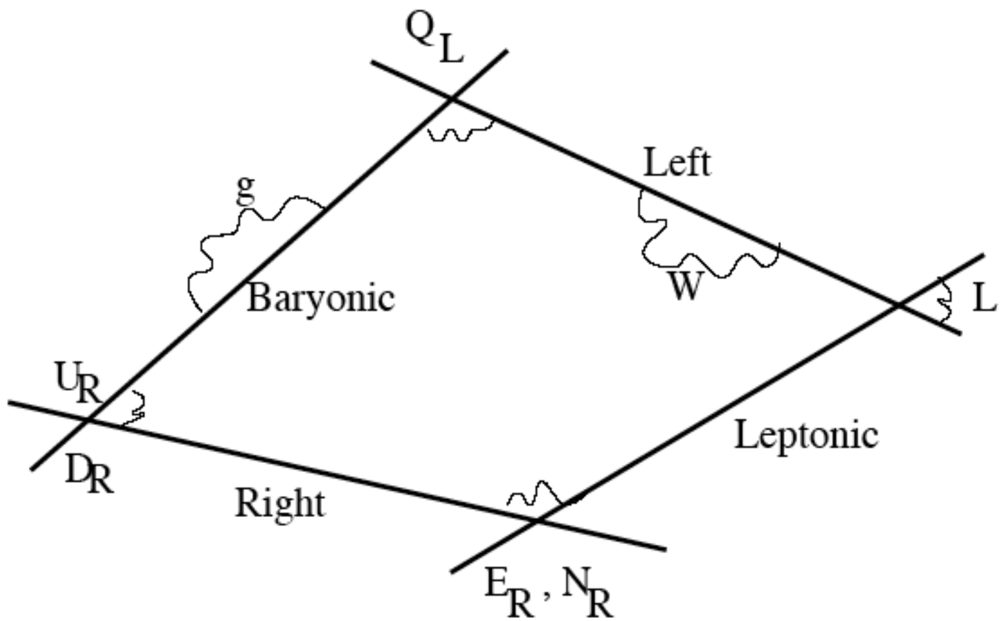


Compactification



Label	Multiplicity	Gauge Group	Name
stack a	$N_a = 3$	$SU(3) \times U(1)_a$	Baryonic brane
stack b	$N_b = 2$	$SU(2) \times U(1)_b$	Left brane
stack c	$N_c = 1$	$U(1)_c$	Right brane
stack d	$N_d = 1$	$U(1)_d$	Leptonic brane

Table 1: Brane content yielding the SM spectrum.



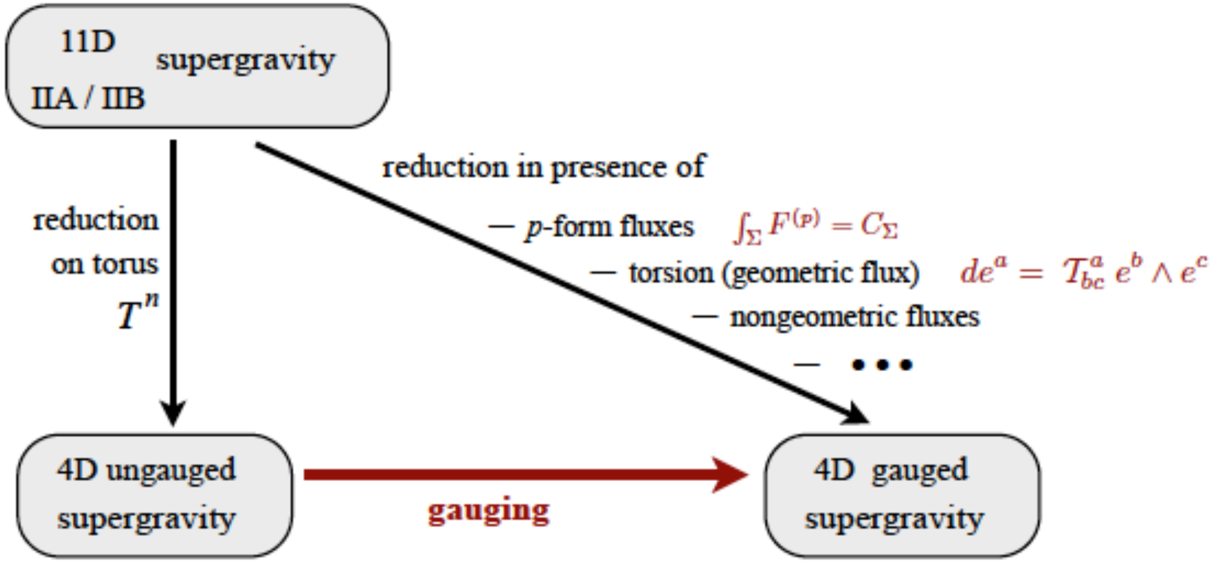


Figure 1: Gauged supergravities and flux compactifications.

From Samtleben review on gauged supergravities

See also a recent thesis by J. De Rydt (Lovain)

Main lessons: the gauging of global symmetries brings in anomalies.
One needs to compensate with counterterms the effective action to restore gauge invariance.

(D=4) with fields of spin 2, 1, 0, 3/2, 1/2

$e^a_\mu,\ A^I_\mu,\ \phi^u,\ \psi_\mu,\ \lambda^A$

$$
\begin{aligned}
e^{-1}\mathcal{L} \;=\; & \tfrac{1}{2}R \\
& +\tfrac{1}{4}(\mathrm{Im}\,\mathcal{N}_{IJ})\mathcal{F}^I_{\mu\nu}\mathcal{F}^{\mu\nu J} - \tfrac{1}{8}(\mathrm{Re}\,\mathcal{N}_{IJ})\varepsilon^{\mu\nu\rho\sigma}\mathcal{F}^I_{\mu\nu}\mathcal{F}^J_{\rho\sigma} \\
& -\tfrac{1}{2}g_{uv}D_\mu\phi^u D^\mu\phi^v - V \\
& \left\{-\bar{\psi}_{\mu i}\gamma^{\mu\nu\rho}D_\nu\psi_\rho{}^i - \tfrac{1}{2}g_A{}^B\bar{\lambda}^A \not{D}\lambda_B + \text{h.c.}\right\} + \ldots
\end{aligned}
$$

It is expected that these theories have a unitarity bound, although this is Essentially related to the Planck scale.

Interactions such as Re N FF are typical of the supersymmetric version of the Low energy models that we will present.

Global U(1) Anomalies

The PQ axion: where the story commences

Why CP is conserved in Strong Interactions ?

$$L_\theta = \frac{\alpha_s \theta}{8\pi} G^a_{\mu\nu} \tilde{G}^{a\mu\nu} \qquad L_m = \frac{\alpha_s Arg(\det M)}{8\pi} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$$

$$\bar{\theta} = \theta + Arg(\det M) \Rightarrow L_{\bar{\theta}} = \frac{\alpha_s \bar{\theta}}{8\pi} \vec{G}_{\mu\nu} \cdot \tilde{\vec{G}}^{\mu\nu}$$

$$L = \sum_{n=1}^{N} \bar{\psi}_n \left(i\gamma^\mu D_\mu - m_n \right) \psi_n - \frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} + \frac{\alpha_s \bar{\theta}}{8\pi} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$$

We do not observe any CP violating effect in the strong interactions, and this shows that THETA has to be small.

Smallness of the neutron electrical dipole moment shows that THETA has to be zero.

Indeed if we calculate the neutron electric dipole moment we get: $d_n \approx 10^{-16}\bar{\theta}$ e·cm

Given the experimental limit: $d_n < 0.63 \times 10^{-25}$ e·cm

→ $\bar{\theta} \approx 10^{-9}$

PECCEI and QUINN proposed in 1977 an extension of the SM With an anomalous U(1) symmetry U(1)_PQ

This is a symmetry of the theory at the lagrangean level, broken

Spontaneously at a large scale (f_a)

and

Explicitly at the QCD hadron transition

The axion is massless until the QCD transition. Instanton effects are held responsible for the generation of the axion potential

$$\mathcal{L}_{\mathrm{QCD+a}} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \frac{1}{2}\partial_\mu a \partial^\mu a + \sum_q \bar{q}(i\gamma^\mu\partial_\mu - m_q)q + \frac{g_s^2}{32\pi^2}(\theta + \frac{a}{f_a})G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

The axion interacts with photons, electrons, hadrons with a strength ~ f_a^{-1}

Initially f_a was tied to the Electroweak scale and the CP breaking with the Electroweak breaking

The realization that the axion scale can be very big (and its coupling very weak) initiated the 'invisible axion' models

DFSZ

- $f_a >> f_{ew}$
- Two Higgs fields and
- one scalar field.
- Fermions: carry PQ charge.

KSVZ

- $f_a >> f_{ew}$
- One Higgs field,
- one scalar field
- one exotic quark with PQ charge.

The axion dark matter scenario

Peccei-Quinn symmetry breaks at $T \sim f_a \leq 10^9$GeV

The axions acquire mass at $T \sim \Lambda_{QCD}$

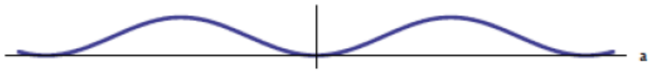
A periodic potential is generated at the QCD hadron transition

$$m_a(T) = \begin{cases} m_a b(\frac{\Lambda}{T})^4, & T \gtrsim \Lambda, \\ m_a, & T \lesssim \Lambda. \end{cases}$$

b=0.018

Periodic potential and oscillations
Due to "vacuum misalignment"

$$V(\theta) = -C(T)\cos(\theta), \text{ where } \theta \equiv a/F_a.$$



Peccei & Quinn(1977), Weinberg(1978), Wilczek, (1978), J. E. Kim(1979),Shifman, Vainstein & Zakharov, (1980),Dine, Fischler & Srednicki(1981),Zhitnitskii(1980).

