Southern Methodist University

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Black-Hole Bombs and Photon-Mass Bounds

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http://blackholes.ist.utl.pt

PP, Cardoso, Gualitieri, Berti, Ishibashi

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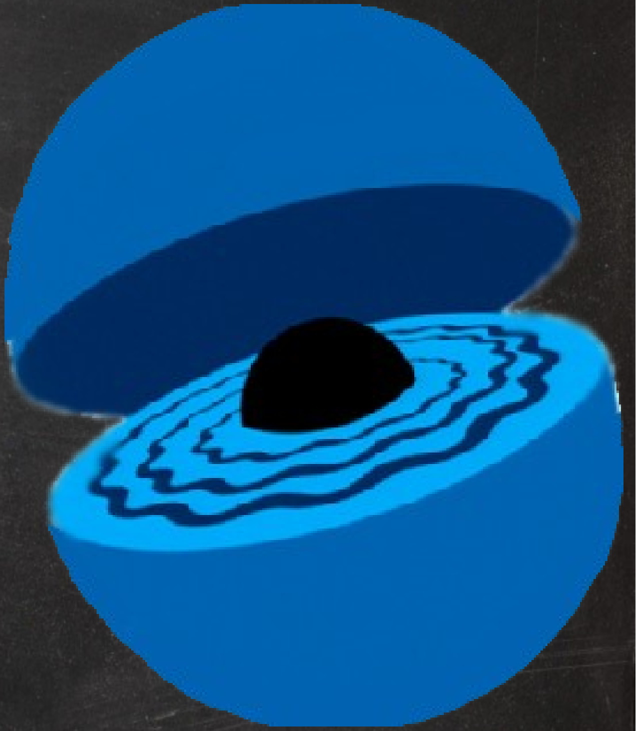
Outline

Dynamics of light massive fields around spinning black holes

- BHs as particle physics labs
- Review on scalar superradiant instability
- Open problems in BH perturbation theory
- Slow-rotation framework
- Massive spin-1 fields around Kerr BHs
- Astrophysical consequencies of the Proca instability



[Credit: Ana Sousa]

BH superradiance

- Simple BH-matter interaction
- Kerr BH: Killing vector ∂_t becomes spacelike in the ergoregion:
- Amplification of scattered waves → angular momentum extraction if:

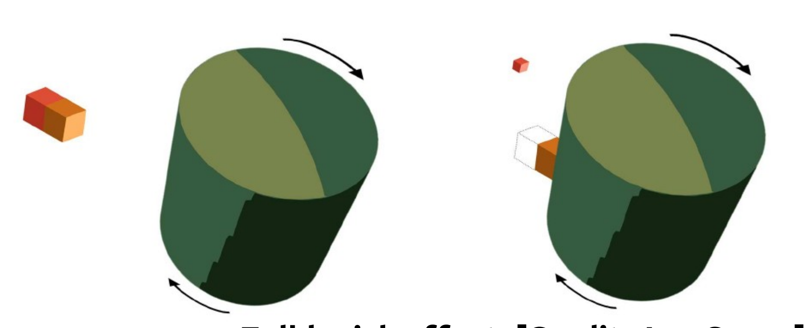
$$\omega < m\Omega_H$$

- Linear effect, but peek to backreaction
- Requires <u>dissipation</u> → needs an event horizon
- ~ tidal heating at the horizon

[Cardoso & Pani, 2012]

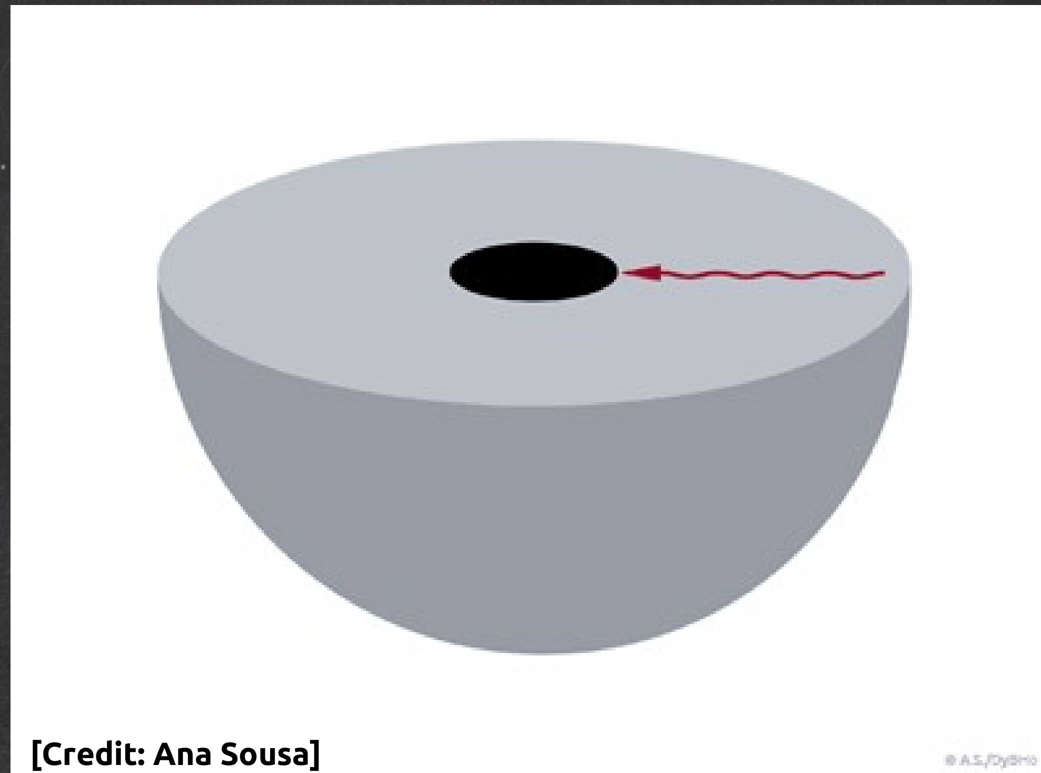
[Thorne, Price, Macdonald's book]

[Richartz et al. 2008]



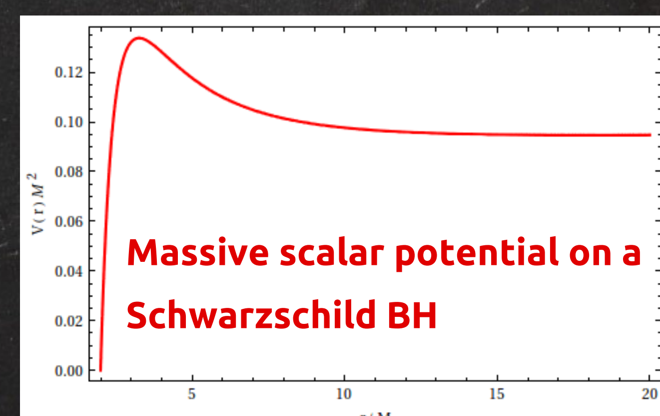
Zel'dovich effect. [Credit: Ana Sousa]

BH bomb [Press and Teukolsky '72]



[Credit: Ana Sousa]

- *"Nature may provide its own mirrors"*
  [Cardoso, Dias, Lemos, Yoshida, 2004]
  - AdS boundaries
  - Massive fields

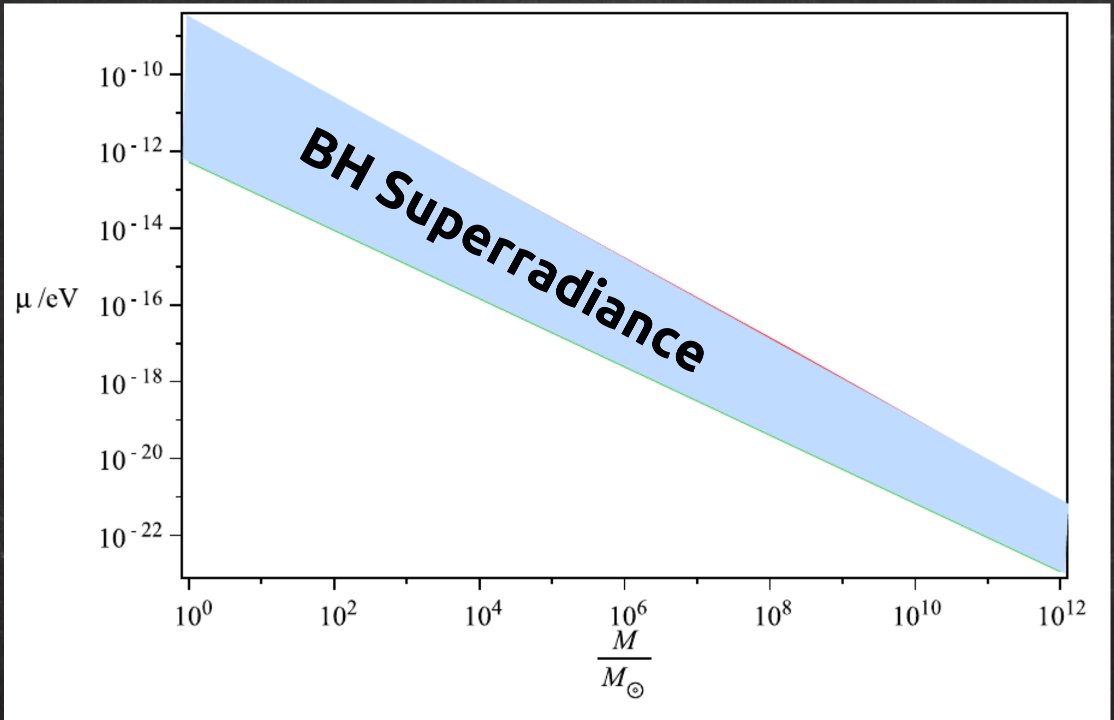


Scalar fields & superradiance

$$\Box\phi - \mu^2\phi = 0$$

- Massive fields around spinning BHs are unstable
- Strongest instability when $\mu M \sim 1 \rightarrow \tau = 10^7\ M$
  - Ultra-light particles ($m \sim 10^{-21} - 10^{-9}$ eV) and (super)massive BHs

