General Mass Scheme for NLO Jet Production in DIS

Piotr Kotko

Institute of Nuclear Physics, Cracow

in collab. with W. Slominski (Jagiellonian Univ.)

Southern Methodist University, Dallas
September 10, 2012

Outline

In standard QCD calculations we have two approaches to heavy quarks

- when energy scale $\mu \gg m_{\mathbf{Q}}$ we treat **Q** as massless; there is PDF for **Q**
- when $\mu \sim m_{\mathbf{Q}}$ we keep **Q** massive; there is no PDF for **Q**

General Mass Scheme: a method of treating heavy quarks which is reliable for $0 < m_{\mathbf{Q}}/\mu < 1$ and contains both approaches as a special cases.

There are such solutions for nucleon structure functions – e.g. ACOT scheme (Aivazis-Collins-Olness-Tung), TR scheme (Thorne-Roberts), FONLL, ...

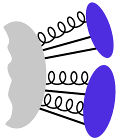
This seminar: general mass scheme for NLO jet production processes.

- Problems in QCD calculations of jet observables
- Dipole subtraction method with massive quarks
- Factorization of quasi-collinear singularities according to ACOT scheme
- Example

Basic papers:

- S. Catani, M. H. Seymour, Nucl. Phys. B 485 (1997)
- S. Catani, S. Dittmaier, M. H. Seymour, Z. Trocsanyi, Nucl. Phys. B 627 (2002)
- S. Dittmaier, Nucl. Phys. B 565 (2000)
- M.A.G. Aivazis, J.C. Collins, F.I. Olness, W.K. Tung, Phys. Rev. D50, 3102 (1994)
- J.C. Collins, Phys. Rev. D58, 094002 (1998)
- P. Kotko, PHD thesis
- P. Kotko, W. Slominski, arXiv:1206.4024

Basics



IDEA: the momenta of partons $\{p_1, \ldots, p_n\}$ are translated into the momenta of jets $\{P_1, \ldots, P_m\}$

$$\{p_1, \ldots, p_n\} \xrightarrow{F_n} \{P_1, \ldots, P_m\}.$$

- jets are characterized by their four-momenta only
- final state hadrons *are not identified*

- the jet function F_n is some mathematical realization of a *jet algorithm* – a way to cluster several partons into a single jet
  - they have to be *infra-red safe* (explained later)
  - two families: k_T-cluster and cone algorithms

PROBLEMS IN THEORETICAL CALCULATIONS

Integration over the phase space (PS) of final state partons possesses cuts on various kinematic variables (transverse momenta, angles etc.) – this can be effectively done only via MC

- at NLO integration may diverge due to collinear or soft emissions – radiative corrections (RC)
- at NLO there are also virtual corrections (VC)

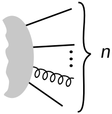
Basics (cont.)

- RC and VC both have singularities which cancel in physical observables (Kinoshita-Lee-Nauenberg theorem)
    - singularities in RC appear *after* PS integration
    - singularities in VC exist at the *integrand* level

CROSS SECTION FOR *n* JETS

- LO cross section

$$\sigma_n^{\mathrm{LO}} = \int d\Gamma_n \, |\mathcal{M}_n|^2 \, F_n$$

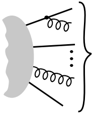
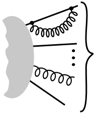


- NLO contribution to the cross section

$$\sigma_n^{\mathrm{NLO}} = \sigma_n^{\mathrm{RC}} + \sigma_n^{\mathrm{VC}} - \sigma_n^{\mathrm{fact}}$$

$$\sigma_n^{\mathrm{RC}} = \int d\Gamma_{n+1} \, \left|\mathcal{M}_{n+1}\right|^2 \, F_{n+1}$$

$$\sigma_n^{\mathrm{VC}} = \int d\Gamma_n \, \mathcal{M}_n^{\mathrm{loop}} \, F_n$$

RC =  $n+1$, VC =  n

- methods of cancelling infra-red (IR) singularities:
    - phase space slicing
    - subtraction method
    - antenna method