Temperature dependence of axion mass : instanton size integration

$$C(T) = \int d\rho\, n(\rho, T).$$

(tHooft(1976), Gross, Yaffe & Pisarski(1981))

Mass and couplings are related by the same scale

$$m_a \simeq 6\ \mu eV \left(\frac{10^{12}\ \text{GeV}}{f_a}\right) .$$

$$g \approx \frac{1}{f_a}$$

$$f_a \geq 10^{10}\,GeV$$

Astrophysical constraint
linked to the stellar evolution

$$f_a \leq 10^{12}\,GeV$$

Cosmological constraint given by the
dark energy amount

$$10^{-6}\,eV \leq m_a \leq 10^{-3}\,eV$$

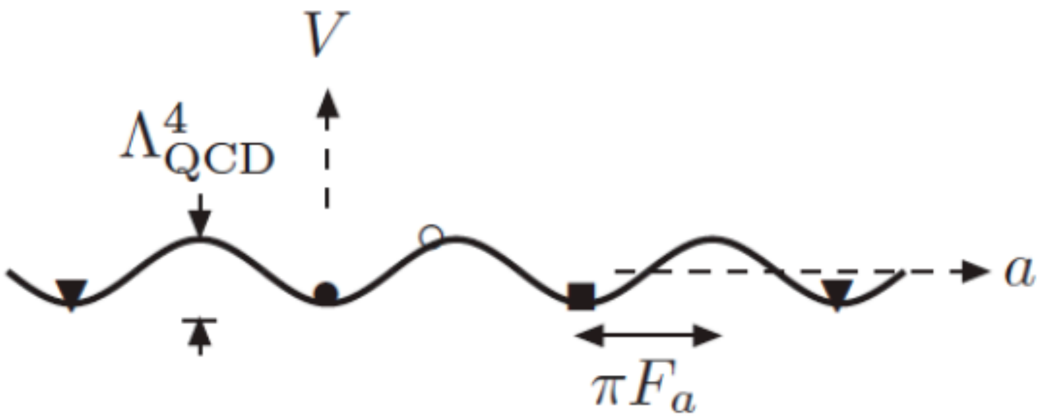
The PQ axion feels the QCD vacuum via the interaction $$\frac{a}{f_a} G\tilde{G}$$

The angle of misalignment is

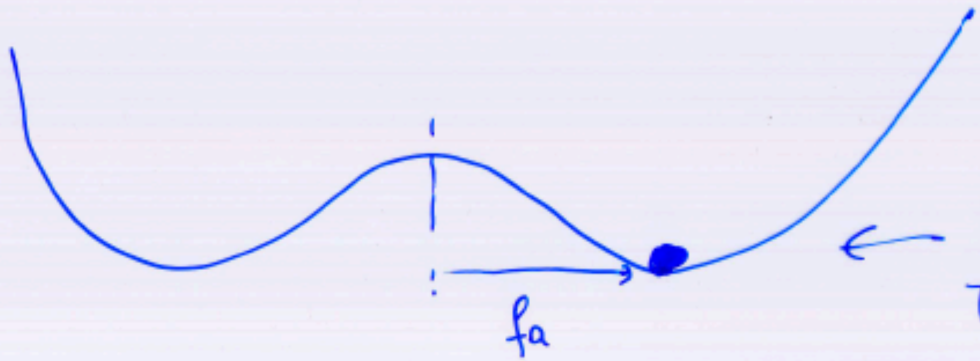
$$\theta = \frac{a(x)}{f_a}$$

The mass is sizeable

$$10^{-3} - 10^{-4} eV$$



BRIEF SUMMARY.



Very early times

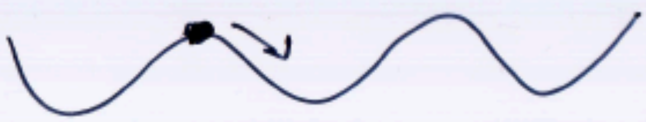
$T \sim 10^{12}$ GeV

← Breaking of the $U(1)_{PQ}$ symmetry

presence of a vacuum valley

At each point the value of the field sits at the vacuum.

⇓ universe expands and cools down to the QCD temperature



the field is misaligned. and rolls down towards the minimum.

The Universe has to be OLD ENOUGH for the field to start oscillating.

Axion Condensates: rotation of the light polarization plane

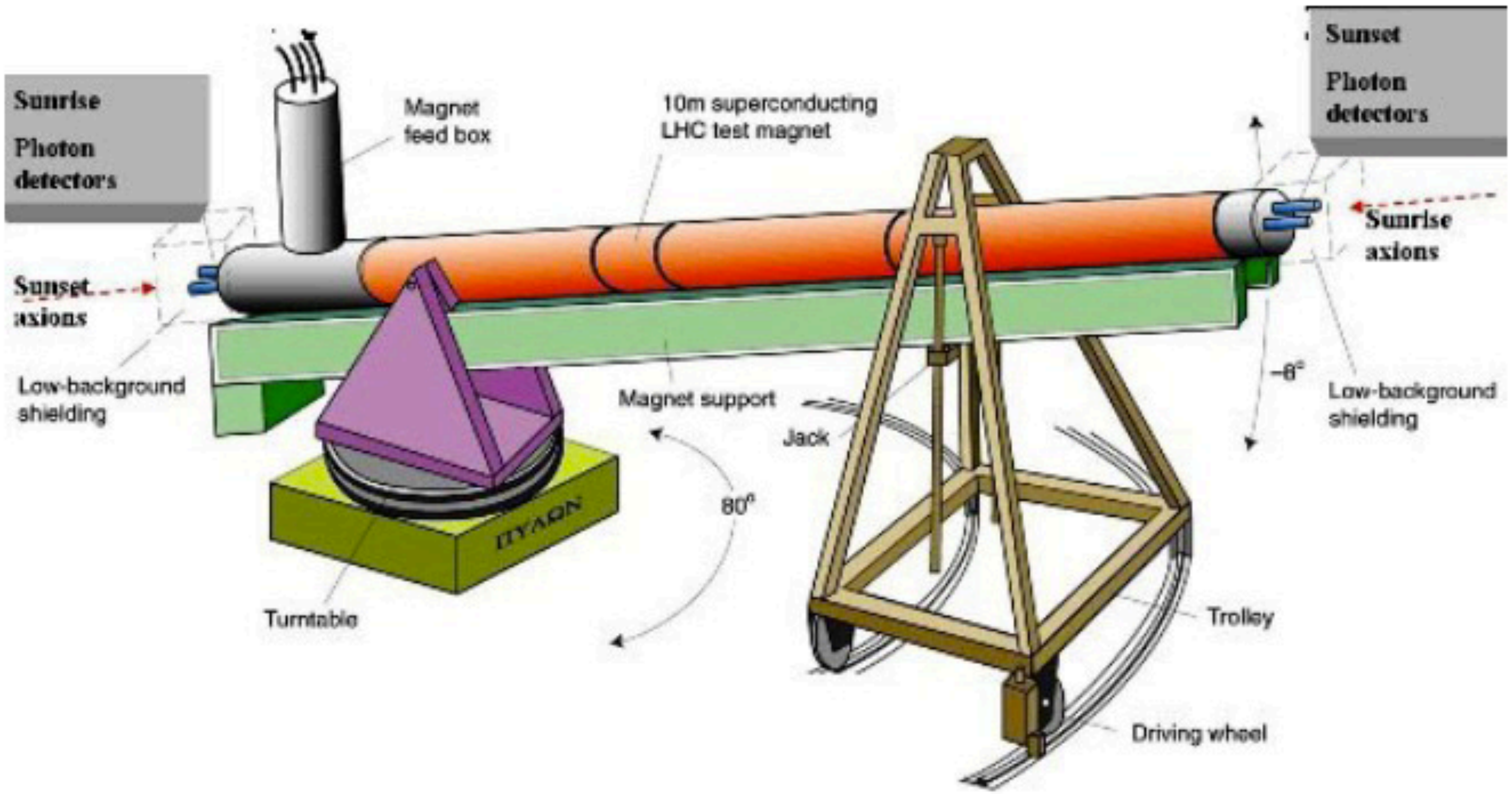
$$\Box(\boldsymbol{E} - \frac{1}{2}\tilde{g}\varphi\boldsymbol{B}) = -\frac{1}{2}\tilde{g}\varphi\Box\boldsymbol{B},$$
$$\Box(\boldsymbol{B} + \frac{1}{2}\tilde{g}\varphi\boldsymbol{E}) = \frac{1}{2}\tilde{g}\varphi\Box\boldsymbol{E}.$$

$$\boldsymbol{D} \equiv \boldsymbol{E} - \frac{1}{2}\tilde{g}\varphi\boldsymbol{B},$$

$$\boldsymbol{H} \equiv \boldsymbol{B} + \frac{1}{2}\tilde{g}\varphi\boldsymbol{E}.$$

$$\Delta\boldsymbol{E} \equiv \boldsymbol{E}(L) - \boldsymbol{E}(0) = \frac{1}{2}\tilde{g}\Delta\varphi\boldsymbol{H}(0).$$

$$\theta = \frac{1}{2}\tilde{g}\Delta\varphi,$$



Sunrise
Photon
detectors
Magnet
feed box
10m superconducting
LHC test magnet
Sunset
Photon
detectors
Sunrise
axions
Sunset
axions
Low-background
shielding
Magnet support
Jack
~8°
Low-background
shielding
80°
Turntable
Trolley
Driving wheel

If this misalignment was caused by electroweak instantons, the axion mass would be much smaller. There could be a very slow rolling of the misplaced field (angle) towards the minimum of the potential.

The field, in this case, could contribute to dark energy.

Oscillations occur only if the condition

$$m_a(T) = 3H(T)$$

is satisfied. This defines (implicitly) the oscillation temperature T_{osc}

The oscillation temperature for the QCD axion is about the QCd scale

The mass induced by the electroweak instanton potential can be of the order

$$m_a^2 \sim \Lambda_{ew}^2 e^{-2\pi/\alpha_w(v)}$$

Where the instanton suppression pushes it towards tiny values

$$e^{-2\pi/\alpha_w(v)} \sim e^{-198}$$

If we had an axion sensitive both to the QCD and EW vacuum the physics would change

The gauging of an anomalous symmetry takes us very far away From the PQ picture, although its main features remain valid.

The pattern gets much more complicated and it will take some time before that we can draw a more complete picture of what is really going on at a phenomenological level.

But there is a lot to learn even from the most simplified models that one can costruct "bottom up", neglecting, at first sight, some formal details.

The models:
MLSOM (non supersymmetric) and **USSM-A** (supersymmetric)

MLSOM----> **a gauged axion generated at the electroweak phase transition**

USSM-A----> a saxion and a gauged axion.

Mariano, Guzzi, C.C.