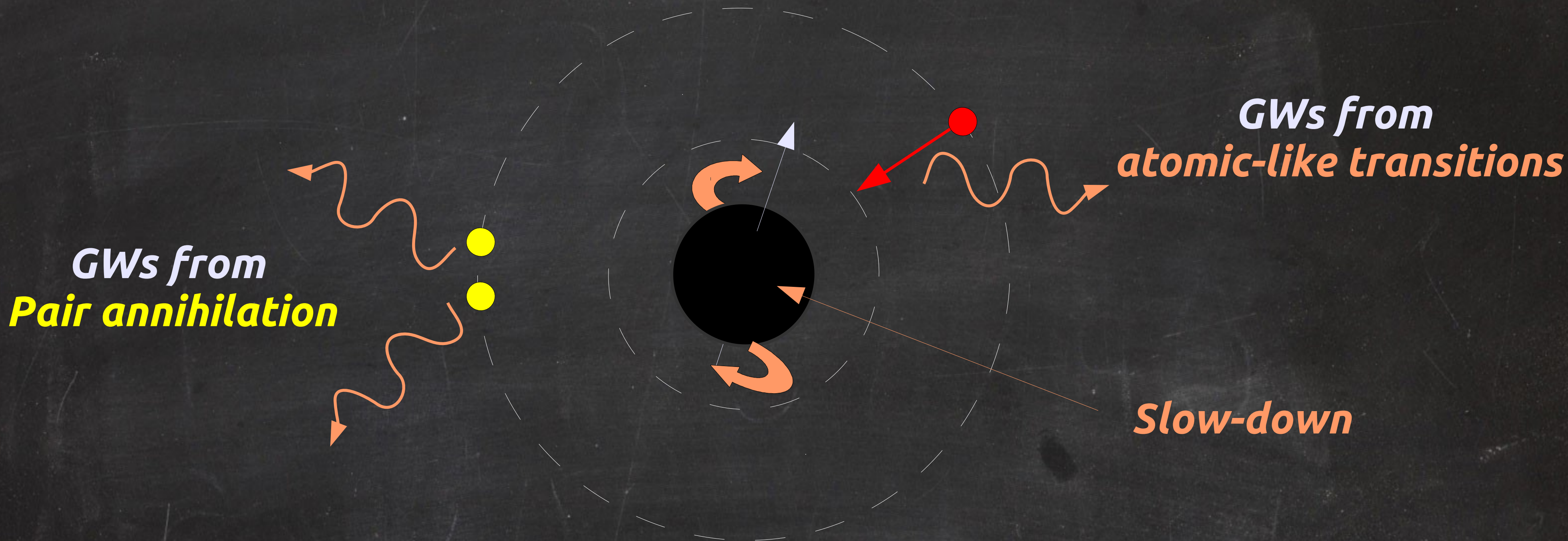
[Damour et al. 1976]
[Detweiler, 1980]
[Earley & Zouros 1979]
[Cardoso & Yoshida 2005]
[Dolan 2007]
[Rosa 2010, 2012]



[Arvanitaki et al. 2010-2011]
[Kodama & Yoshino 2011-2012]

- Peccei-Quinn QCD axion , Axiverse scenarios

Scalar fields & superradiance



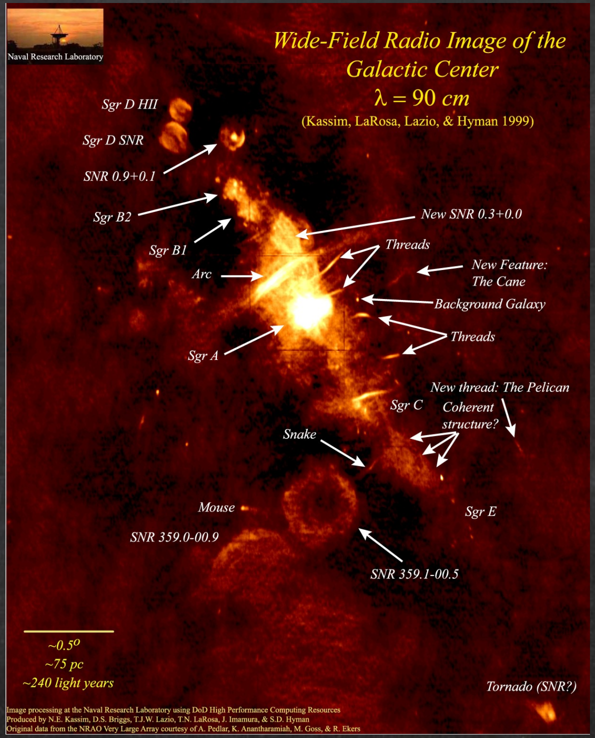
- **Nonlinear interactions are important**
  - **Saturation of the instability ?**
  - **Bosenova explosion ?**
- **Numerical simulation are challenging**

$$\Box\phi - \mu^2 \sin^2\phi = 0$$

[Kodama & Yoshino 2011-2012]

[Witek et al, in preparation]

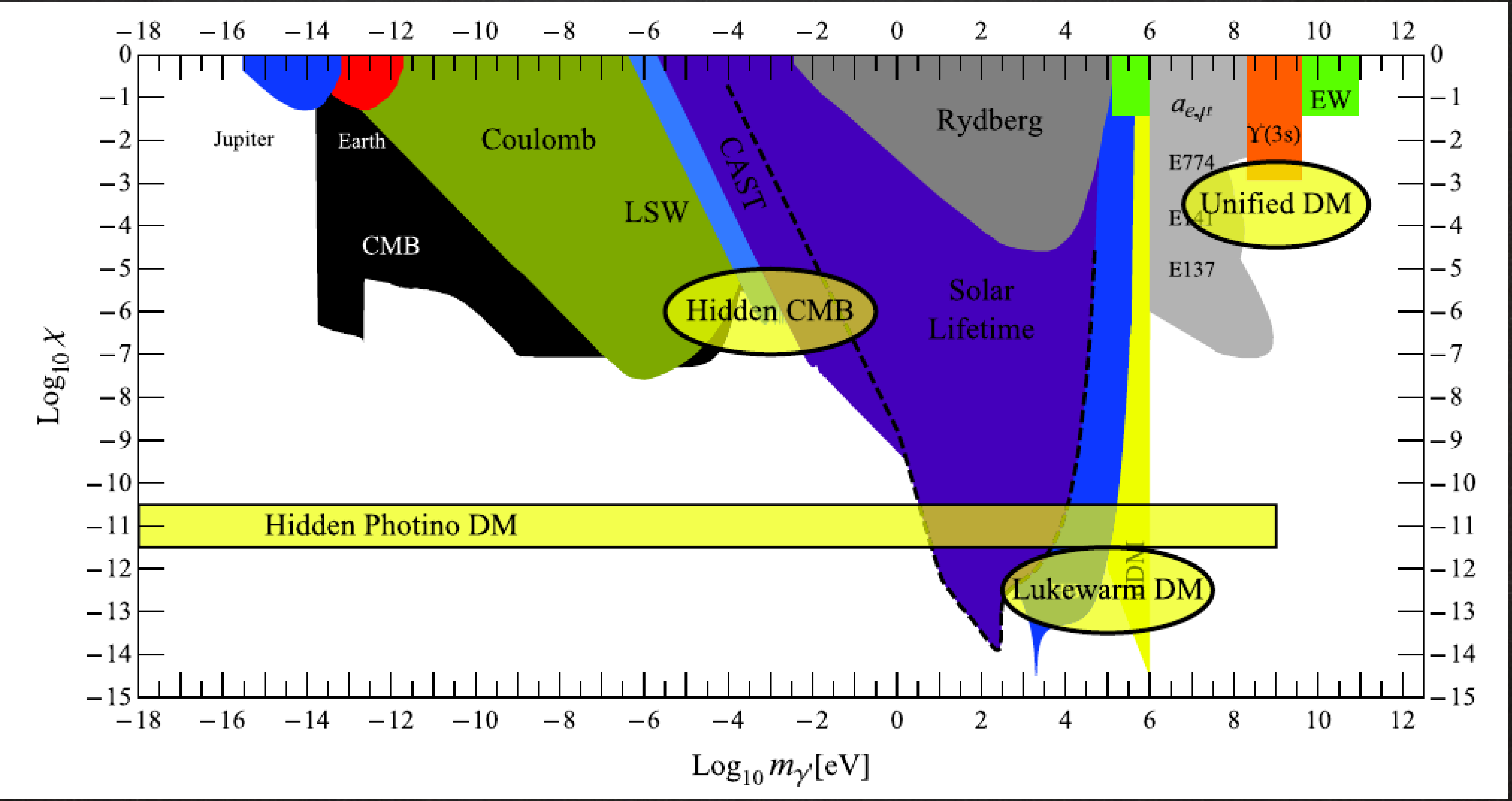
If ultralight scalars exist and they are superradiantly unstable, we shouldn't observe highly-spinning BHs