Subtraction Method

- First UV renormalization has to be done (using dimensional regularization with $D = 4 - 2\varepsilon$ dimensions). There are remaining IR singularities in VC appearing as $(1/\varepsilon)^m$ poles.
- Construct a mapping $\Gamma_{n+1} \to \Gamma_n$, i.e. a set of new momenta $\tilde{\Gamma}_n = \{\tilde{p}_1, \ldots, \tilde{p}_n\}$ which are expressed in terms of the old ones $\Gamma_{n+1} = \{p_1, \ldots, p_{n+1}\}$ such that

$$\tilde{\Gamma}_n\Big|_{\mathcal{S}} = \Gamma_{n+1}\Big|_{\mathcal{S}},$$

  where $\mathcal{S}$ is a 'singular subspace' of the full PS Γ_{n+1}.

  On the level of differential PS the following relation can be stated

$$d\Gamma_{n+1} = d\tilde{\Gamma}_n \otimes d\phi,$$

  where $d\phi$ parametrizes $\mathcal{S}$ (leads to singularities after integration).
- Construct an auxiliary function such that it mimics all the singularities of $\left|\mathcal{M}_{n+1}\right|^2$, i.e.

$$\left|\mathcal{M}^{\rm sub}_{n+1}\right|^2\Big|_{\mathcal{S}} = \left|\mathcal{M}_{n+1}\right|^2\Big|_{\mathcal{S}}.$$

  It has the general 'factorized' form

$$\left|\mathcal{M}^{\rm sub}_{n+1}\right|^2 = \hat{V} \otimes |\mathcal{M}_n|^2\,,$$

  where $\mathcal{M}_n$ is calculated using $\tilde{\Gamma}_n$.

Subtraction method (cont.)

- Auxiliary cross section is constructed as

$$\sigma_n^{\rm sub} = \int d\Gamma_{n+1} \left|\mathcal{M}_{n+1}^{\rm sub}\right|^2 F_n.$$

- Add and subtract $\sigma_n^{\rm sub}$ from $\sigma_n^{\rm NLO}$

$$\sigma_n^{\rm NLO} = \left(\sigma_n^{\rm RC} - \sigma_n^{\rm sub}\right) + \left(\sigma_n^{\rm VC} + \sigma_n^{\rm sub}\right)$$

$$\sigma^{\rm NLO} = \int d\Gamma_{n+1}\left\{\left|\mathcal{M}_{n+1}\right|^2 F_{n+1} - \left|\mathcal{M}_{n+1}^{\rm sub}\right|^2 F_n\right\}$$

$$+ \int d\Gamma_n\left\{\mathcal{M}_n^{\rm loop} + \int d\phi \left|\mathcal{M}_{n+1}^{\rm sub}\right|^2 - \mathcal{C}_n\right\} F_n$$

- The jet function has to be IR safe

$$F_{n+1}\big|_{\mathcal{S}} = F_n|_{\mathcal{S}},$$

so the red curly bracket is integrable in 4 dimensions via MC.

- In the green bracket the integral over $d\phi$ has to be done analytically

$$\int d\phi \left|\mathcal{M}_{n+1}^{\rm sub}\right|^2 = \left(\int d\phi\, \hat{V}\right) \otimes |\mathcal{M}_n|^2.$$

Resulting IR poles $(1/\varepsilon)^m$ are cancelled against similar poles buried in $\mathcal{M}_n^{\rm loop}$.

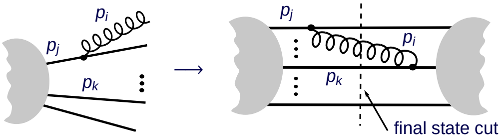
Singularities of Matrix Elements

In order to construct $\mathcal{M}_{n+1}^{\text{sub}}$ one has to investigate singularities of tree-level matrix elements.