Anomalous U(1) extensions of the Standard Model (MLSOM)

The SU(3)xSU(2)xU(1)xU(1) Model

kinetic

Higgs doublets

$$\mathcal{L} = -\frac{1}{2}\operatorname{tr} G_{\mu\nu}G^{\mu\nu} - \frac{1}{2}\operatorname{tr} W_{\mu\nu}W^{\mu\nu} - \frac{1}{4}F^{l}_{\mu\nu}F^{\mu\nu,l}$$
$$- \left|\left(\partial_\mu + i\frac{g_2}{2}\tau^a W^a_\mu + i q_l^{(H_u)} g_l A^l_\mu\right) H_u\right|^2 - \left|\left(\partial_\mu + i\frac{g_2}{2}\tau^a W^a_\mu + i q_l^{(H_d)} g_l A^l_\mu\right) H_d\right|^2$$
$$+ Q^\dagger_{Li}\sigma^\mu \mathcal{D}_\mu Q_{Li} + u^\dagger_{Ri}\bar{\sigma}^\mu \mathcal{D}_\mu u_{Ri} + d^\dagger_{Ri}\bar{\sigma}^\mu \mathcal{D}_\mu d_{Ri}$$
$$+ L^\dagger_{Li}\sigma^\mu \mathcal{D}_\mu L_{Li} + e^\dagger_{Ri}\bar{\sigma}^\mu \mathcal{D}_\mu e_{Ri} + \nu^\dagger_{Ri}\bar{\sigma}^\mu \mathcal{D}_\mu \nu_{Ri}$$
$$+ \gamma^u_{ij} H^T_u \tau^2 (Q^t_{Li}\sigma^2 u_{Rj}) + \gamma^d_{ij} H^\dagger_d (Q^t_{Li}\sigma^2 d_{Rj}) + \text{c.c.}$$
$$+ \gamma^e_{ij} H^\dagger_u (L^t_{Li}\sigma^2 e_{Rj}) + \gamma^\nu_{ij} H^T_d \tau^2 (L^t_{Li}\sigma^2 \nu_{Rj}) + \text{c.c.}$$
$$- \frac{1}{2}\sum_I (\partial_\mu a^I + g_l M^I_l A^l_\mu)^2 + E_{lmn}\epsilon^{\mu\nu\rho\sigma} A^l_\mu A^m_\nu F^n_{\rho\sigma}$$
$$+ \sum_I (D_I a^I \operatorname{tr}\{G\wedge G\} + F_I a^I \operatorname{tr}\{W\wedge W\} + C_{Imn} a^I F^m \wedge F^n)$$
$$+ V(H_u, H_d, a^I), \qquad (3.1)$$

L/R fermion

CS

GS

Higgs-axion mixing

Stueckelberg

Irges, Kiritsis, C.

$$
\begin{aligned}
\mathcal{L} \; = \; & - \; \frac{1}{2} Tr\, [F^G_{\mu\nu} F^{G\mu\nu}] - \frac{1}{2} Tr[\, F^W_{\mu\nu} F^{W\mu\nu}] - \frac{1}{4} F^B_{\mu\nu} F^{B\mu\nu} - \frac{1}{4} F^Y_{\mu\nu} F^{Y\mu\nu} \\
& + \; |(\partial_\mu + i g_2 \frac{\tau^j}{2} W^j_\mu + i g_Y q^Y_u A^Y_\mu + i g_B \frac{q^B_u}{2} B_\mu) H_u|^2 \\
& + \; |(\partial_\mu + i g_2 \frac{\tau^j}{2} W^j_\mu + i g_Y q^Y_d A^Y_\mu + i g_B \frac{q^B_d}{2} B_\mu) H_d|^2 \\
& + \; \overline{Q}_{Li}\, i\gamma^\mu \left(\partial_\mu + i g_3 \frac{\lambda^a}{2} G^a_\mu + i g_2 \frac{\tau^j}{2} W^j_\mu + i g_Y q^{(Q_L)}_Y A^Y_\mu + i g_B q^{(Q_L)}_B B_\mu \right) Q_{Li} \\
& + \; \overline{u}_{Ri}\, i\gamma^\mu \left(\partial_\mu + i g_Y q^{(u_R)}_Y A^Y_\mu + i g_B q^{(u_R)}_B B_\mu \right) u_{Ri} \\
& + \; \overline{d}_{Ri}\, i\gamma^\mu \left(\partial_\mu + i g_Y q^{(d_R)}_Y A^Y_\mu + i g_B q^{(d_R)}_B B_\mu \right) d_{Ri} \\
& + \; \overline{L}_i\, i\gamma^\mu \left(\partial_\mu + i g_2 \frac{\tau^j}{2} W^j_\mu + i g_Y q^{(L)}_Y A^Y_\mu + i g_B q^{(L)}_B B_\mu \right) L_i \\
& + \; \overline{e}_{Ri}\, i\gamma^\mu \left(\partial_\mu + i g_Y q^{(e_R)}_Y A^Y_\mu + i g_B q^{(e_R)}_B B_\mu \right) e_{Ri} \\
& + \; \overline{\nu}_{Ri}\, i\gamma^\mu \left(\partial_\mu + i g_Y q^{(\nu_R)}_Y A^Y_\mu + i g_B q^{(\nu_R)}_B B_\mu \right) \nu_{Ri} \\
& - \; \Gamma^d\, \overline{Q}_L H_d d_R - \Gamma^u\, \overline{Q}_L (i\sigma_2 H^*_u) u_R + c.c. \\
& - \; \Gamma^e\, \overline{L} H_d e_R - \Gamma^\nu\, \overline{L} (i\sigma_2 H^*_u) \nu_R + c.c. \\
& + \; \frac{1}{2} (\partial_\mu b + M_1 B_\mu)^2 \\
& + \; \frac{C_{BB}}{M} b F_B \wedge F_B + \frac{C_{YY}}{M} b F_Y \wedge F_Y + \frac{C_{YB}}{M} b F_Y \wedge F_B \\
& + \; \frac{F}{M} b Tr[F^W \wedge F^W] + \frac{D}{M} b Tr[F^G \wedge F^G] \\
& + \; d_1 BY \wedge F_Y + d_2 YB \wedge F_B + c_1 \epsilon^{\mu\nu\rho\sigma} B_\mu C^{SU(2)}_{\nu\rho\sigma} + c_2 \epsilon^{\mu\nu\rho\sigma} B_\mu C^{SU(3)}_{\nu\rho\sigma} \\
& + \; V(H_u, H_d, b),
\end{aligned} \tag{5.1}
$$

The Higgs covariant derivatives responsible for the gauge boson mixing together with the Stueckelberg terms

$$|\mathcal{D}_\mu H_u|^2 + |\mathcal{D}_\mu H_d|^2 + \frac{1}{2}(\partial_\mu b + M_I B_\mu)^2$$

$$\begin{aligned}
\mathcal{D}_\mu H_u &= \left(\partial_\mu + \frac{i}{\sqrt{2}} g_2 \left(T^+ W^+ + T^- W^-\right) + i g_2 \frac{\tau_3}{2} W^3_\mu + i g_Y \frac{1}{2} A^Y_\mu + i g_B \frac{q^B_u}{2} B_\mu\right) H_u \\
\mathcal{D}_\mu H_d &= \left(\partial_\mu + \frac{i}{\sqrt{2}} g_2 \left(T^+ W^+ + T^- W^-\right) + i g_2 \frac{\tau_3}{2} W^3_\mu + i g_Y \frac{1}{2} A^Y_\mu + i g_B \frac{q^B_d}{2} B_\mu\right) H_d
\end{aligned}$$

V/M drives the breaking

$v_{u,}\ v_d << M$

The neutral sector shows a mixing between W3, hypercharge and the anomalous gauge boson, B

$$\mathcal{L}_{mass} = (W_3, A^Y, B)\, \mathrm{M}^2 \begin{pmatrix} W_3 \\ A^Y \\ B \end{pmatrix}$$

Higgs-axion mixing and NG-bosons

The gauge invariant Higgs potential is then

$$V_{PQ} = \sum_{a=u,d} \left(\mu_a^2 H_a^\dagger H_a + \lambda_{aa} (H_a^\dagger H_a)^2 \right) - 2\lambda_{ud} (H_u^\dagger H_u)(H_d^\dagger H_d) + 2\lambda'_{ud} |H_u^T \tau_2 H_d|^2$$

as before, plus the new terms

$$\begin{aligned} V_{\not P \not Q} \;=\; & b_1 \left(H_u^\dagger H_d e^{-i(q_u^B - q_d^B)\frac{b}{M_I}} \right) + \lambda_1 (H_u^\dagger H_d e^{-i(q_u^B - q_d^B)\frac{b}{M_I}})^2 \\ + \; & \lambda_2 (H_u^\dagger H_u)(H_u^\dagger H_d e^{-i(q_u^B - q_d^B)\frac{b}{M_I}}) + \lambda_3 (H_d^\dagger H_d)(H_u^\dagger H_d e^{-i(q_u^B - q_d^B)\frac{b}{M_I}}) + c.c. \end{aligned}$$

The quadratic sector is given by

$$\begin{aligned} V_q(H) + V'_q(H,b) \;=\; & \left({H_u}^-, {H_d}^- \right) \mathcal{N}_1 \begin{pmatrix} {H_u}^+ \\ {H_d}^+ \end{pmatrix} + \left(Re{H_u}^0, Re{H_d}^0 \right) \mathcal{N}_2 \begin{pmatrix} Re{H_u}^0 \\ Re{H_d}^0 \end{pmatrix} \\ & + \left(Im{H_u}^0, Im{H_d}^0, b \right) \mathcal{N}_3 \begin{pmatrix} Im{H_u}^0 \\ Im{H_d}^0 \\ b \end{pmatrix} . \qquad (235) \end{aligned}$$