Constraints on axion parameters from BH observations and future GW detection

What about vector fields?

$$\mathcal{L} = -\frac{1}{4g_a^2} F^{(a)}_{\mu\nu} F^{\mu\nu}_{(a)} - \frac{1}{4g_b^2} F^{(b)}_{\mu\nu} F^{\mu\nu}_{(b)} + \frac{\chi_{ab}}{2g_a g_b} F^{(a)}_{\mu\nu} F^{(b)\mu\nu} + \frac{m_{ab}^2}{g_a g_b} A^{(a)}_{\mu} A^{(b)\mu}$$



[Goodsell et al. 2009]

Vector fields & superradiance

Alexandru Proca

$$\nabla_\sigma F^{\sigma\nu} - \mu^2 A^\nu = 0$$

$$\implies \quad \nabla_\sigma A^\sigma = 0\,, \qquad \Box A^\nu - \mu^2 A^\nu = 0$$

- The massive spin-1 around Kerr BHs still uncharted territory
- Proca eq. (apparently) nonseparable in a Kerr background
- Note that EM (massless) perturbations in Kerr-(A)dS are separable!

$$\nabla_\sigma F^{\sigma\nu} = 0 \quad \implies \quad \Box A^\nu - \nabla^\nu(\nabla_\sigma A^\sigma) + \Lambda A^\nu = 0$$

Vector fields & superradiance

$$\nabla_\sigma F^{\sigma\nu} - \mu^2 A^\nu = 0$$

$$\Longrightarrow \quad \nabla_\sigma A^\sigma = 0\,, \qquad \Box A^\nu - \mu^2 A^\nu = 0$$

Alexandru Proca

- **Massive hidden U(1) vector fields are generic features of extensions of SM**

  [Goodsell et al. 2009]

- **Proxy for gravitational theories with higher-curvature terms:**

$$\mathcal{L} = \sqrt{-g}\left(R + \alpha R_{[ab]} R^{[ab]}\right)$$

[Buchdahl '70]
[Vitagliano, Sotiriou, Liberati '10]

- **Conjecture of a stronger instability?**

[Rosa & Dolan 2011]
[Herdeiro, Sampaio, Wang 2012]
[Konoplya 2006]

Part 0

BH perturbations in a nutshell

BH perturbations. Spherical symmetry

[Kokkotas & Schmidt 1998]
[Berti et al. 2009]
[Konoplya & Zhidenko 2011]

$$ds^2 = \underbrace{-f(r)dt^2 + h(r)^{-1}dr^2 + r^2 d\Omega_2}_{\text{background}} + \underbrace{(\delta_{\rm RW} g_{\mu\nu})e^{-i\omega t}dx^\mu dx^\nu}_{\text{perturbations}}$$

- Regge-Wheeler formalism:

Polar

Axial

$$||\delta_{\rm RW} g_{\mu\nu}|| = \begin{bmatrix} f(r)H_0(r)Y_{lm} & H_1(r)Y_{lm} & -h_0(r)\frac{1}{\sin\theta}\frac{\partial Y_{lm}}{\partial\varphi} & h_0(r)\sin\theta\frac{\partial Y_{lm}}{\partial\theta} \\ * & \frac{H_2(r)Y_{lm}}{h(r)} & -h_1(r)\frac{1}{\sin\theta}\frac{\partial Y_{lm}}{\partial\varphi} & h_1(r)\sin\theta\frac{\partial Y_{lm}}{\partial\theta} \\ * & * & r^2K(r)Y_{lm} & 0 \\ * & * & * & r^2\sin^2\theta K(r)Y_{lm} \end{bmatrix}$$

- The axial and polar sectors decouple → master equations

$$\mathcal{A}_\ell = 0 \qquad \mathcal{P}_\ell = 0$$

Linear equations involving axial *or* polar perturbations only

- Solved with suitable boundary conditions → eigenvalue problem

- Any spherically symmetric background, any theory, any field

Non-separable (?) problems

- Separability in Kerr is almost a miracle!

[Teukolsky ~ 1973]
[Teukolsky and Press]
[Chandra's book]

- Four dimensions
  - Massive vector (Proca) fields on a Kerr background
  - Gravito-EM perturbations of Kerr-Newman BHs
  - Rotating objects in alternative theories
- Higher dimensions
  - Myers-Perry BHs with generic spins
  - Other rotating solutions
- Stability, greybody factors, quasinormal modes?

Part I

Perturbations of slowly-rotating BHs:

General framework

Method. *Perturbations of slowly rotating spacetimes*

[Kojima 1992, 1993, 1997]

[Pani et al., 2012]

- **Slowly-rotating background metric:**

$$ds_0^2 = -F(r)dt^2 + B(r)^{-1}dr^2 + r^2 d^2\Omega - 2\varpi(r)\sin^2\theta d\varphi dt$$

- **Expand any equation (scalar, vector, tensor...) in spherical harmonics**

$$\delta X_{\mu_1\ldots}(t,r,\vartheta,\varphi) = \delta X^{(i)}_{\ell m}(r)\mathcal{Y}^{\ell m\,(i)}_{\mu_1\ldots}e^{-i\omega t}$$

- **For any metric, any theory and any perturbations: system of radial ODEs:**

$$\mathcal{A}_{\ell m} + \tilde{a}m\bar{\mathcal{A}}_{\ell m} + \tilde{a}(\mathcal{Q}_{\ell m}\tilde{\mathcal{P}}_{\ell-1m} + \mathcal{Q}_{\ell+1m}\tilde{\mathcal{P}}_{\ell+1m}) = 0$$

$$\mathcal{P}_{\ell m} + \tilde{a}m\bar{\mathcal{P}}_{\ell m} + \tilde{a}(\mathcal{Q}_{\ell m}\tilde{\mathcal{A}}_{\ell-1m} + \mathcal{Q}_{\ell+1m}\tilde{\mathcal{A}}_{\ell+1m}) = 0$$

- **Zeeman splitting**
- **Laporte-like selection rule**
- **Propensity rule**

$$\mathcal{Q}_{\ell m} = \sqrt{\frac{\ell^2 - m^2}{4\ell^2 - 1}}$$

$\mathcal{A}, \mathcal{P} \rightarrow$ **Linear combinations of axial and polar perturbations**

Perturbative scheme

- **To first order in the rotation, modes do not depend on the couplings:**

$$\mathcal{A}_{\ell m} + \tilde{a} m \bar{\mathcal{A}}_{\ell m} = 0 \qquad \mathcal{P}_{\ell m} + \tilde{a} m \bar{\mathcal{P}}_{\ell m} = 0$$

- **Second order: particularly advantageous**
  - **Cauchy horizon, even horizons, ergosphere**

$$r_+ = 2M\left(1-\frac{\tilde{a}^2}{4}\right) \qquad r_- = \frac{M\tilde{a}^2}{2} \qquad r_{\mathrm{ER}} = 2M\left(1-\cos^2\vartheta\frac{\tilde{a}^2}{4}\right)$$

  - **The superradiance regime is now consistent** $\omega < m\Omega_H \sim \tilde{a}$

$$\omega = \omega_0 + \tilde{a} m \omega_1 + \tilde{a}^2 \omega_2 + \mathcal{O}(\tilde{a}^3)$$

Perturbative scheme

$0 = \mathcal{A}_\ell$

Zeroth order: decoupled

$\mathcal{P}_{L+3}$

$0 = \mathcal{P}_\ell$

$\mathcal{A}_{L+2}$

$\mathcal{P}_{L+1}$

$\mathcal{A}_{L}$

$\mathcal{P}_{L-1}$

$\mathcal{A}_{L-2}$

$\mathcal{P}_{L-3}$

Perturbative scheme

$$0 = \mathcal{A}_\ell$$

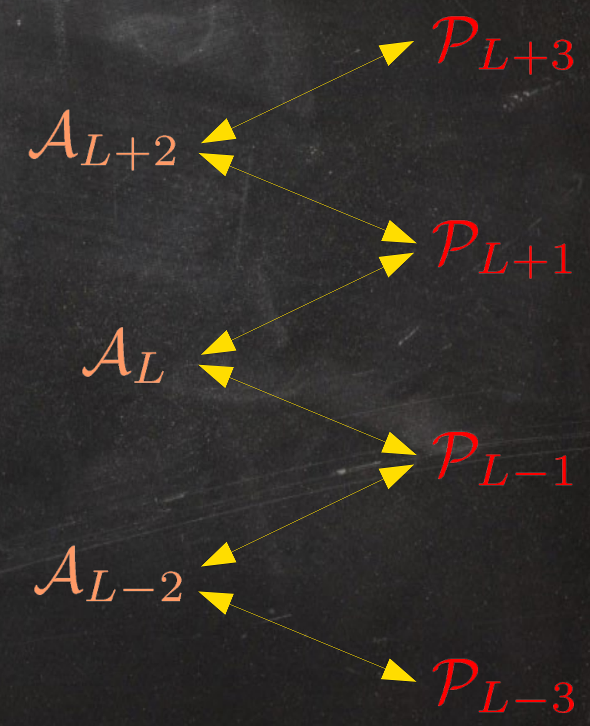
$$+\tilde{a}m\bar{\mathcal{A}}_\ell + \tilde{a}(\mathcal{Q}_\ell\tilde{\mathcal{P}}_{\ell-1} + \mathcal{Q}_{\ell+1}\tilde{\mathcal{P}}_{\ell+1})$$

