Hadron Structure in Transverse Momentum Dependent Resummation and Evolution

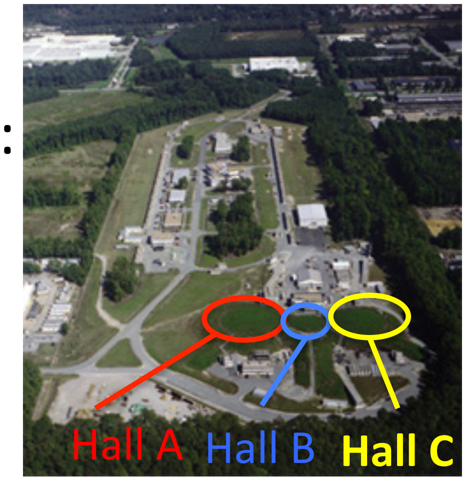
T. C. Rogers

C.N. Yang Institute for Theoretical Physics, SUNY Stony Brook

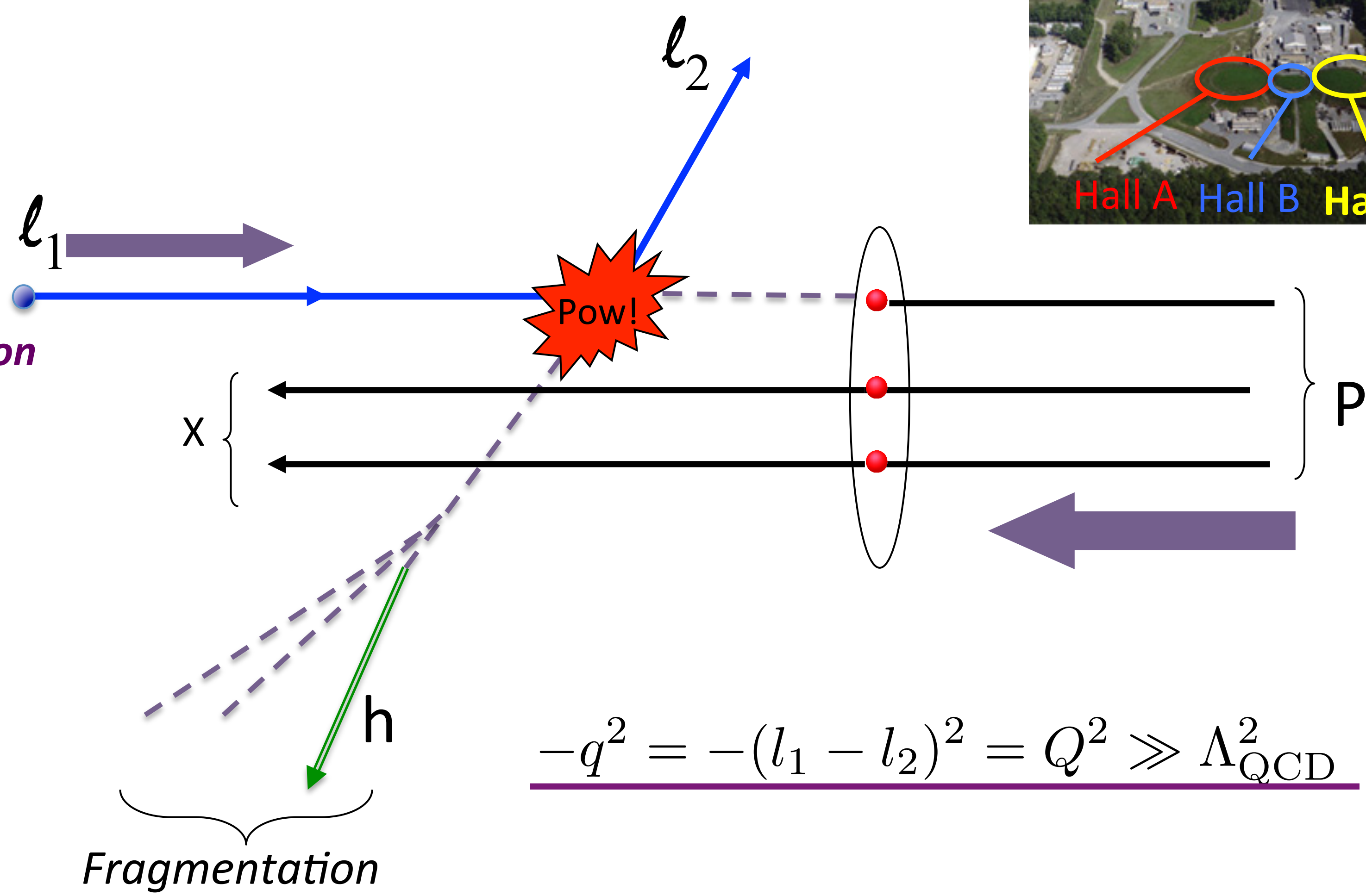
- Perturbative QCD and Collinear Factorization.
- Transverse Momentum Dependent (TMD) Factorization.
- TMD project: Implementing TMD-factorization.