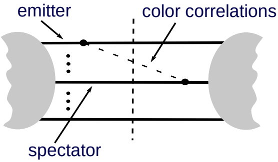
SOFT SINGULARITIES

A denominator can become zero if a final state parton i (gluon) has vanishing energy implying $p_i \to 0$. In that limit $\left|\mathcal{M}_{n+1}\right|^2$ behaves as

$$\left|\mathcal{M}_{n+1}\right|^2 \rightsquigarrow \alpha_s \sum_{j\neq i} \frac{-1}{p_i \cdot p_j} \sum_{k\neq j} \left(\frac{p_j \cdot p_k}{p_i \cdot \left(p_j + p_k\right)} - \frac{m_j^2}{2p_i \cdot p_j} \right) \langle \mathcal{M}_n |\, \hat{T}_j \cdot \hat{T}_k \,| \mathcal{M}_n \rangle$$



- nomenclature: emitter, spectator; both can be initial state (IS) or final state (FS)
- i is always the emitted parton, j, k are any final state partons, a is an initial state parton



Singularities of ME (cont.)

COLLINEAR SINGULARITIES

They appear when two massless FS partons i, j or FS and IS i, a are collinear.

We aim at *massive calculations* with, however, control over the potential collinear singularities (CS). This is done using the quasi-collinear limit.

Consider initial state emissions. Momentum p_i is decomposed to transverse momentum k_T and longitudinal component (with fraction u) with respect to collinear direction given by p_a.

- Uniform rescaling k_T and the masses

$$k_T \equiv \lambda k_T, \quad m_q \equiv \lambda m_q, \quad \lambda \to 0.$$

- Tree level $\left|\mathcal{M}_{n+1}\right|^2$ behaves as

$$\left|\mathcal{M}_{n+1}\right|^2 \rightsquigarrow \alpha_s \frac{1}{\lambda^2} \frac{-1}{2p \cdot k} \frac{1}{1-u} \langle \mathcal{M}_n | \hat{P}_{ai}(u, m_a; \varepsilon) | \mathcal{M}_n \rangle,$$

where $\hat{P}_{ai}$ are splitting matrices:

$$\hat{P}_{qq} = C_F \left(\frac{1+\bar{u}^2}{u} - \varepsilon u + \frac{2\bar{u} m_a^2}{p_{ai}^2 - m_a^2} \right), \qquad \left(\hat{P}_{qg}\right)^{\mu\nu} = C_F (1-\varepsilon) \left(-\bar{u} g^{\mu\nu} - \frac{4 k_T^\mu k_T^\nu}{\bar{u} p_{ai}^2} \right),$$

$$\hat{P}_{gq} = T_R \left[1 - \frac{2}{1-\varepsilon} \left(u\bar{u} + \frac{\bar{u} m_i^2}{p_{ai}^2 - m_i^2} \right) \right], \qquad \left(\hat{P}_{qg}\right)^{\mu\nu} = 2C_A \left[-g^{\mu\nu} \left(\frac{\bar{u}}{u} + \frac{u}{\bar{u}} \right) - (1-\varepsilon) \frac{2\bar{u} k_T^\mu k_T^\nu}{p_{ai}^2} \right],$$

where $p_{ai} = p_a - p_i$ and $\bar{u} = 1 - u$.

Dipole Method

Particular realization of subtraction method → Dipole Subtraction Method. $\left|\mathcal{M}^{\mathrm{sub}}_{n+1}\right|^2$ is given as a sum of distinct 'dipoles' for all possible emissions and emitter/spectator combinations

$$\left|\mathcal{M}^{\mathrm{sub}}_{n+1}\right|^2 = \sum_{\mathrm{comb}} \left(\mathcal{D}^{\mathrm{FE-FS}}_{i,j,k} + \mathcal{D}^{\mathrm{FE-IS}}_{i,j,a} + \mathcal{D}^{\mathrm{IE-FS}}_{i,a,j}\right)$$

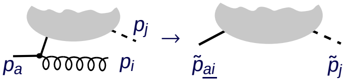
EXAMPLE: dipole $\mathcal{D}^{\mathrm{IE-FS}}_{g,q,j}$ (initial state $q(p_a) \to q\, g(p_i)$ splitting)

- PS mapping:

$$\Gamma_{n+1}\left(p_a; p_1, \ldots, p_i, \ldots, p_j, \ldots, p_{n+1}\right) \to \tilde{\Gamma}_n\left(\tilde{p}_{\underline{ai}}; p_1, \ldots, \tilde{p}_j, \ldots, p_n\right),$$

  where in present case $i = g$, $\underline{ai} = q$ the new 'dipole momenta' are

$$\tilde{p}^\mu_j = \tilde{w}\left(p^\mu_i + p^\mu_j\right) - \tilde{u} p^\mu_a,$$
$$\tilde{p}^\mu_{\underline{ai}} = (\tilde{w} - 1)\left(p^\mu_i + p^\mu_j\right) - (\tilde{u} - 1)\, p^\mu_a.$$



  The soft limit is approached when $\tilde{u} \to 0$ ($\tilde{w} \to 1$).