CP even

CP odd

1 physical axion,
The Axi-Higgs

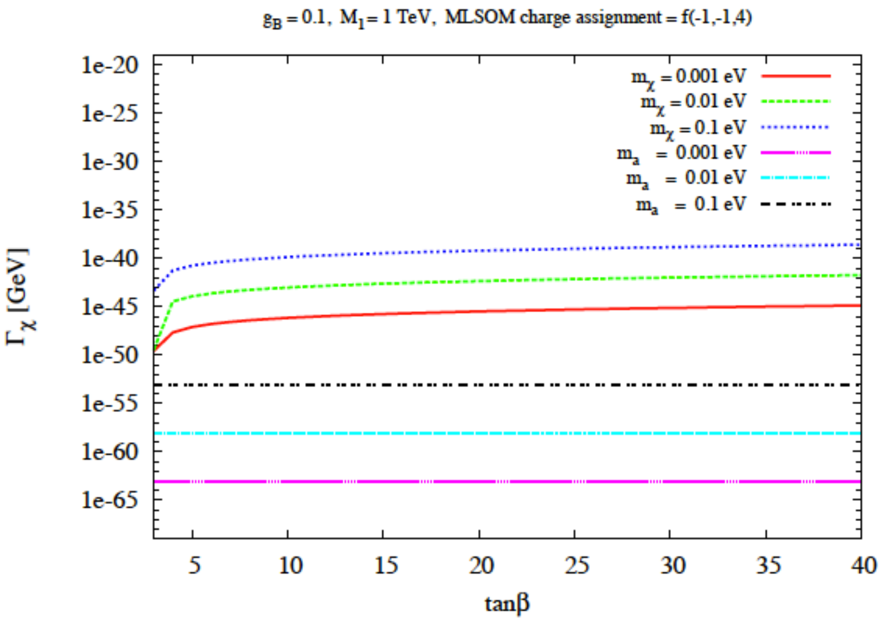
$$\begin{pmatrix} \mathrm{Im}\, H_u^0 \\ \mathrm{Im}\, H_d^0 \\ \cdot \\ a_I' \\ \cdot \end{pmatrix} = O^{\chi} \begin{pmatrix} \chi \\ G_1^0 \\ G_2^0 \\ \cdot \\ \cdot \end{pmatrix}$$

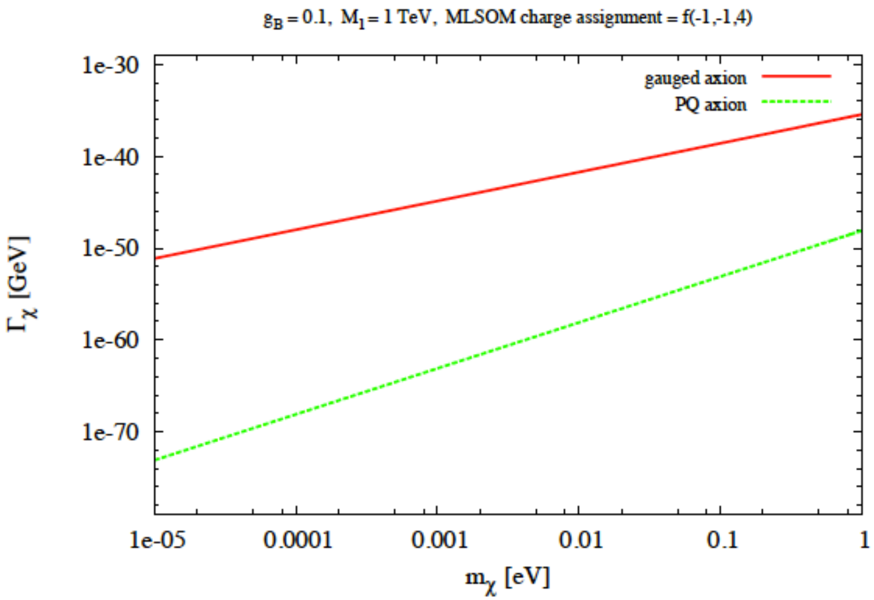
GS Axions

N Nambu-Goldstone modes

Independently of the number of the anomalous U(1)'s one gets only 1 physical axion

Decay rates of the axi-Higgs





Relic Densities

$$V = V_{PQ}(H_u, H_d) + V_{\not{P}\not{Q}}(H_u, H_d, b).$$

$$V_{PQ} = \mu_u^2 H_u^\dagger H_u + \mu_d^2 H_d^\dagger H_d + \lambda_{uu}(H_u^\dagger H_u)^2 + \lambda_{dd}(H_d^\dagger H_d)^2 - 2\lambda_{ud}(H_u^\dagger H_u)(H_d^\dagger H_d) + 2\lambda'_{ud}|H_u^T \tau_2 H_d|^2$$

$$\begin{aligned} V_{\not{P}\not{Q}} = \; & \lambda_0 (H_u^\dagger H_d e^{-i g_B (q_u - q_d)\frac{b}{2M}}) + \lambda_1 (H_u^\dagger H_d e^{-i g_B (q_u - q_d)\frac{b}{2M}})^2 + \lambda_2 (H_u^\dagger H_u)(H_u^\dagger H_d e^{-i g_B (q_u - q_d)\frac{b}{2M}}) + \\ & \lambda_3 (H_d^\dagger H_d)(H_u^\dagger H_d e^{-i g_B (q_u - q_d)\frac{b}{2M}}) + \text{h.c.}, \end{aligned} \tag{12}$$

$(\mathrm{Im} H_d^0, \mathrm{Im} H_u^0, b)$ CP-odd basis

$$\begin{pmatrix} G_0^1 \\ G_0^2 \\ \chi \end{pmatrix} = O^\chi \begin{pmatrix} \mathrm{Im} H_d^0 \\ \mathrm{Im} H_u^0 \\ b \end{pmatrix},$$

The physical axion expanded in the CP-odd sector

$$\chi = \frac{1}{\sqrt{g_B^2 (q_d - q_u)^2 v_u^2 v_d^2 + 2M^2 (v_d^2 + v_u^2)}} \left(\sqrt{2} M v_u, -\sqrt{2} M v_d, g_B (q_d - q_u) v_d v_u \right)$$

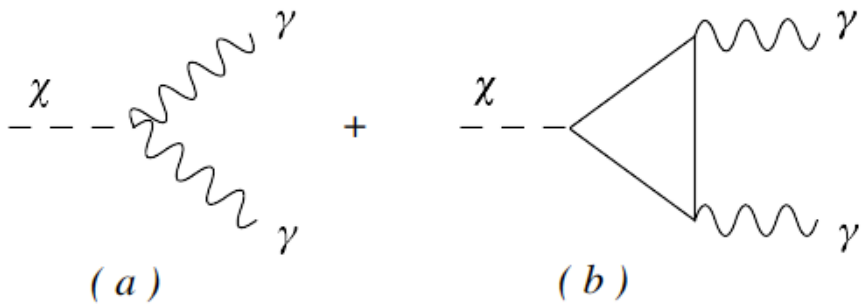
$$b = O^{\chi}_{13}G^1_0 + O^{\chi}_{23}G^2_0 + O^{\chi}_{33}\chi,$$

$$\chi = O^{\chi}_{31}\mathrm{Im}H_d + O^{\chi}_{32}\mathrm{Im}H_u + O^{\chi}_{33}b.$$

$$V' = 4v_u v_d\left(\lambda_2 v_d^2 + \lambda_3 v_u^2 + \lambda_0\right)\cos\left(\frac{\chi}{\sigma_\chi}\right) + 2\lambda_1 v_u^2 v_d^2 \cos\left(2\frac{\chi}{\sigma_\chi}\right),$$

with a mass for the physical axion χ given by

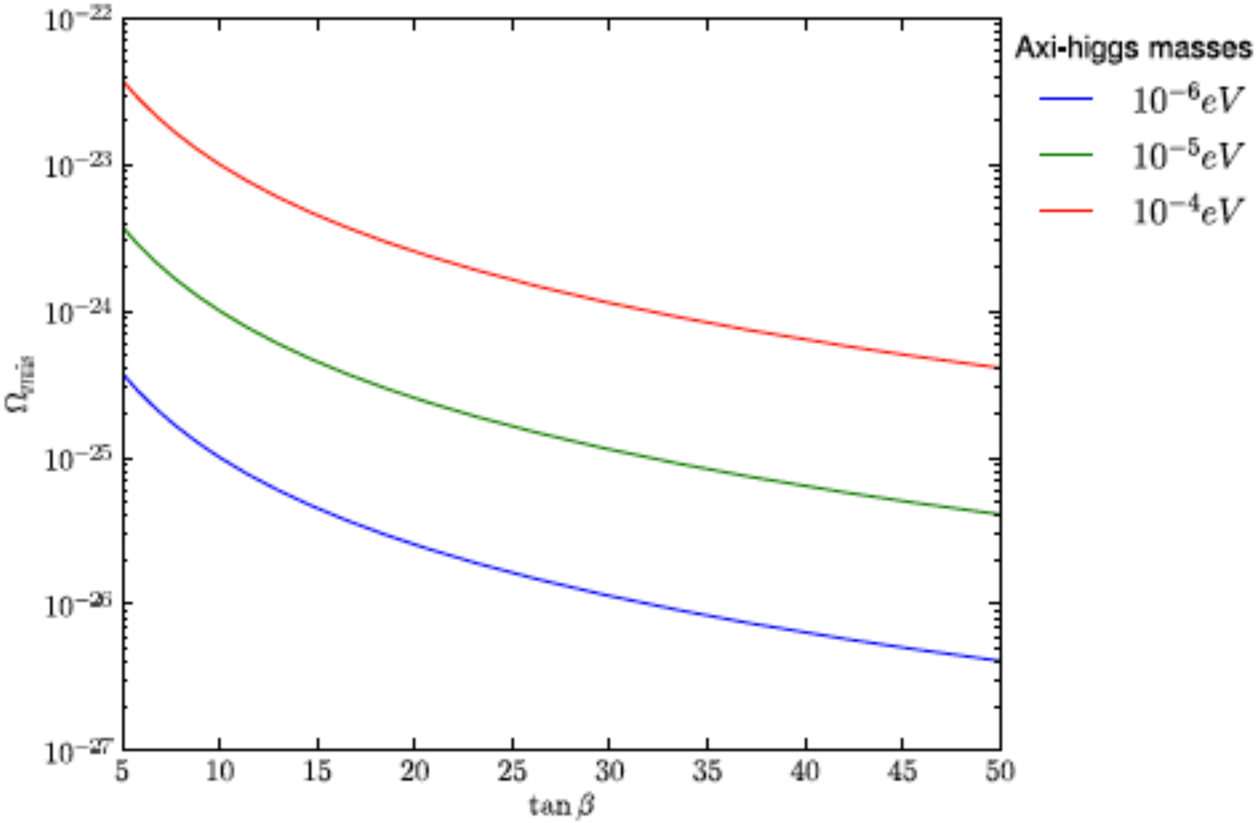
$$m_\chi^2 = \frac{2v_u v_d}{\sigma_\chi^2}\left(\bar{\lambda}_0 v^2 + \lambda_2 v_d^2 + \lambda_3 v_u^2 + 4\lambda_1 v_u v_d\right) \approx \lambda v^2.$$