Zeroth order: decoupled

First order: polar-axial l±1

$$0 = \mathcal{P}_\ell$$

$$+\tilde{a}m\bar{\mathcal{P}}_\ell + \tilde{a}(\mathcal{Q}_\ell\tilde{\mathcal{A}}_{\ell-1} + \mathcal{Q}_{\ell+1}\tilde{\mathcal{A}}_{\ell+1})$$



Perturbative scheme

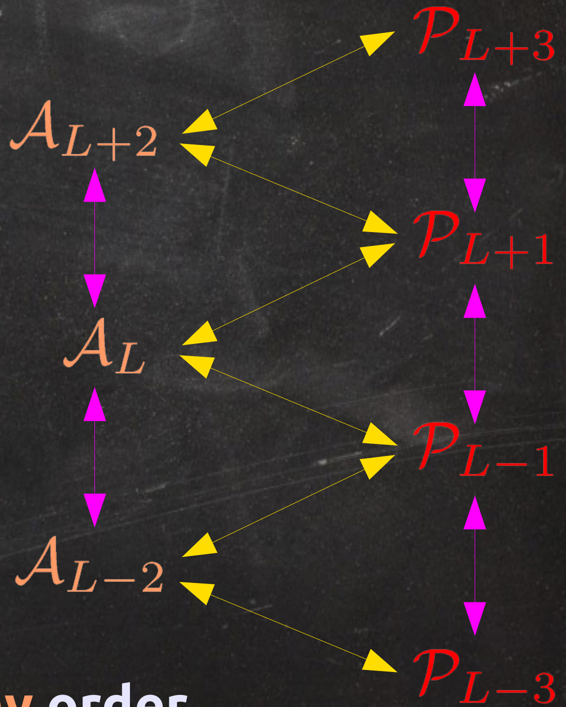
$$
\begin{aligned}
0 = {} & \mathcal{A}_\ell \\
& + \tilde{a} m \bar{\mathcal{A}}_\ell + \tilde{a}\left(\mathcal{Q}_\ell \tilde{\mathcal{P}}_{\ell-1} + \mathcal{Q}_{\ell+1} \tilde{\mathcal{P}}_{\ell+1}\right) \\
& + \tilde{a}^2 \left[\hat{\mathcal{A}}_\ell + \mathcal{Q}_{\ell-1}\mathcal{Q}_\ell \breve{\mathcal{A}}_{\ell-2} + \mathcal{Q}_{\ell+2}\mathcal{Q}_{\ell+1}\breve{\mathcal{A}}_{\ell+2}\right]
\end{aligned}
$$

Zeroth order: decoupled

First order: polar-axial l±1

Second order: l±2

$$
\begin{aligned}
0 = {} & \mathcal{P}_\ell \\
& + \tilde{a} m \bar{\mathcal{P}}_\ell + \tilde{a}\left(\mathcal{Q}_\ell \tilde{\mathcal{A}}_{\ell-1} + \mathcal{Q}_{\ell+1} \tilde{\mathcal{A}}_{\ell+1}\right) \\
& + \tilde{a}^2 \left[\hat{\mathcal{P}}_\ell + \mathcal{Q}_{\ell-1}\mathcal{Q}_\ell \breve{\mathcal{P}}_{\ell-2} + \mathcal{Q}_{\ell+2}\mathcal{Q}_{\ell+1}\breve{\mathcal{P}}_{\ell+2}\right]
\end{aligned}
$$

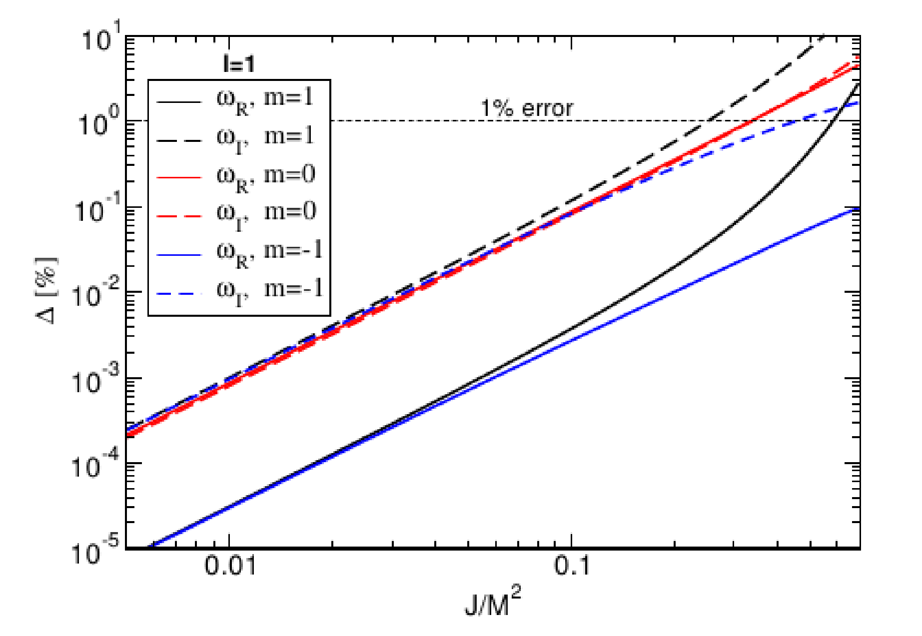


- **Generic: any metric, any perturbation, any theory, any order**

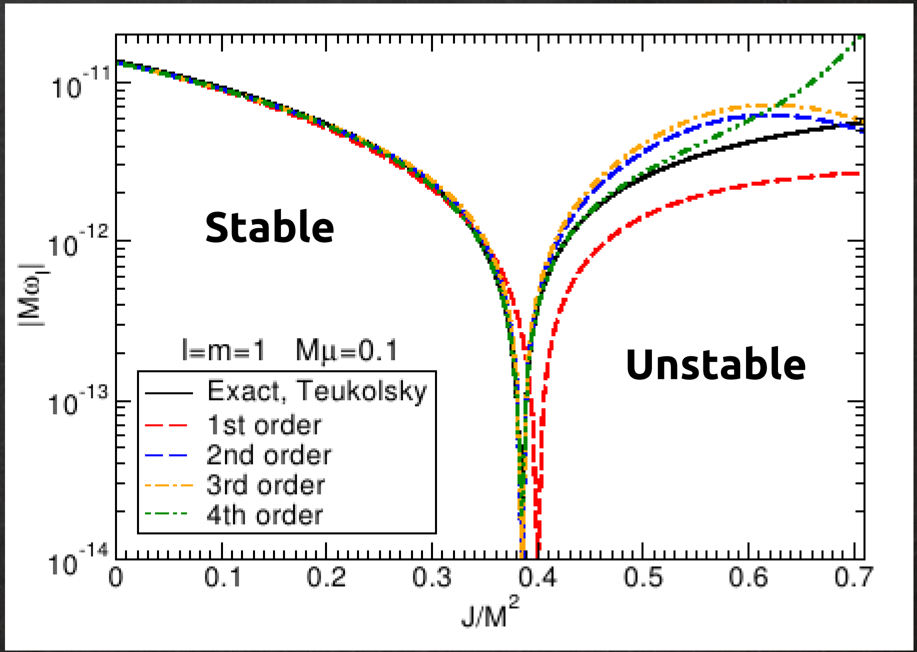
Slow-rotation method. *tests*

Numerics in the slow-rotation scheme are "easy" to perform

- direct integration (bound states)
- continued fractions (QNMs, bound states)
- Breit-Wigner method (QNMs, bound states)
- WKB (?)



EM (massless) QNMs of a Kerr BH



Massive scalar modes of a Kerr BH

- Good results even for moderately large spin

Part III

Proca perturbations of a Kerr BH

Proca in slowly-rotating Kerr

- **The Proca problem becomes tractable in the slow-rotation approximation**
- **Let us decompose the Proca field in vector spherical harmonics:**

$$Y_a^{\ell m} = \left(\partial_\vartheta Y^{\ell m}, \partial_\varphi Y^{\ell m}\right) \qquad S_a^{\ell m} = \left(\frac{1}{\sin\vartheta}\partial_\varphi Y^{\ell m}, -\sin\vartheta\,\partial_\vartheta Y^{\ell m}\right)$$

$$\delta A_\mu(t,r,\vartheta,\varphi) = \underbrace{\sum_{\ell,m}\begin{bmatrix} 0 \\ 0 \\ u_{(4)}^{\ell m}(t,r) S_a^{\ell m} \end{bmatrix}}_{\text{Axial parity}} + \underbrace{\sum_{\ell,m}\begin{bmatrix} u_{(1)}^{\ell m}(t,r) Y^{\ell m} \\ u_{(2)}^{\ell m}(t,r) Y^{\ell m} \\ u_{(3)}^{\ell m}(t,r) Y_a^{\ell m} \end{bmatrix}}_{\text{Polar parity}}$$