- PS factorization:

$$d\Gamma_{n+1} = \int d\tilde{u}\, \tilde{\Gamma}_n(\tilde{u})\, d\phi(\tilde{u}), \qquad d\phi = \mathcal{A}(\tilde{z})\, d\tilde{z},$$

  where $\tilde{z} = p_a \cdot p_i / \left(p_i + p_j\right) \cdot p_a$.

Dipole Method (cont.)

- unintegrated 'dipole'

$$\mathcal{D}^{\mathrm{IE-FS}}_{g,q,j} = \frac{-1}{p_{ai}^2 - m^2} \frac{1}{1-\tilde{u}} \langle \mathcal{M}_n | \frac{\hat{T}_{\underline{ai}} \cdot \hat{T}_j}{\hat{T}^2_{\underline{ai}}} \hat{V}(\tilde{u},\tilde{z}) | \mathcal{M}_n \rangle$$

so called 'dipole splitting function' (below $\nu = \sqrt{1 - m^2 m_j^2/\gamma^2}$ with $\tilde{\gamma} = \tilde{p}_j \cdot p_a$)

$$\hat{V}(\tilde{u},\tilde{z}) = 8\pi\mu_r^{2\varepsilon}\alpha_s C_F \left[\frac{2}{\tilde{u}\nu^2 + \tilde{z}} + (1-\varepsilon)\tilde{u} - 2 - \frac{(1-\tilde{u})\, m^2}{p_i \cdot p_a} \right]$$

- integrated 'dipole splitting function'

$$\int d\Gamma_{n+1} \mathcal{D}^{\mathrm{IE-FS}}_{g,q,j} F_n = \underbrace{\left(\int d\phi\, V \right)}_{=I} \otimes \int d\tilde{\Gamma}_n \langle \mathcal{M}_n | \frac{\hat{T}_{\underline{ai}} \cdot \hat{T}_j}{\hat{T}^2_{\underline{ai}}} | \mathcal{M}_n \rangle F_n$$

  - massless case ($m = 0$)

$$I \sim \left(\frac{1}{\varepsilon} a_1 + \frac{1}{\varepsilon^2} a_2 + a_3 \right) \delta(\tilde{u}) - \frac{1}{\varepsilon} P_{qq}(1-\tilde{u}) + \ldots$$

  - massive case ($m \equiv m_{\mathbf{Q}}$)

$$I \sim \left(\frac{1}{\varepsilon} b_1 + \frac{1}{\varepsilon} \log \eta^2\, b_2 + b_3 \right) \delta(\tilde{u}) + \log \eta^2\, P_{qq}(1-\tilde{u}) + \mathcal{O}\left(\eta^2\right) + \ldots,$$

where $\eta^2 = m^2_{\mathbf{Q}}/2\tilde{\gamma}$ and $P_{qq}(z) = C_F \left(1 + z^2/1 - z\right)_+$.

Factorization

If $\tilde{\gamma} \gg m_{\mathbf{Q}}^2$ the logs of η^2 are harmful. They form quasi-collinear singularity that should be factorized out.