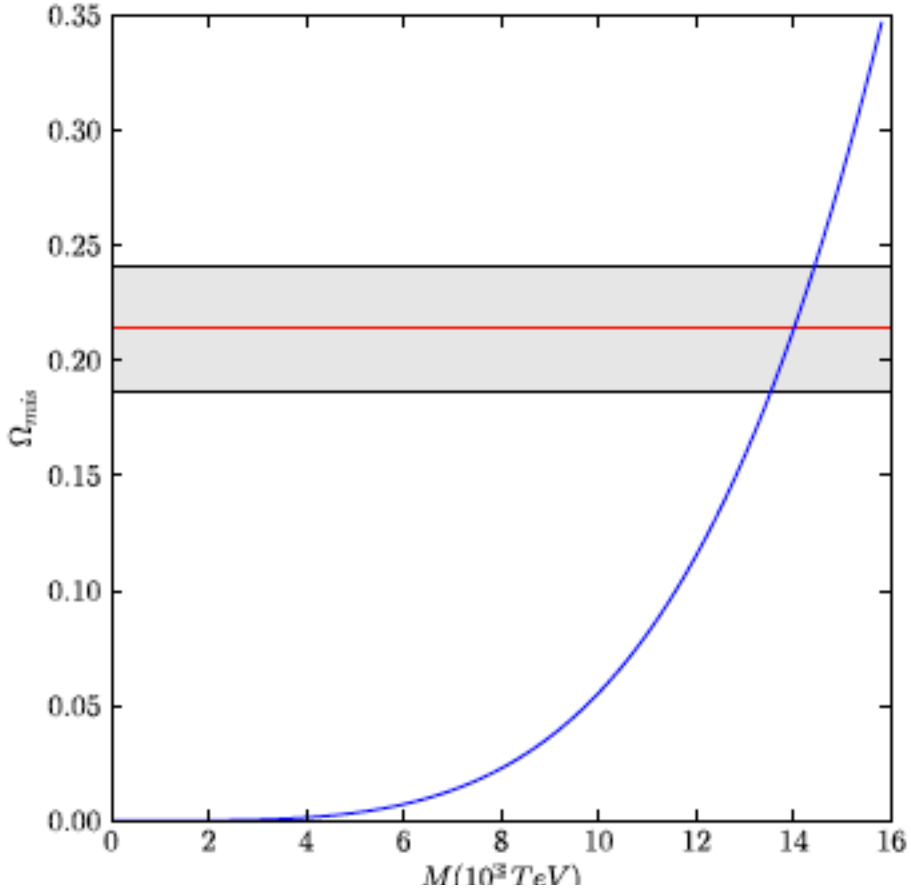
$$\sigma_\chi \equiv \frac{2v_u v_d M}{\sqrt{g_B^2(q_d - q_u)^2 v_d^2 v_u^2 + 2M^2(v_d^2 + v_u^2)}}.$$

The axion, in this extension, feels both the electroweak and the QCD phase transitions:

SEQUENTIAL MISALIGNMENT. The size of the two potentials are different, with the



Small contributions
If M~ 1TeV



Relic density of the axi-Higgs as a function of M. The grey bar represents the measured value of

$$\Omega_{DM}h^2 = 0.1123 \pm 0.0035$$

Supersymmetric generalizations

The Stuckelberg multiplet turns into a supermultiplet:
Axion, Axino, Gaugino (of the anomalous gauge boson)
There is also an extra singlet superfield (NMSSM-type)

The axion has a real and an imaginary component: Re b and Im b.
The Stuckelberg axion, in this case, is the imaginary part, Im b
The real part is called "the saxion" (Re b)

Several neutralinos in the spectrum.
The neutralino has also an axino component which is absent in the NMSSM.

The MSSM with an extra anomalous U(1) symmetry does not allow a
Physical axion but only an axino. The Stuckelberg field is a
Nambu-Goldstone mode.
Higgs-axion mixing doe not occur.

$$\mathcal{L}_{USSM} = \mathcal{L}_{lep} + \mathcal{L}_{quark} + \mathcal{L}_{Higgs} + \mathcal{L}_{gauge} + \mathcal{L}_{SMT} + \mathcal{L}_{GMT}$$

$$\mathcal{W} = \lambda \hat{S}\hat{H}_1 \cdot \hat{H}_2 + y_e \hat{H}_1 \cdot \hat{L}\hat{R} + y_d \hat{H}_1 \cdot \hat{Q}\hat{D}_R + y_u \hat{H}_2 \cdot \hat{Q}\hat{U}_R.$$

The superfield $\hat{\mathbf{b}}$ describes the Stückelberg multiplet,

$$\hat{\mathbf{b}} = b + i\sqrt{2}\theta\psi_{\mathbf{b}} - i\theta\sigma^\mu\bar{\theta}\partial_\mu b + \frac{\sqrt{2}}{2}\theta\theta\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi_{\mathbf{b}} - \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box b + \theta\theta F_{\mathbf{b}},$$

Superfields	SU(3)	SU(2)	$U(1)_Y$	$U(1)_B$
$\hat{\mathbf{b}}(x,\theta,\bar{\theta})$	**1**	**1**	0	s
$\hat{S}(x,\theta,\bar{\theta})$	**1**	**1**	0	B_S
$\hat{L}(x,\theta,\bar{\theta})$	**1**	**2**	-1/2	B_L
$\hat{R}(x,\theta,\bar{\theta})$	**1**	**1**	1	B_R
$\hat{Q}(x,\theta,\bar{\theta})$	**3**	**2**	1/6	B_Q
$\hat{U}_R(x,\theta,\bar{\theta})$	$\bar{\mathbf{3}}$	**1**	-2/3	B_{U_R}
$\hat{D}_R(x,\theta,\bar{\theta})$	$\bar{\mathbf{3}}$	**1**	+1/3	B_{D_R}
$\hat{H}_1(x,\theta,\bar{\theta})$	**1**	**2**	-1/2	B_{H_1}
$\hat{H}_2(x,\theta,\bar{\theta})$	**1**	**2**	1/2	B_{H_2}

USSM-A

NMSSM +
Extra U(1)
with an anomaly
And
A gauged axion
supermultiplet

Mariano, Guzzi, C.C.

Susy Stuckelberg term and Susy counterterms

$$\mathcal{L}_{St} = \frac{1}{4}\int d^4\theta (\hat{\mathrm{b}} + \hat{\mathrm{b}}^\dagger + 2M_{St}\hat{B})^2$$

$$\mathcal{L}_{WZ} = -\frac{1}{2}\int d^4\theta \left\{ \left[\frac{1}{2}\frac{c_G}{M_{St}}\mathrm{Tr}(\mathcal{G}\mathcal{G})\hat{\mathrm{b}} + \frac{1}{2}\frac{c_W}{M_{St}}\mathrm{Tr}(WW)\hat{\mathrm{b}} + \frac{c_Y}{M_{St}}\hat{\mathrm{b}}W^Y_\alpha W^{Y,\alpha} + \frac{c_B}{M_{St}}\hat{\mathrm{b}}W^B_\alpha W^{B,\alpha} + \frac{c_{YB}}{M_{St}}\hat{\mathrm{b}}W^Y_\alpha W^{B,\alpha} \right] \delta(\bar{\theta}^2) + h.c. \right\},$$

$$\delta_B \hat{B} = \hat{\Lambda} + \hat{\Lambda}^\dagger$$

$$\delta_B \hat{\mathrm{b}} = -2M_{St}\hat{\Lambda}$$

Anomalies

$$\{U(1)_B^3\},\ \{U(1)_B, U(1)_Y^2\},\ \{U(1)_B^2, U(1)_Y\},\ \{U(1)_B, SU(2)^2\},\ \{U(1)_B, SU(3)^2\}.$$

$$c_B = -\frac{\mathcal{A}_{BBB}}{384\pi^2} \qquad c_Y = -\frac{\mathcal{A}_{BYY}}{128\pi^2} \qquad c_{YB} = -\frac{\mathcal{A}_{BYB}}{128\pi^2}$$

$$c_W = -\frac{\mathcal{A}_{BWW}}{64\pi^2} \qquad c_G = -\frac{\mathcal{A}_{BGG}}{64\pi^2}.$$