- **One spurious degree of freedom → three physical perturbation functions**

Proca in SR Kerr. *Field equations*

- **Polar** and **axial** sector are **coupled**:

$$
\begin{cases}
\hat{\mathcal{D}}_2 u^\ell_{(2)} - \dfrac{2F}{r^2}\left(1-\dfrac{3M}{r}\right)\left[u^\ell_{(2)} - u^\ell_{(3)}\right] = \\
\qquad = \dfrac{2\tilde{a}M^2 m}{\Lambda r^5 \omega}\left[\Lambda\left(2r^2\omega^2+3F^2\right)u^\ell_{(2)} + 3F\left(r\Lambda F u'^\ell_{(2)} - \left(r^2\omega^2+\Lambda F\right)u^\ell_{(3)}\right)\right] \\
\qquad - \dfrac{6i\tilde{a}M^2 F\omega}{\Lambda r^3}\left[(\ell+1)\mathcal{Q}_{\ell m} u^{\ell-1}_{(4)} - \ell\mathcal{Q}_{\ell+1 m} u^{\ell+1}_{(4)}\right] \\
\hat{\mathcal{D}}_2 u^\ell_{(3)} + \dfrac{2F\Lambda}{r^2} u^\ell_{(2)} = \dfrac{2\tilde{a}M^2 m}{r^5\omega}\left[2r^2\omega^2 u^\ell_{(3)} + 3rF^2 u'^\ell_{(3)} - 3\left(\Lambda + r^2\mu^2\right)F u^\ell_{(2)}\right] \\
\hat{\mathcal{D}}_2 u^\ell_{(4)} - \dfrac{4\tilde{a}M^2 m\omega}{r^3} u^\ell_{(4)} = -\dfrac{6i\tilde{a}M^2 F}{r^5\omega}\left[(\ell+1)\mathcal{Q}_{\ell m}\,\psi^{\ell-1} - \ell\mathcal{Q}_{\ell+1 m}\,\psi^{\ell+1}\right]
\end{cases}
$$

- **Where we have used the Lorenz condition and defined:**

$$
\hat{\mathcal{D}}_2 = \frac{d^2}{dr_*^2} + \omega^2 - F\left[\frac{\ell(\ell+1)}{r^2} + \mu^2\right], \qquad \psi^\ell = \left(\Lambda + r^2\mu^2\right)u^\ell_{(2)} - (r-2M)u'^\ell_{(3)}
$$

Proca in SR Kerr. *Analytical results*

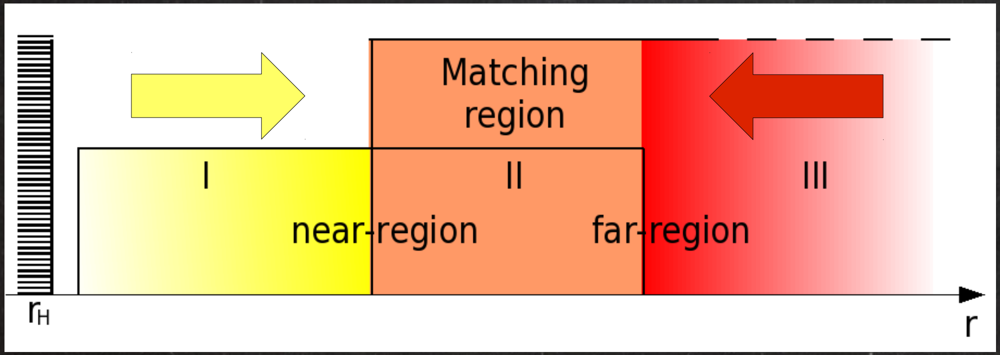
- In the axial case → **master equation** (scalar → **s=0** , axial vector → **s=1**)

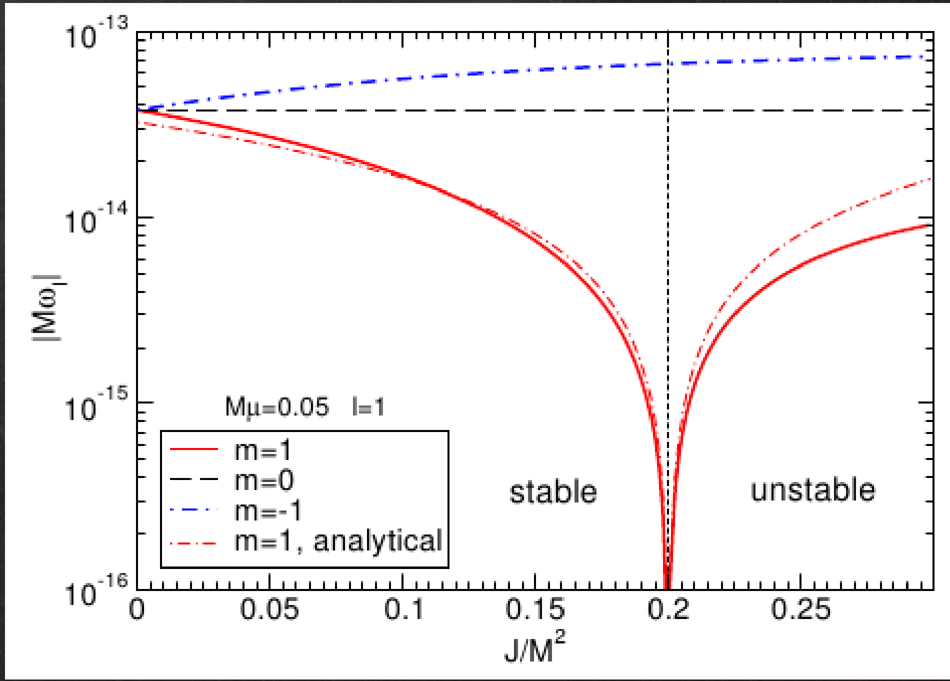
$$\frac{d^2\Psi}{dr_*^2}+\left[\omega^2-\frac{2m\varpi(r)\omega}{r^2}-F\left(\frac{\Lambda}{r^2}+\mu^2+(1-s^2)\left\{\frac{B'}{2r}+\frac{BF'}{2rF}\right\}\right)\right]\Psi=0$$

$$ds_0^2=g^{(0)}_{\mu\nu}dx^\mu dx^\nu=-F(r)dt^2+B(r)^{-1}dr^2+r^2d^2\Omega-2\varpi(r)\sin^2\theta d\varphi dt$$

- **Suitable for analytical methods**
- **Matching asymptotics**

[Starobisky 1973]
[Detweiler 1980]





$$M\omega_I\sim\gamma_{s\ell}\left(\tilde{a}m-2r_+\mu\right)(M\mu)^{4\ell+5}$$

Proca in SR Kerr. *Field eqs. at second order*

- System of second order ODEs → solved numerically

$$\mathcal{D}_A\Psi_A^\ell + V_A\Psi_A^\ell = 0 \qquad \mathcal{D}_P\Psi_P^\ell + V_P\Psi_P^\ell = 0$$

$$\Psi_A = (u_{(4)}^\ell, u_{(2)}^{\ell\pm1}, u_{(3)}^{\ell\pm1}, u_{(4)}^{\ell\pm2}) \qquad \Psi_P = (u_{(2)}^\ell, u_{(3)}^\ell, u_{(4)}^{\ell\pm1}, u_{(2)}^{\ell\pm2}, u_{(3)}^{\ell\pm2})$$

- **Behavior at infinity:** $u_{(i)} \sim e^{-ik_H r_*}$ $\quad k_H \sim \underbrace{\omega - m\Omega_H}_{\text{Superradiance}} \simeq \omega - \frac{m\tilde{a}}{4M} + \mathcal{O}(\tilde{a}^3)$

- **Near-horizon behavior:**

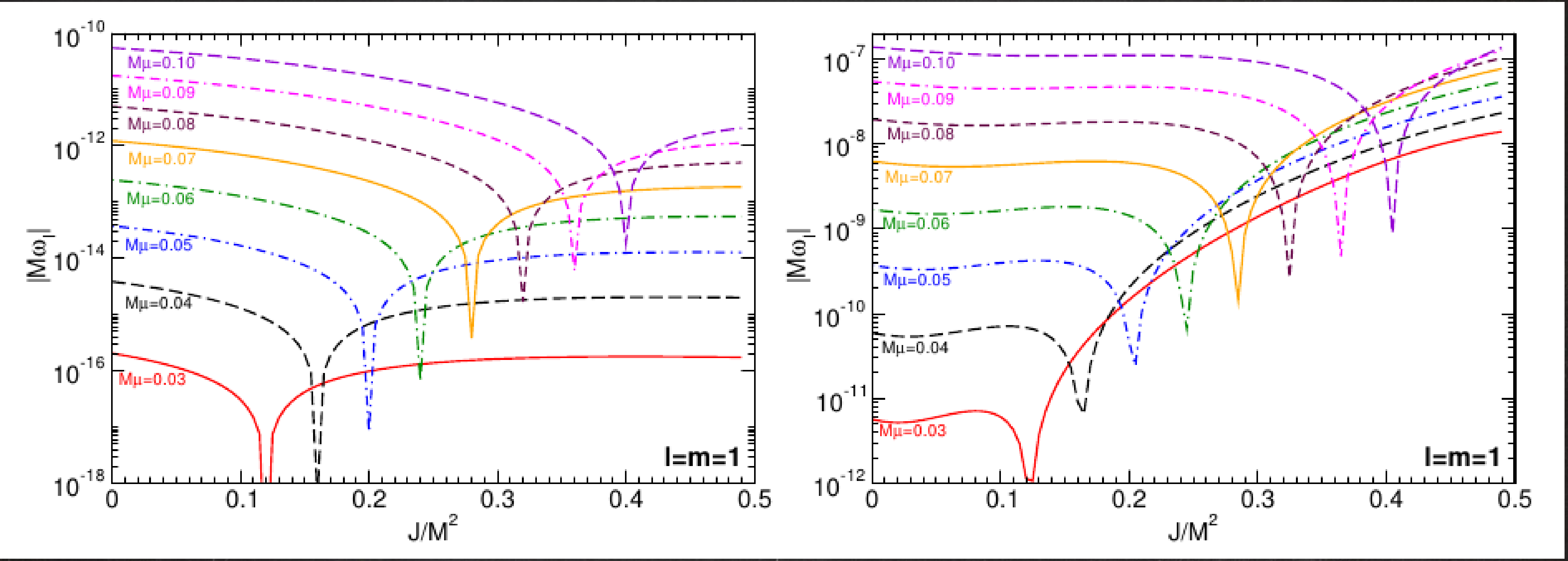
$$u_{(i)} \sim B_{(i)} e^{-k_\infty r} r^{-\frac{M(\mu^2-2\omega^2)}{k_\infty}} + C_{(i)} e^{k_\infty r} r^{\frac{M(\mu^2-2\omega^2)}{k_\infty}} \qquad k_\infty = \sqrt{\mu^2-\omega^2}$$