- in massles case factorization is done via subtracting

$$C_a = \sum_b f_{ab} \otimes \sum_j \langle \mathcal{M}_n | \frac{\hat{T}_b \cdot \hat{T}_j}{\hat{T}_b^2} | \mathcal{M}_n \rangle$$

where $f_{ab}(z) = \frac{\alpha_s}{2\pi}(-1/\varepsilon)\, P_{ab}(z)$ are massless densities of partons inside a parton renormalized in $\overline{\text{MS}}$ with given number of active flavours N_f. The splitting functions are

$$P_{gq}(z) = T_R\,[1-2z(1-z)], \quad P_{qq}(z) = C_F \left(\frac{1+z^2}{1-z}\right)_+, \quad P_{qg}(z) = C_F \frac{1+(1-z)^2}{z}$$

$$P_{gg}(z) = 2C_A\left[\left(\frac{1}{1-z}\right)_+ + \frac{1-z}{z} - 1 + z(1-z)\right] + \delta(1-z)\left(\frac{11}{6}C_A - \frac{2}{3}N_f T_R\right)$$

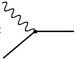
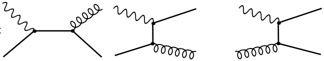
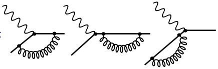
- in massive case one uses the ACOT (Aivazis-Collins-Olness-Tung) scheme.

Partonic densities have to be calculated with full mass dependence and renormalized in special CWZ (Collins-Wilczek-Zee) scheme: for given number of active flavours (including possible heavy quarks) diagrams with loops containing still heavier quarks are renormalized by zero-momentum subtraction (at the order α_s this is actually trivial).

$$f_{g\mathbf{Q}}(z) = \frac{\alpha_s}{2\pi}\log\rho\, P_{gq}(z), \quad f_{\mathbf{Q}g}(z) = \frac{\alpha_s}{2\pi} C_F\, P_{qg}(z)\,[\log\rho - 2\log z - 1], \qquad \rho = \mu_r^2/m_{\mathbf{Q}}^2$$

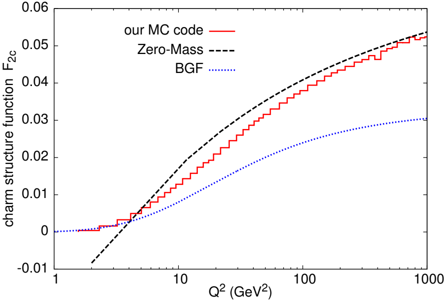
$$f_{\mathbf{QQ}}(z) = \frac{\alpha_s}{2\pi} C_F \left\{P_{qq}(z)\,[\log\rho - 2\log z - 1]\right\}_+, \quad f_{gg}(z) = \frac{\alpha_s}{2\pi}\left[\left(-\frac{1}{\varepsilon}\right)P_{gg}(z) - \frac{2}{3}\delta(1-z)\, T_R \log\rho\right].$$

Example: structure function $F_2^{(\mathbf{Q})}$ at NLO

- LO contribution: $\mathcal{M}_1 =$  $\mathbf{Q}, \overline{\mathbf{Q}}$
- NLO contribution
  - real emissions: $\mathcal{M}_2 =$ 
  - virtual corrections: $\mathcal{M}_1^{\text{loop}} =$ 
- Needed 'dipoles': $\mathcal{D}^{\text{IE-FS}}_{g,\mathbf{Q},\mathbf{Q}}$, $\mathcal{D}^{\text{FE-IS}}_{g,\mathbf{Q},\mathbf{Q}}$, $\mathcal{D}^{\text{IE-FS}}_{\mathbf{Q},g,\overline{\mathbf{Q}}}$
- Quasi-collinear factorization terms: $\mathcal{C}_{\mathbf{Q}}$, $\mathcal{C}_g$
- Numerical integration using MC program

  The result interpolates between completely massless calculation at high Q^2 and boson-gluon fusion (BGF) at low Q^2.

  It recovers the result of [Kretzer, Schienbein].



Summary

GENERAL MASS SCHEME FOR JETS:

1. dipole subtraction method with quark masses taken into account
2. massive factorization scheme (ACOT)

- We have checked that for very large scale all possible quasi-collinear singularities are properly factorized at NLO and the jet cross sections become exactly known massless cross sections.
- The full MC program is under development, we have checked its demo version against F_2 structure function and found agreement with existing calculations in the ACOT scheme.
- Possible extensions: hadron-hadron collisions, identified final states (fragmentation functions)
- Issues not discussed: e.g. an ambiguity in the ACOT scheme