Seminar: Southern Methodist University – October 7, 2013

Example:



- Semi-Inclusive Deep Inelastic Scattering (SIDIS):

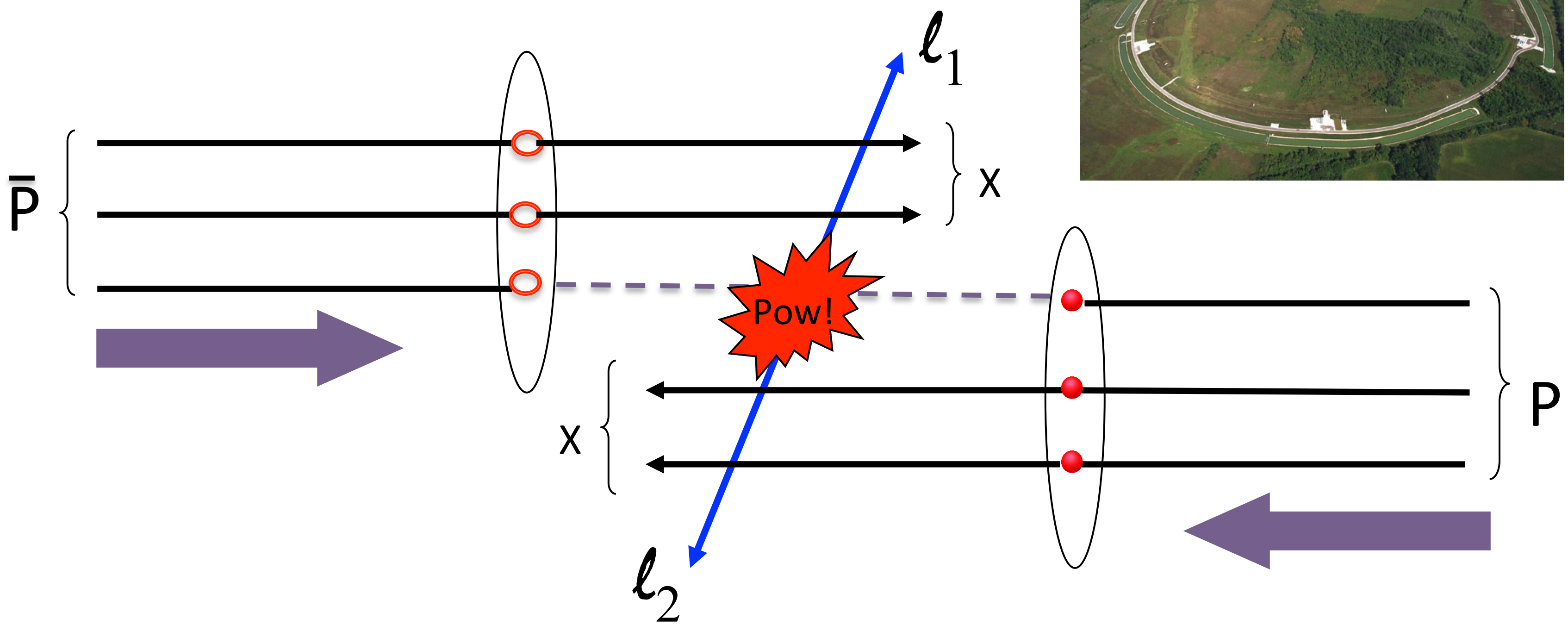


$$-q^2 = -(l_1 - l_2)^2 = Q^2 \gg \Lambda^2_{\rm QCD}$$

Example 2:

Tevatron

- Drell-Yan:

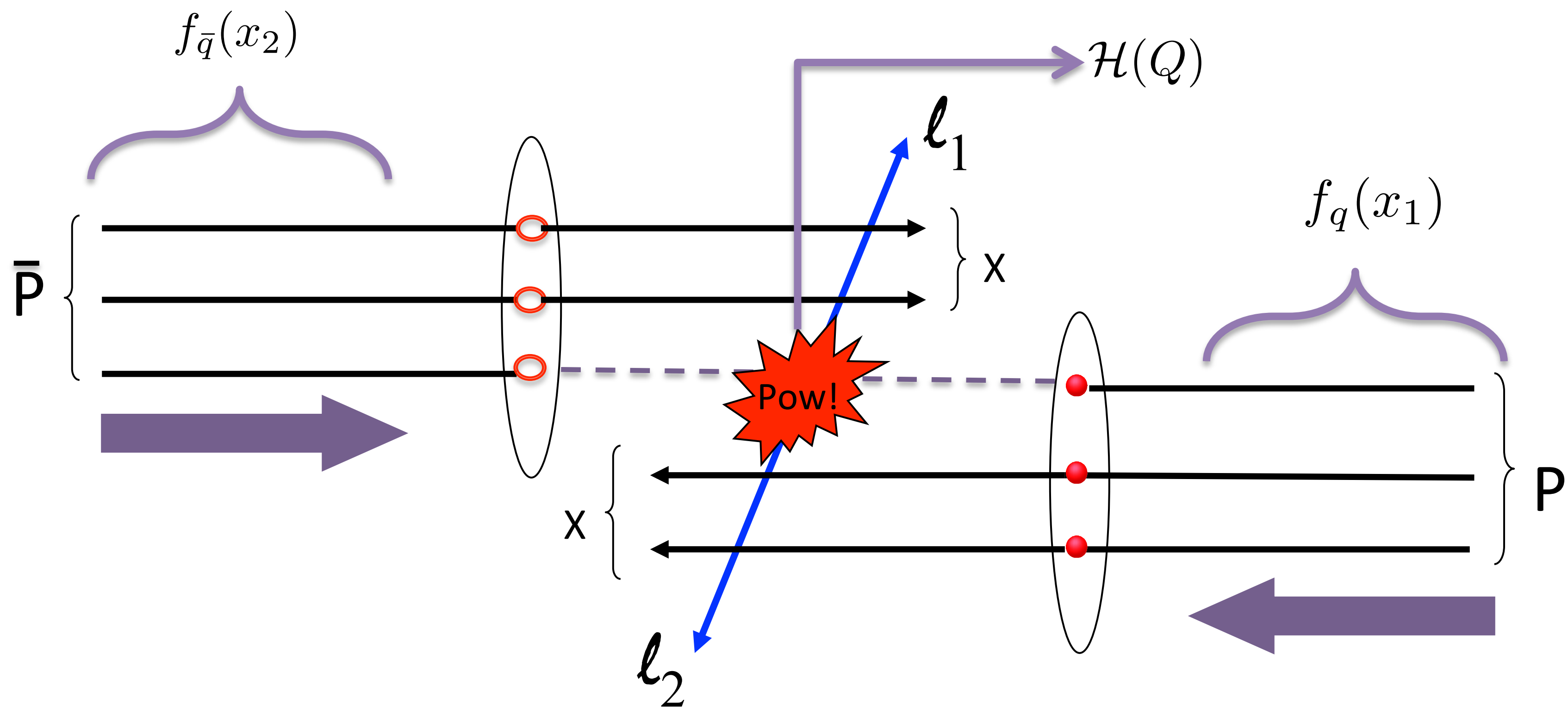


$$q^2 = (l_1 + l_2)^2 = Q^2 \gg \Lambda^2_{\mathrm{QCD}}$$

$$\sim 1/Q$$

Small Scales

Example 2:



$q^2 = (l_1 + l_2)^2 = Q^2 \gg \Lambda^2_{\mathrm{QCD}}$

$\sim 1/Q$

Small Scales

Collinear (Standard) Case

- Parton Model Picture