B=0 → quasinormal modes (purely outgoing waves at infinity)

C=0 → bound states (exponential decay, spacially localized near the BH)

Proca in SR Kerr. Results

Axial modes (S=0) | Polar modes (S=+1,-1)

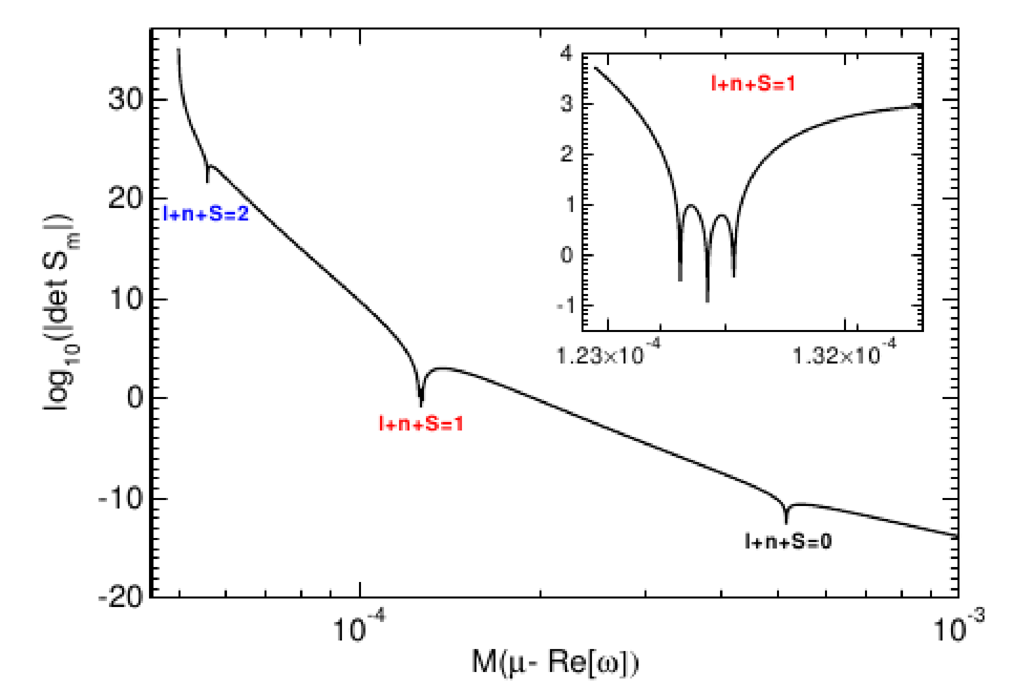


$$\omega_R \sim \mu - \frac{\mu(M\mu)^2}{2(\ell+n+S+1)} \qquad M\omega_I \sim \gamma_{S\ell}\left(\tilde{a}m - 2r_+\mu\right)(M\mu)^{4\ell+5+2S}$$

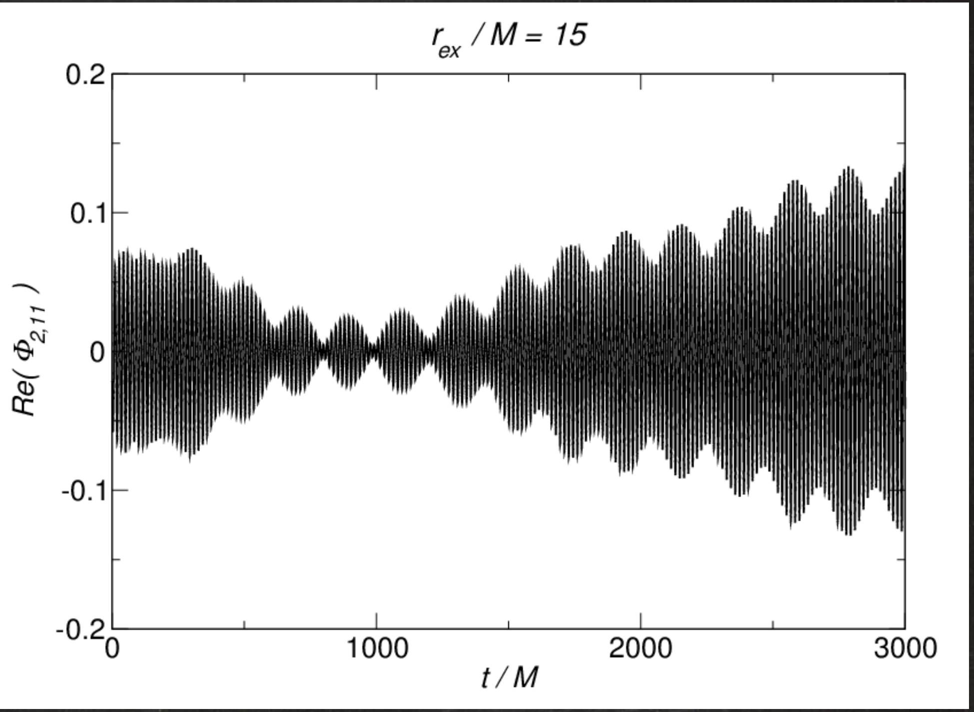
- Vector axial modes are very similar to scalar ones
- Vector polar modes are more unstable

Proca in SR Kerr. Fully coupled system

$$\omega_R \sim \mu - \frac{\mu(M\mu)^2}{2(\ell+n+S+1)} \qquad M\omega_I \sim \gamma_{S\ell}\left(\tilde{a}m - 2r_+\mu\right)(M\mu)^{4\ell+5+2S}$$



Breit-Wigner resonances



Confirmed by time evolution
[Witek et al., in preparation]

Part IV

Astrophysical consequences of the Proca instability

Proca instability

- Can we extrapolate these results to higher rotation?
- Scalar case (l=1) $M\omega_I \sim \frac{1}{48}(\tilde{a}m - 2r_+\mu)(M\mu)^9$

[Dolan 2007]

TABLE III. Maximum instability growth rates of the $l = 1$, $m = 1$ state.

a	0.7	0.8	0.9	0.95	0.98	0.99
μ	0.187	0.231	0.293	0.343	0.393	0.421
τ^{-1}	3.33×10^{-10}	2.16×10^{-9}	1.55×10^{-8}	4.88×10^{-8}	1.11×10^{-7}	1.50×10^{-7}
	3.43×10^{-10}	2.37×10^{-9}	1.94×10^{-8}	6.82×10^{-8}	1.75×10^{-7}	2.53×10^{-7}

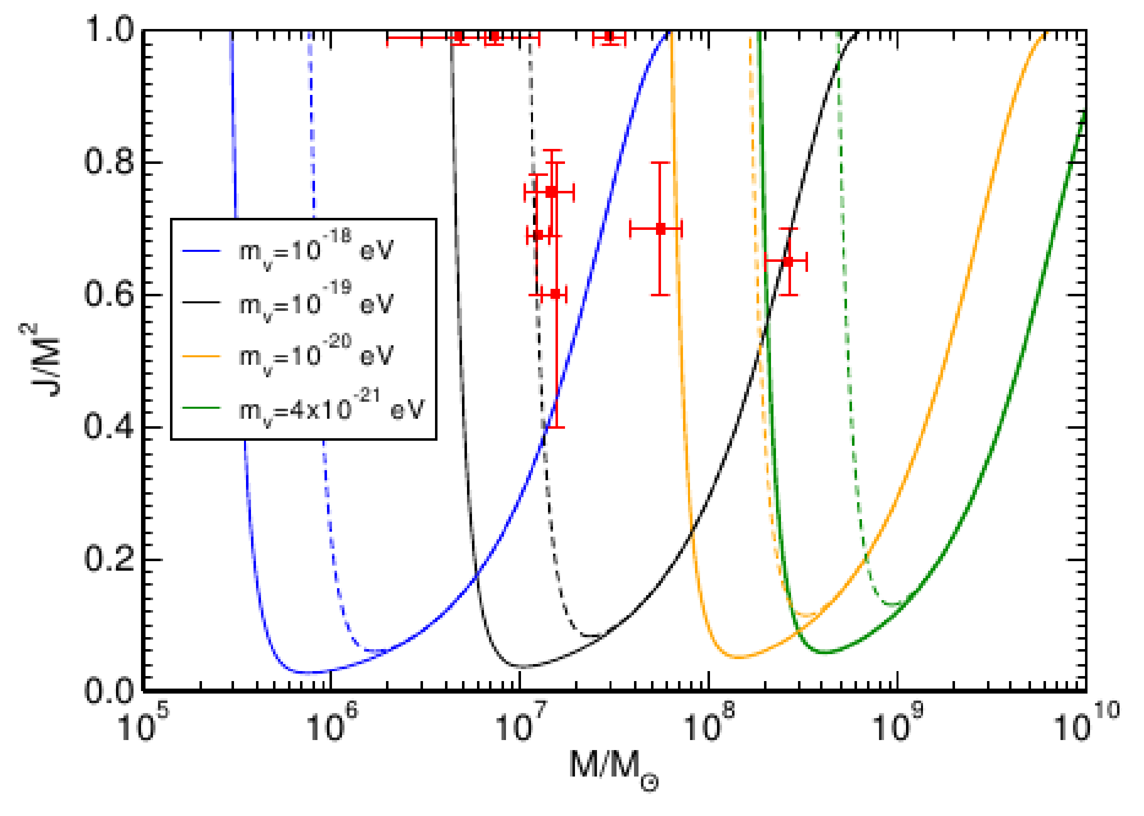
- Extrapolation should provide an order of magnitude for the instability
- Massive vectors: stronger instability for polar modes with S = -1 and l=1:

$$\tau_{\rm vector} = \omega_I^{-1} \sim \frac{M(M\mu)^{-7}}{\gamma_{-11}(\tilde{a} - 2\mu r_+)} \sim 5 \times 10^4 M$$