$$
\begin{aligned}
\mathcal{L}_{axion/saxion} =& \frac{1}{2}\left(\partial_\mu \mathrm{Im}\, b + M_{st} B_\mu\right)^2 + \frac{1}{2}\partial_\mu \mathrm{Re} b\, \partial^\mu \mathrm{Re} b + \frac{i}{2}\psi_{\mathbf{b}}\sigma^\mu \partial_\mu \bar{\psi}_{\mathbf{b}} + \frac{i}{2}\bar{\psi}_{\mathbf{b}}\bar{\sigma}^\mu \partial_\mu \psi_{\mathbf{b}} + \frac{1}{2}F_{\mathbf{b}}F_{\mathbf{b}}^\dagger + \\
& M_{St}\mathrm{Re} b D_B - \frac{M_{st}}{\sqrt{2}}(\psi_{\mathbf{b}}\lambda_B + h.c.) + \frac{1}{8}\frac{c_G}{M_{St}}\, G^a_{\mu\nu}G^{a\,\mu\nu}\mathrm{Re}\, b + \\
& \frac{1}{8}\frac{c_W}{M_{St}}\, W^i_{\mu\nu}W^{i\,\mu\nu}\mathrm{Re}\, b + \frac{1}{2}\frac{c_Y}{M_{St}}\, F^Y_{\mu\nu}F^{Y\,\mu\nu}\mathrm{Re}\, b + \frac{1}{2}\frac{c_B}{M_{St}}\, F^B_{\mu\nu}F^{B\,\mu\nu}\mathrm{Re}\, b + \\
& \frac{1}{2}\frac{c_{YB}}{M_{St}}\, F^Y_{\mu\nu}F^{B\,\mu\nu}\mathrm{Re}\, b - \frac{1}{16}\frac{c_G}{M_{St}}\, \epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma}\mathrm{Im}\, b - \frac{1}{16}\frac{c_W}{M_{St}}\, \epsilon^{\mu\nu\rho\sigma}W^i_{\mu\nu}W^i_{\rho\sigma}\mathrm{Im}\, b - \\
& \frac{1}{4}\frac{c_Y}{M_{St}}\epsilon^{\mu\nu\rho\sigma}F^Y_{\mu\nu}F^Y_{\rho\sigma}\mathrm{Im}\, b - \frac{1}{4}\frac{c_B}{M_{St}}\epsilon^{\mu\nu\rho\sigma}F^B_{\mu\nu}F^B_{\rho\sigma}\mathrm{Im}\, b - \frac{1}{4}\frac{c_{YB}}{M_{St}}\epsilon^{\mu\nu\rho\sigma}F^Y_{\mu\nu}F^B_{\rho\sigma}\mathrm{Im}\, b + \\
& \frac{1}{4}\frac{c_W}{M_{St}}\Big[\frac{1}{4}\mathrm{Im}\, b\lambda_{W^i}\sigma^\mu D_\mu \bar{\lambda}_{W^i} - \frac{i}{4\sqrt{2}}\psi_{\mathbf{b}}\lambda_{W^i}\sigma^\mu\bar{\sigma}^\nu W^i_{\mu\nu} + \\
& \frac{1}{4}F_{\mathbf{b}}\lambda_{W^i}\lambda_{W^i} + \frac{1}{2\sqrt{2}}\psi_{\mathbf{b}}\lambda_{W^i}D^i + h.c.\Big] + \frac{1}{4}\frac{c_G}{M_{St}}\Big[\frac{1}{4}\mathrm{Im}\, b\ \lambda_{g^a}\sigma^\mu D_\mu \bar{\lambda}_{g^a} - \\
& \frac{i}{4\sqrt{2}}\psi_{\mathbf{b}}\ \lambda_{g^a}\sigma^\mu\bar{\sigma}^\nu G^a_{\mu\nu} + \frac{1}{4}F_{\mathbf{b}}\ \lambda_{g^a}\ \lambda_{g^a} + \frac{1}{2\sqrt{2}}\psi_{\mathbf{b}}\ \lambda_{g^a}D^a + h.c.\Big] + \\
& \frac{c_Y}{M_{St}}[\mathrm{Im}\, b\lambda_Y\sigma^\mu D_\mu\bar{\lambda}_Y - \frac{i}{2\sqrt{2}}\psi_{\mathbf{b}}\lambda_Y\sigma^\mu\bar{\sigma}^\nu F^Y_{\mu\nu} + \frac{1}{2}F_{\mathbf{b}}\lambda_Y\lambda_Y + \frac{1}{\sqrt{2}}\psi_{\mathbf{b}}\lambda_Y D_Y + h.c.] + \\
& \frac{c_B}{M_{St}}[\mathrm{Im}\, b\lambda_B\sigma^\mu D_\mu\bar{\lambda}_B - \frac{i}{2\sqrt{2}}\psi_{\mathbf{b}}\lambda_B\sigma^\mu\bar{\sigma}^\nu F^B_{\mu\nu} + \frac{1}{2}F_{\mathbf{b}}\lambda_B\lambda_B + \frac{1}{\sqrt{2}}\psi_{\mathbf{b}}\lambda_B D_B + h.c.] + \\
& \frac{c_{YB}}{M_{St}}[(\mathrm{Im}\, b\lambda_Y\sigma^\mu\partial_\mu\bar{\lambda}_B - i\mathrm{Re}\, b\lambda_Y\sigma^\mu\partial_\mu\bar{\lambda}_B - \frac{i}{2\sqrt{2}}\lambda_Y\sigma^\mu\bar{\sigma}^\nu F^B_{\mu\nu}\psi_{\mathbf{b}} + \frac{1}{2}F_{\mathbf{b}}\lambda_Y\lambda_B + \frac{1}{\sqrt{2}}\psi_{\mathbf{b}}\lambda_Y D_B - \\
& \mathrm{Re} b\, D_Y\, D_B) + (Y \leftrightarrow B) + h.c.].
\end{aligned}
\tag{21}
$$

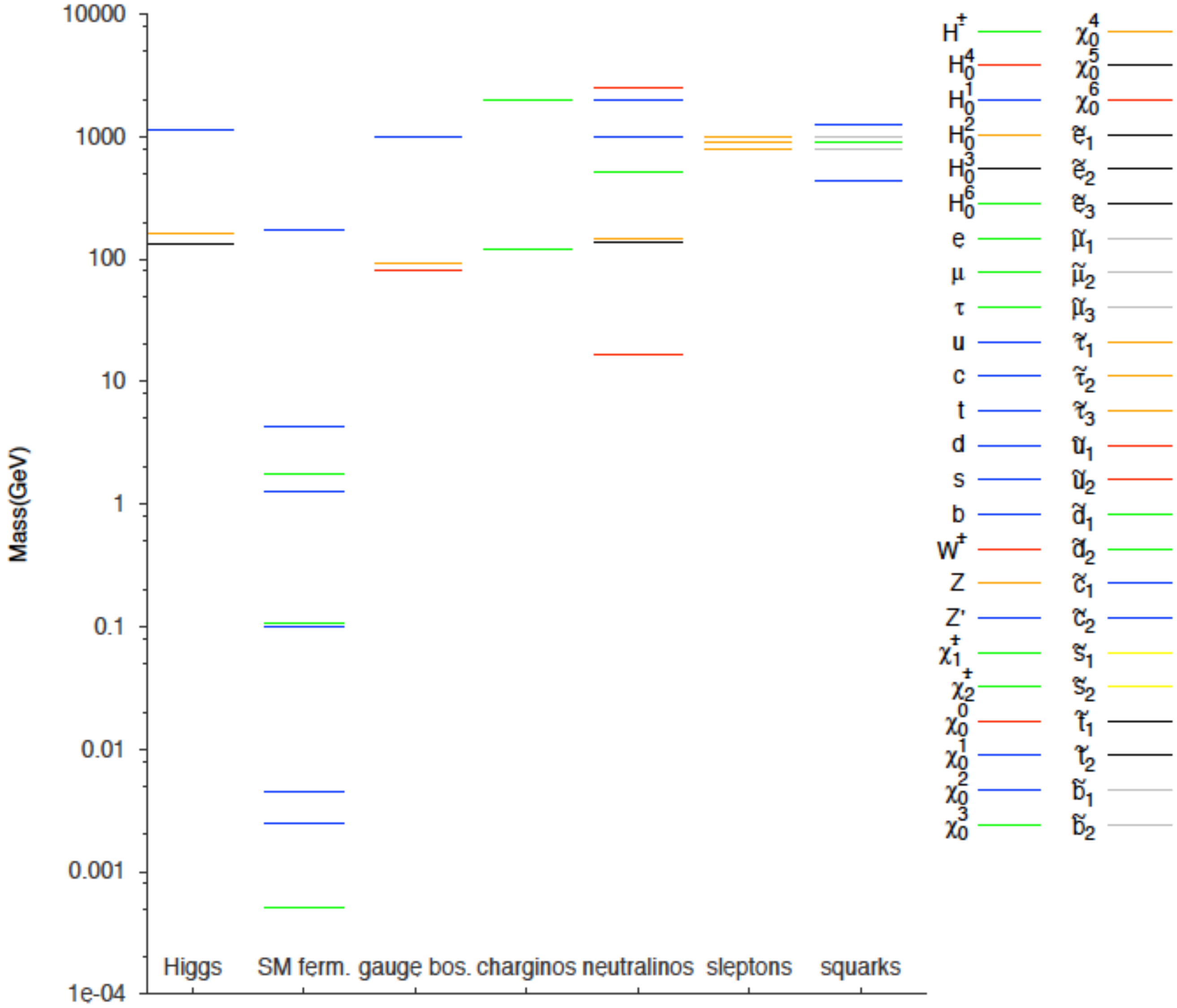
Stuckelberg kinetic

Peccei Quinn

Kinetic saxion

Saxion FF

Axino-gaugino



ure 9: Mass spectrum of all the particles of the model obtained with the parameter values illustrated in Sec. 10

Potential of the Axion-Saxion

An extra U(1) symmetry combined with an extra singlet superpotential allows to extract a physical axion from the spectrum. In this case the potential is a combination of
The Standard NMSSM-type potential plus the extra PQ.breaking potential

$$V = V_D + V_F + V_{SMT}$$

$$V_D = -\frac{1}{2}\Big[\bar{D}^2_{B,OS} - 8\,\mathrm{Re} b \frac{c_{YB}}{M_{St}} \bar{D}_{Y,OS}\bar{D}_{B,OS} + \bar{D}^2_{Y,OS}\Big] + \frac{g_2^2}{8}\Big(|H_1|^2 + |H_2|^2\Big)^2 + \frac{g_2^2}{2}|H_1^\dagger H_2|^2$$

$$V_F = |\lambda H_1 \cdot H_2|^2 + |\lambda S|^2(|H_1|^2 + |H_2|^2)$$

$$V_{SMT} = m_1^2|H_1|^2 + m_2^2|H_2|^2 + m_S^2|S|^2 + (a_\lambda S H_1 \cdot H_2 + h.c.)$$

$$V_1 = a_1 S^4 e^{-i4g_B B_S \frac{\mathrm{Im} b}{2M_{St}}} + h.c.$$

$$V_2 = e^{-ig_B B_S \frac{\mathrm{Im} b}{2M_{St}}}\Big(a_2 H_1 \cdot H_2 S^2 + b_2 H_1^\dagger H_1 S + b_3 H_2^\dagger H_2 S + b_4 S^\dagger S^2 + d_1 S\Big) + h.c.$$

$$V_3 = e^{-ig_B 2B_S \frac{\mathrm{Im} b}{2M_{St}}}\Big(a_3 H_1^\dagger H_1 S^2 + a_4 H_2^\dagger H_2 S^2 + a_5 S^\dagger S^3 + c_1 S^2\Big) + h.c.$$

$$V_4 = a_6 (H_1 \cdot H_2)^2 e^{ig_B 2B_S \frac{\mathrm{Im} b}{2M_{St}}} + h.c.$$

$$V_5 = b_1 S^3 e^{-ig_B 3B_S \frac{\mathrm{Im} b}{2M_{St}}} + h.c.$$

$$V_6 = a\, H \quad H\, e^{ig_B B_S \frac{\mathrm{Im} b}{2M_{St}}} + h.c.$$

$$S(x) = \frac{1}{\sqrt{2}}\left(\rho_S(x) + v_S\right) e^{i\Phi_S(x)}$$

Ho to identify the axion: analysis of the CP odd sector

The Stuckelberg field Im b turns developes a physical component

$$\chi = \frac{1}{N_\chi}\left[2M_{st}v_1 v_2^2\,\mathrm{Im}H_1^0 + 2M_{St}v_1^2 v_2\,\mathrm{Im}H_2^0 - 2M_{st}v^2 v_S\,\mathrm{Im}\,S + B_S\,g_B\sec\alpha(v^2 v_S^2 + v_1^2 v_2^2)\mathrm{Im}\,b\right]$$

$$N_\chi = \sqrt{4M_{St}^2 v^2(v^2 v_S^2 + v_1^2 v_2^2) + B_S^2 g_B^2\sec^2\alpha(v^2 v_S^2 + v_1^2 v_2^2)^2}.$$