$$\sigma \sim \int \underbrace{\mathcal{H}(Q)}_{\text{Elementary collision}} \otimes \underbrace{f_{q/P}(x_1)}_{} \otimes \underbrace{f_{\bar{q}/\bar{P}}(x_2)}_{}$$

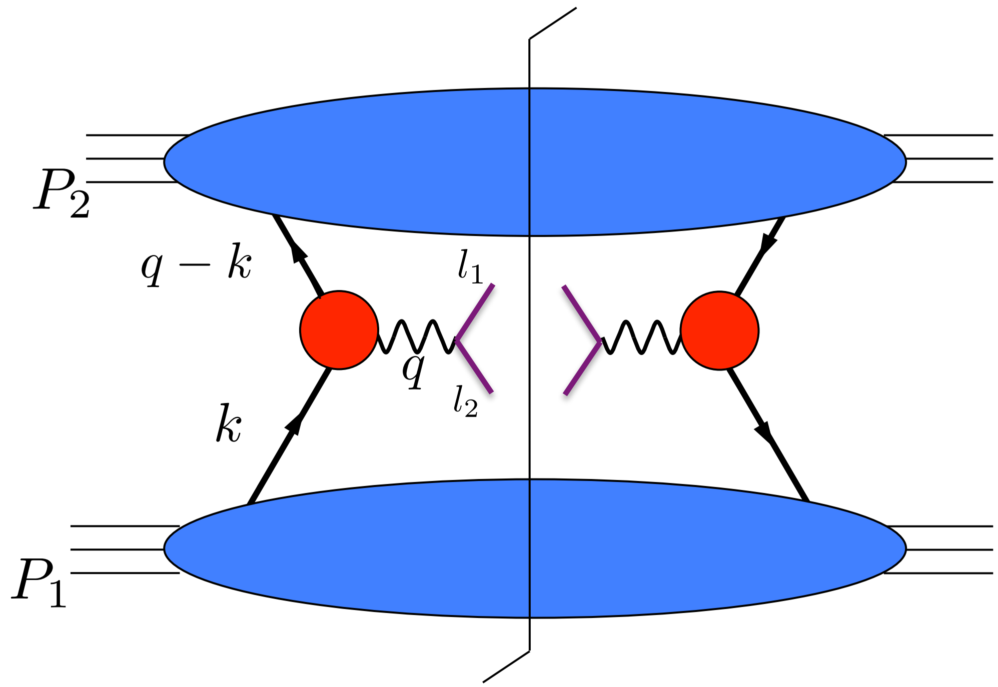
Elementary collision

Number densities

"Parton Distribution Functions" (PDFs)

Example 2:

- Drell-Yan:



Collinear case

Get: $\int d\mathbf{q}_T \frac{d\sigma}{d\mathbf{q}_T \cdots}$

(Or large q_T)

$$q^2 = (l_1 + l_2)^2 = Q^2 \gg \Lambda^2_{\text{QCD}}$$

$$k_1 \equiv k$$

$$k_2 \equiv q - k$$

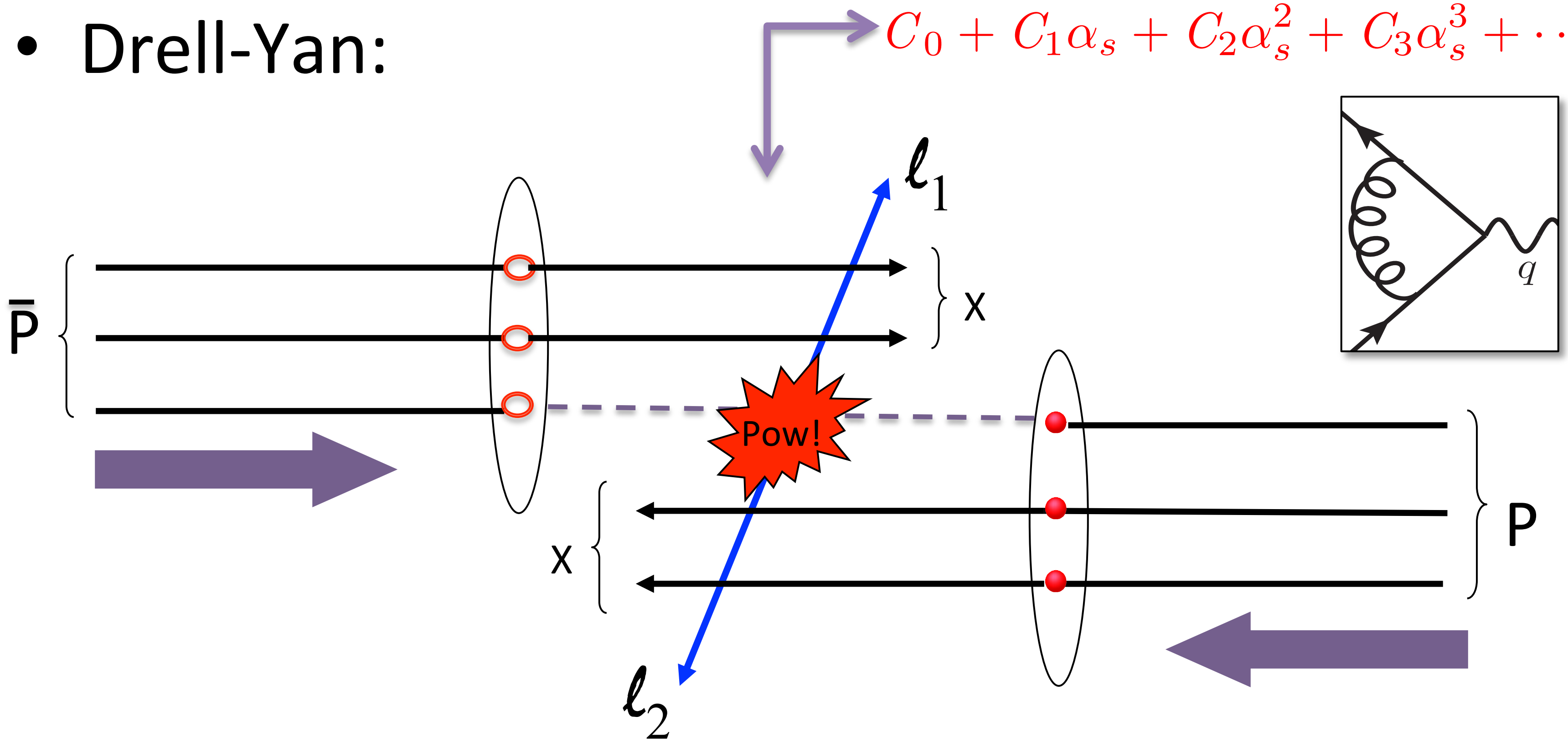
$$\mathbf{k}_{1T} + \mathbf{k}_{2T} = \mathbf{q}_T$$