Within a factor 2 from exact evolution with a=0.99 M [Witek et al., in preparation]

Proca instability. *Regge plane*

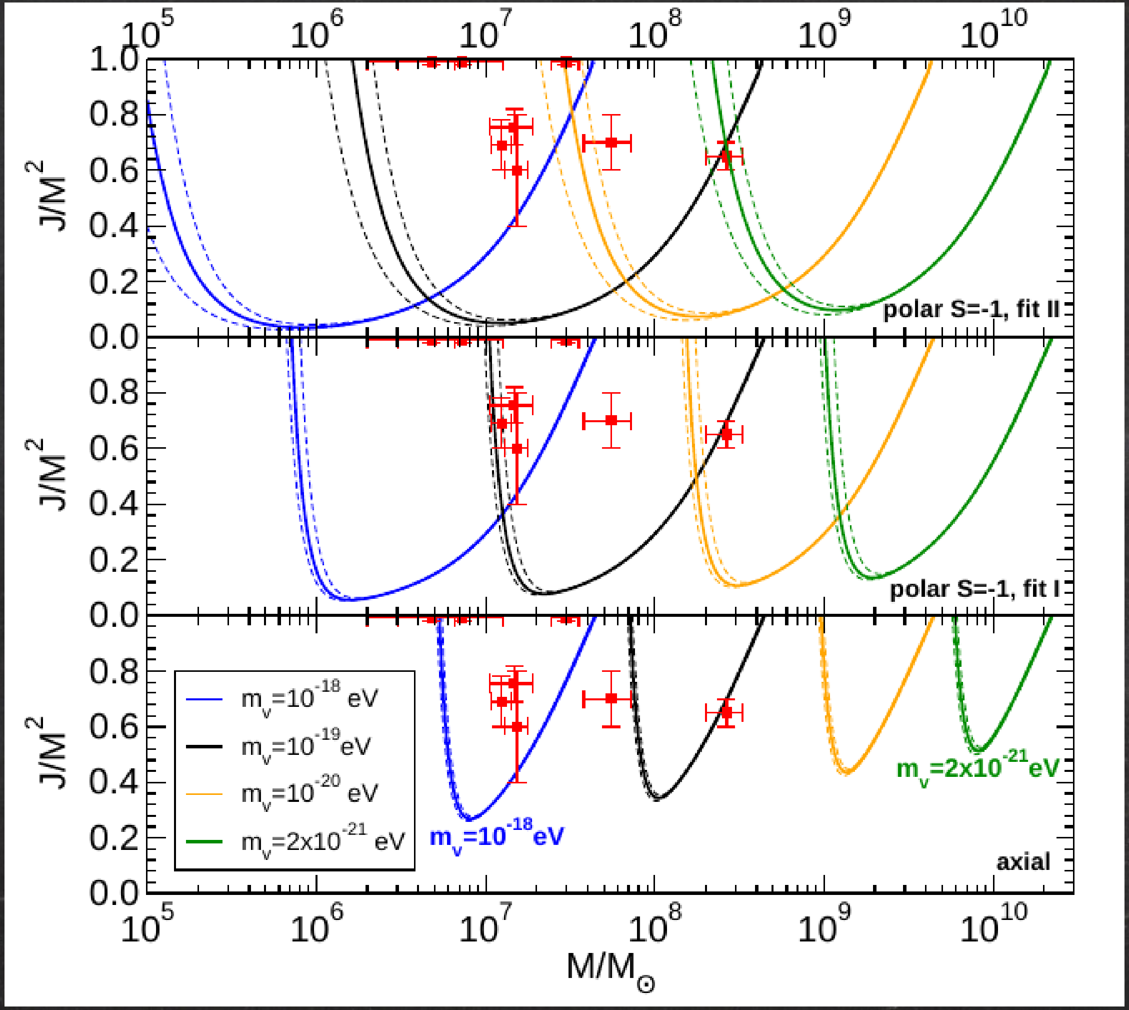
- Instability is effective roughly for any non-vanishing spin!



[Data taken from Brenneman et. Al, ApJ 2011]

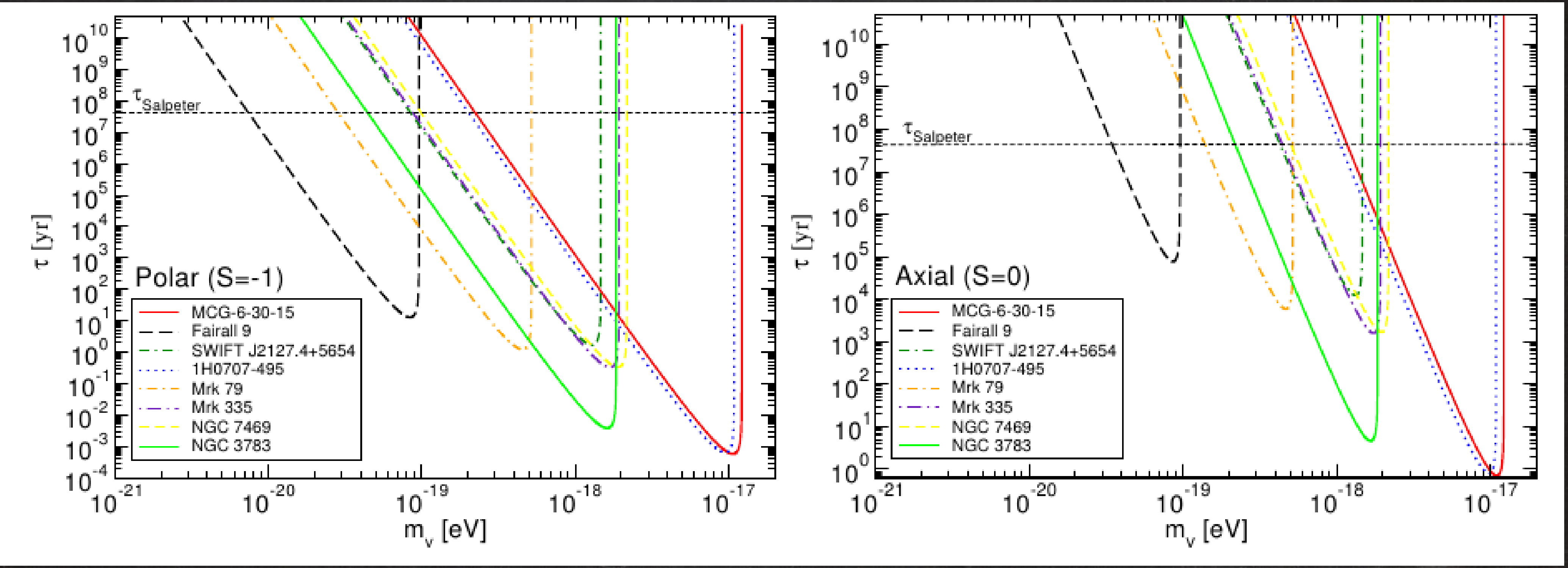
- Depend very mildly on the fit coefficient and on the threshold
- $\tau_{Salpater}$ → timescale for accretion at the Eddington limit