Misalignment is due to the extra potential potential

$$V_1 \sim \lambda_{eff} v^4\cos(\overline{\theta}_1).$$

$$V_3 = e^{-ig_B B_S\frac{\mathrm{Im}b}{M_{St}}} a_3 H_1^\dagger H_1 S^2 + \dots$$

$$\bar{\theta}_3 \equiv \frac{2\Phi_S(x)}{v_S} - \frac{g_B B_S \mathrm{Im}b(x)}{M_{St}},$$

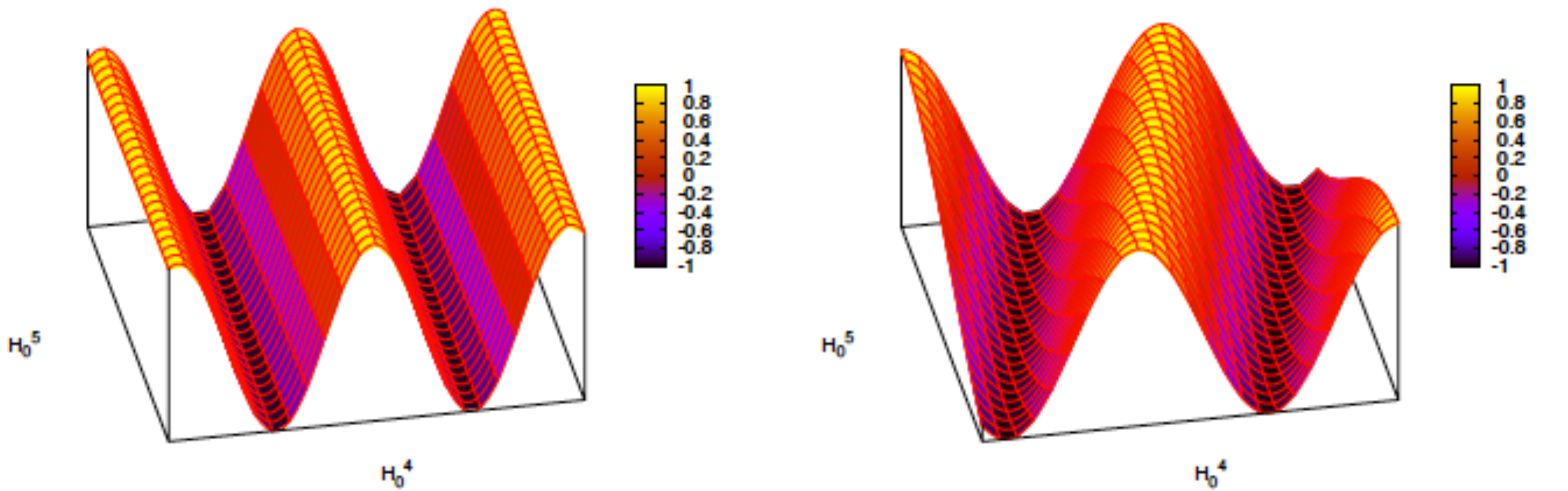
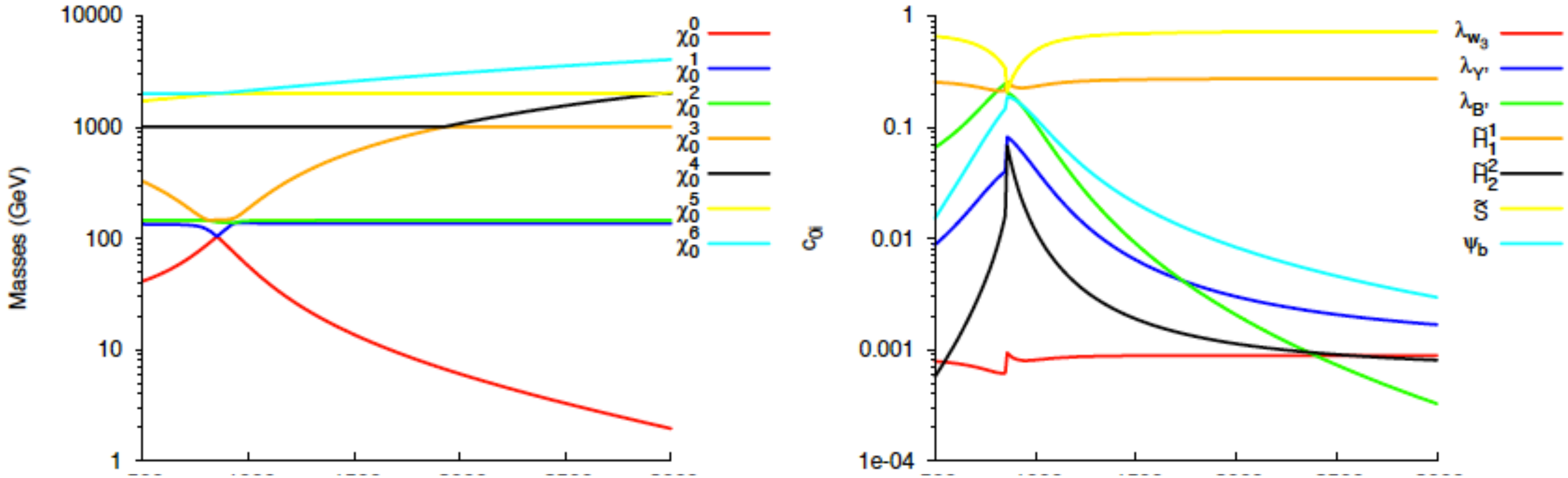


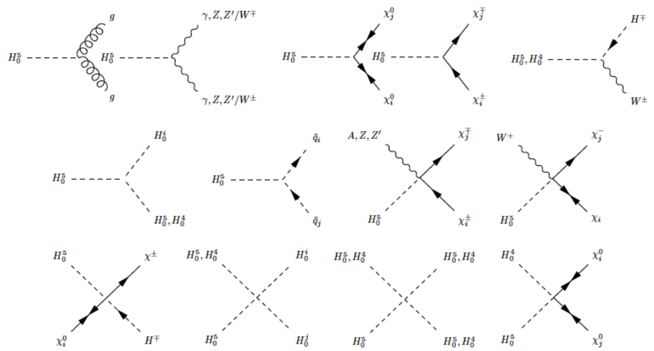
Figure 5: Shape of the extra potential V' at the electroweak scale in the CP-odd sector in the $(H_0^4, \chi \equiv H_0^5)$ plane. χ is an almost flat direction for a strength induced by the instanton vacuum at the electroweak scale (left panel) and acquires a curvature for an axion mass in the MeV region (curvature in the χ direction, right panel).

Singlino dominance of the neutralino

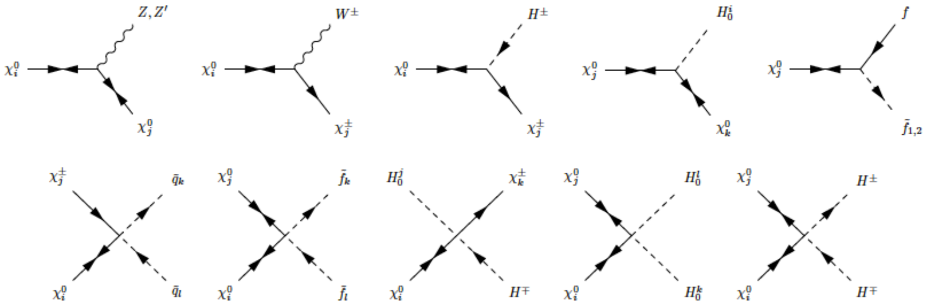


Masses of the neutralinos (left panel) and components of the LSP (right panel) as functions of M_{st}

At large tanBeta the neutralino tends to become very light as the Stuckelberg mass grows. This is respnsible for an overproduction of neutralinos and large relic Densities which need to be made compatible with WMAP.



Axion interactions



in	s-channel	out
$\chi_i^0\,\chi_j^0$	Z, Z'	$H^\pm H^\mp, H_0^k H_0^4, H_0^k H_0^5, Z/Z' H_0^k, \bar{f} f, \tilde{f}^\dagger \tilde{f}$
	H_0^k	$H^\pm H^\mp, H_0^l H_0^m, H_0^4 H_0^4, H_0^4 H_0^5, H_0^5 H_0^5, Z/Z' H_0^4/H_0^5,$ $W^\pm H^\mp, Z/Z' Z/Z', W^\pm W^\mp, \bar{f} f, \tilde{f}^\dagger \tilde{f}$
	H_0^4, H_0^5	$H_0^k H_0^4, H_0^k H_0^5, Z/Z' H_0^k, W^\pm H^\mp, \bar{f} f, \tilde{f}^\dagger \tilde{f}$

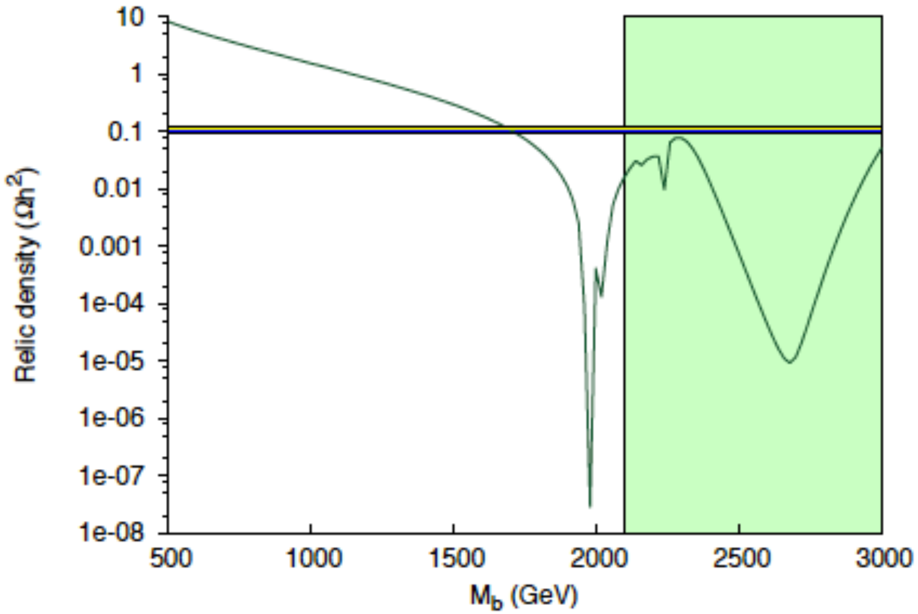
in	t/u-channel	out
$\chi_i^0\,\chi_j^0$	χ_k^0	$H_0^l H_0^m, H_0^l H_0^4/H_0^5, H_0^4/H_0^5 H_0^4/H_0^5,$ $Z/Z' H_0^l/H_0^4/H_0^5, Z/Z' Z/Z'$
	$\chi_k^\pm$	$W^\pm/H^\pm W^\mp/H^\mp$
	$\tilde{f}$	$\bar{f} f$

Neutralino interactions

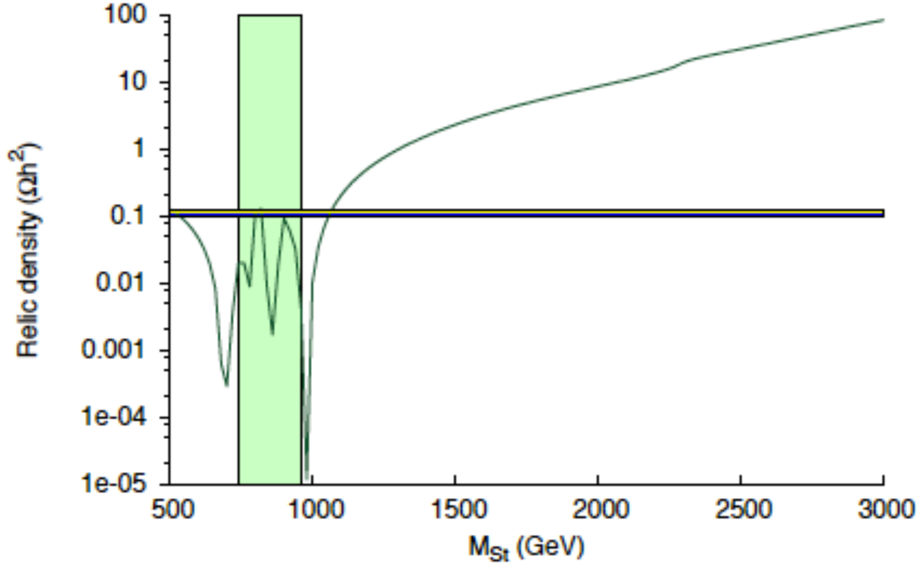
The numerical analysis of these models is very involved and requires
The writing of several interfaces between existing codes.

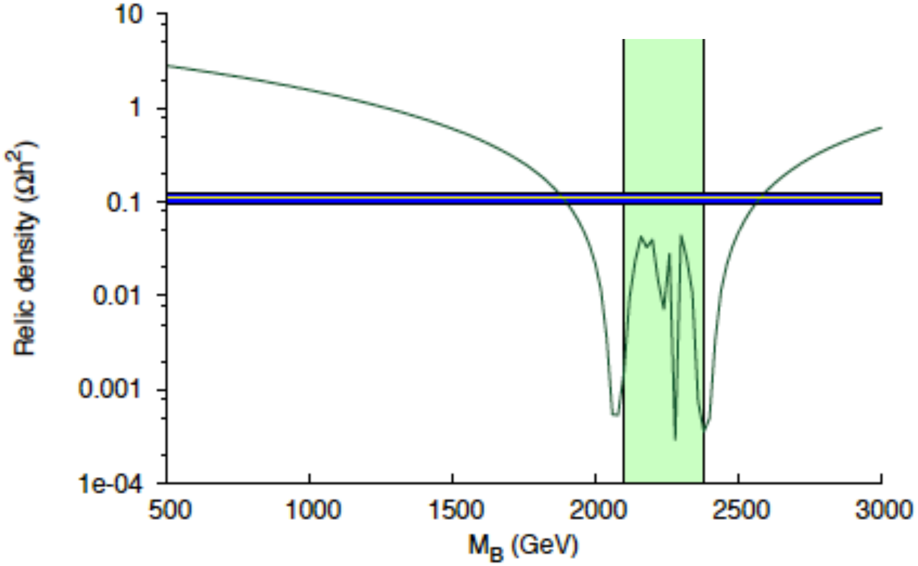
1) Computation of the 2-to-2 scattering amplitudes
2) Interfacing the model with Micromega's (Boudjema et al.)
   for the computation of the corresponding densities.

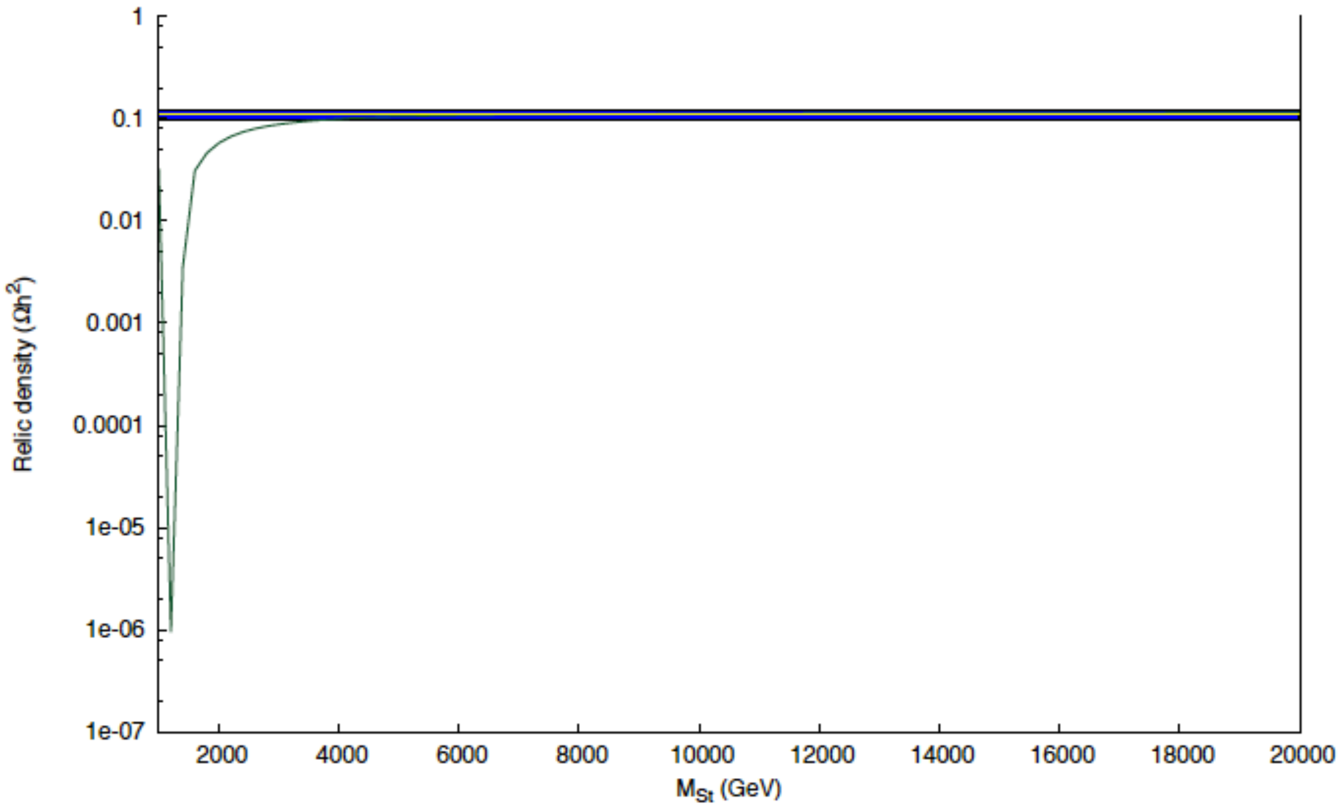
Relic Densities



High tan-beta







Relic densities of neutralinos at low tan-beta

SUMMARY

The mass of the axion in anomalous abelian models is essentially a free parameter, closely linked to the strength of an “extra potential” which can be generated Non-perturbatively at the several phase transitions of the early universe.

The mass of the axion is unrelated to the coupling of the axion to the gauge fields and as such this is an important variant respect to the PQ axion.
Limits on the PQ axion are essentially derived form this strong constraint coming from this special relations

We can satisfy the WMAP constraints in the supersymmetric case both at High and at low tan-beta, with two intervals on the allowed Stuckelberg mass.

In the non-supersymmetric case there are no bounds emerging for the Stuckelberg mass since there are no relic densities.

CONCLUSIONS

The models that we have presented contain the field theory of the Green-Schwarz mechanism in a simplified non-stringy form.

We have also pointed out that these models may appear as the low energy theory of GUT's when some particles of an anomaly-free multiplet decouple. The simpleset example being a heavy RH neutrino.

Theories of axion-like particles have necessarily to involve GAUGE ANOMALIES rather than GLOBAL anomalies.

It is possible to put significant bounds on these models from numerical simulations of relic densities.
Being the phsyical axion charged both under SU(2) and SU(3) the misalignment of this field can be sequential.

THANK YOU for your ATTENTION