Proca instability. *Axial and Polar modes*



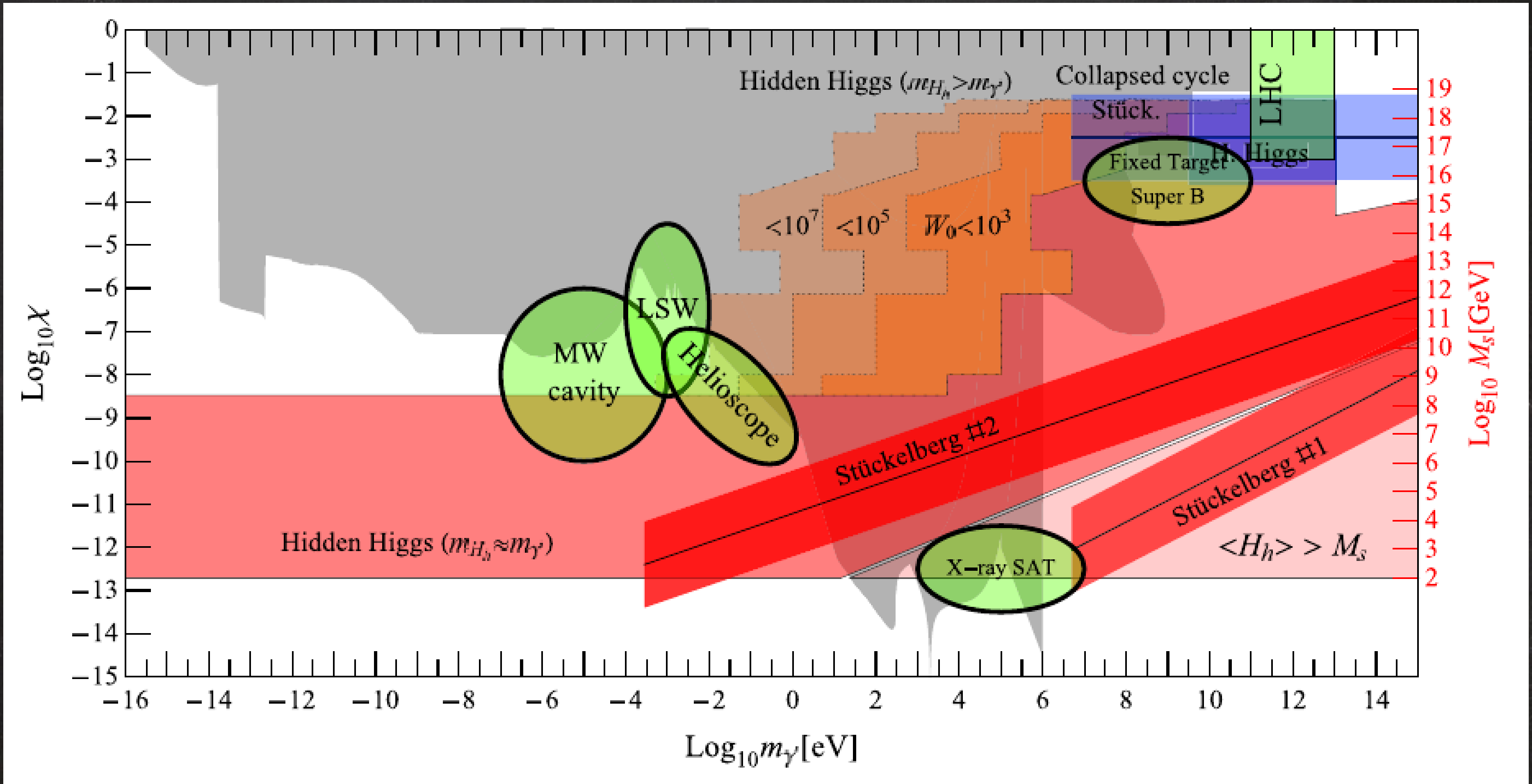
Proca instability

- Not strongly dependent on the timescale nor on type of mode

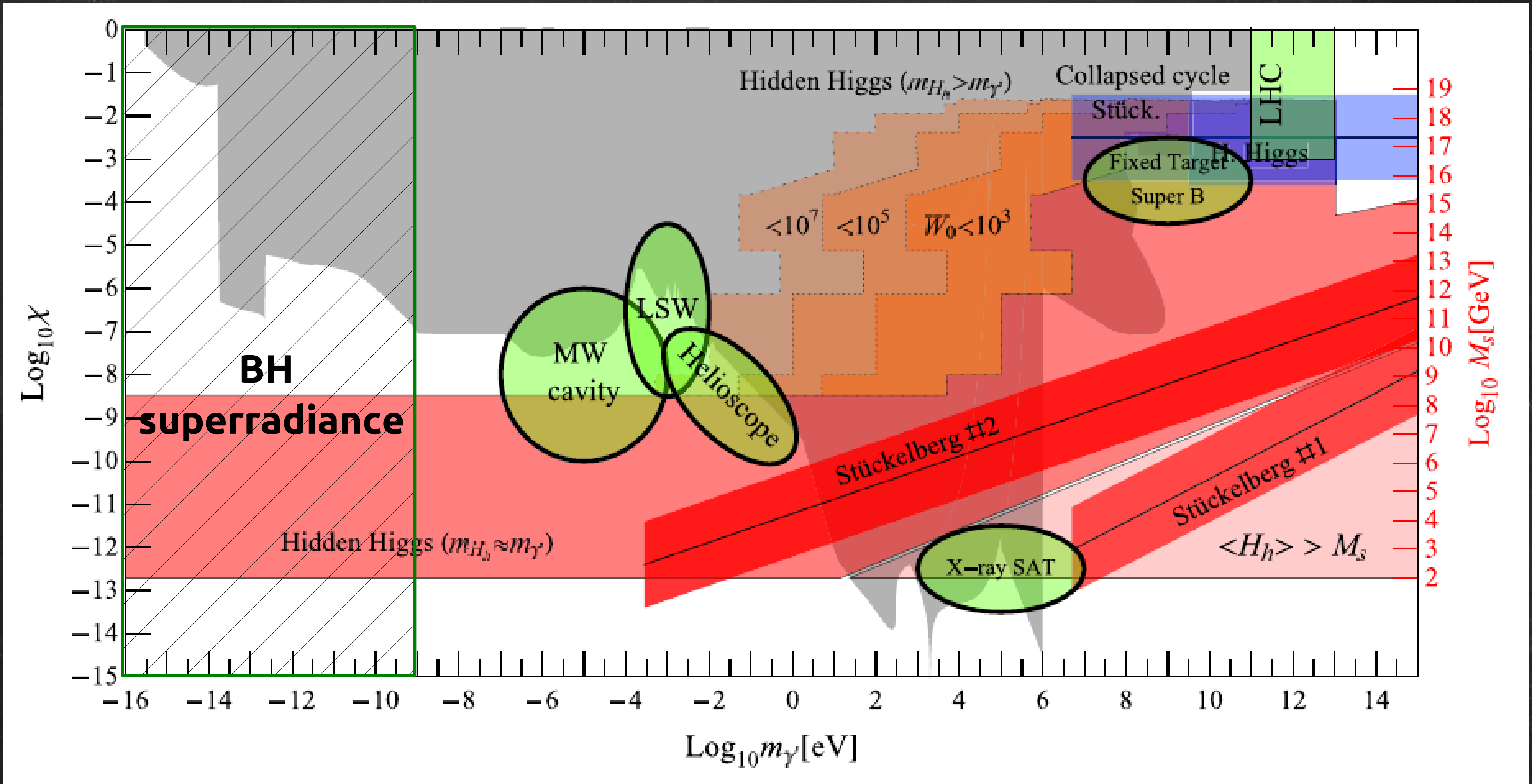


From the existence of spinning BHs → $10^{-21}\text{eV} \lesssim m_\gamma \lesssim 10^{-17}\text{eV}$

Current bound on the photon mass [PDG] → $m_\gamma < 10^{-18}\text{eV}$



[Goodsell et al. 2009]



[Goodsell et al. 2009]

Proca instability. Limitations

- **Nonlinear effects:**
  - Photon self-interaction is very weak → gradual slow down
  - Might be important for exotic fields → Bosenova?
- **Accretion disk**
  - Hidden U(1) fields are weakly coupled to matter
  - Might be relevant for massive photons, but
    - Superradiant mode are coherent and $\lambda \sim$ BH size
    - Disks are charge neutral and matter coupling incoherent
    - Equatorial disks can at most quench some unstable modes

Conclusions

- Spinning BHs as labs for exotic particles and modified gravity
- Perturbation theory of rotating objects is challenging
- Slowly-rotating approximation: general method
- Proca perturbations of Kerr BHs in GR
  - Instability for massive vector fields → Strong (est?) instability
  - Bounds on the photon mass and on exotic ultralight spin-1 fields
- Extensions
  - BHs in alternative theories (e.g. quadratic curvature) [Yunes & Pretorius 2009] [Pani et al. 2011] [Yagi, Yunes, Tanaka 2012]
  - Kerr-Newman BHs
  - Higher dimensions

Obrigado!

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MARIE CURIE ACTIONS

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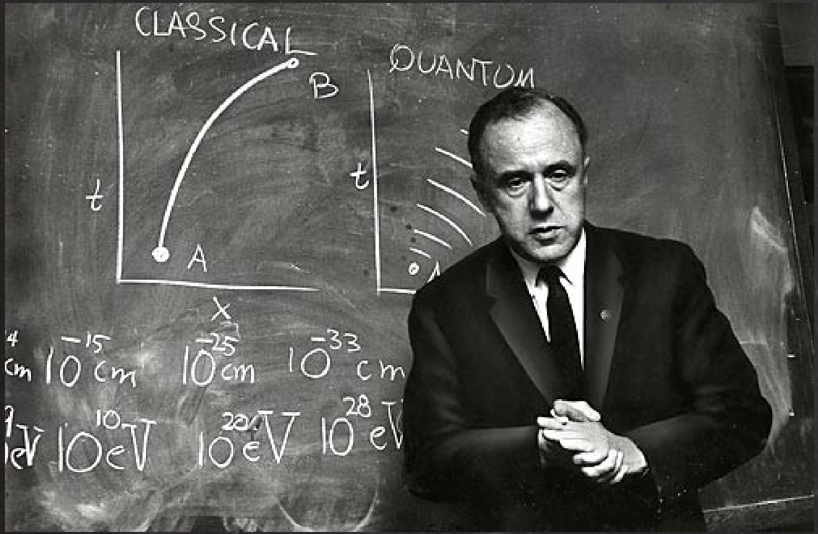
John Archibald Wheeler (1998):



"Black holes teach us that space can be crumpled like a piece of paper into an infinitesimal dot, that time can be extinguished like a blown-out flame, and that the laws of physics that we regard as 'sacred', as immutable, are anything but."