Example 2:

- Drell-Yan:

$$C_0 + C_1\alpha_s + C_2\alpha_s^2 + C_3\alpha_s^3 + \cdots$$

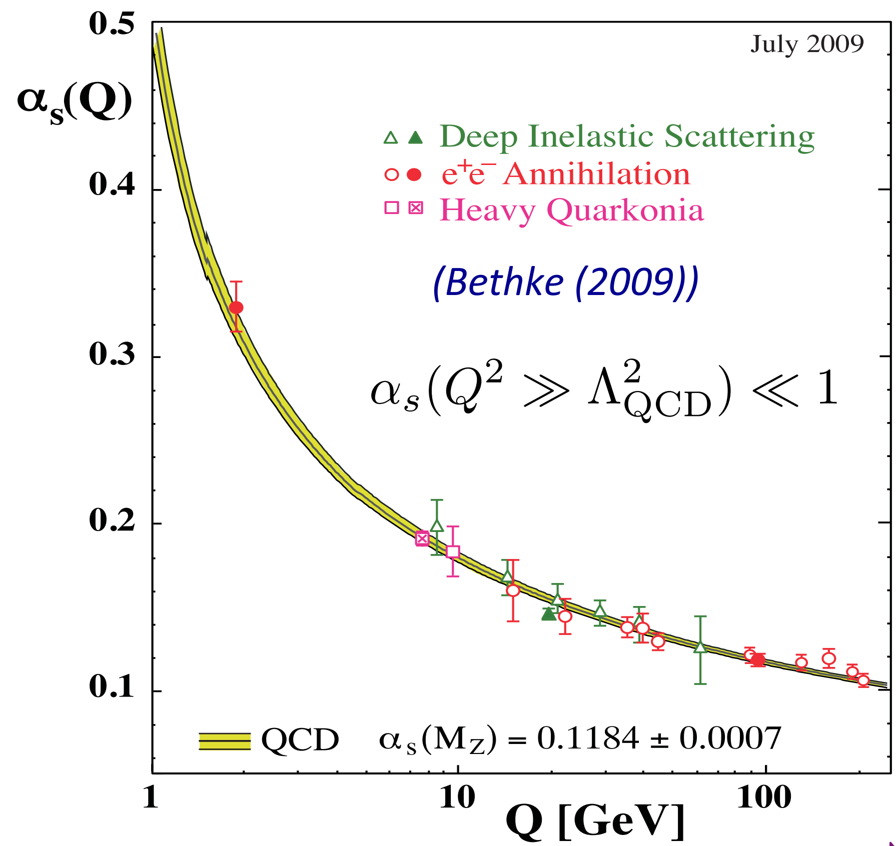
??



α_s = QCD coupling strength

QCD Factorization

- Short Distances; Asymptotic Freedom
  - Perturbation Theory



- Large Distance Scales:
  - Hadron Structure
    Confinement

- Physical processes involve both.
  - Need to be separated: *QCD Factorization*

$d \sim 1/Q$

Collinear (Standard) Case

- Parton Model

$$\sigma \sim \int \underbrace{\mathcal{H}(Q)} \otimes \underbrace{f_{q/P}(x_1) \otimes f_{\bar{q}/\bar{P}}(x_2)}$$

Elementary collision
Short distance scales
$\sim 1/Q$

Hadron Structure: large distance scales

Collinear (Standard) Case

- Perturbative QCD factorization theorem:

$$\sigma \sim \int \mathcal{H}(Q, \mu/Q, \alpha_s(\mu)) \otimes f_{q/P}(x_1;\mu) \otimes f_{\bar{q}/\bar{P}}(x_2;\mu)$$

Small Coupling: Perturbation Theory

$$C_0 + C_1\alpha_s(\mu) + C_2\alpha_s(\mu)^2 + C_3\alpha_s(\mu)^3 + \cdots$$

$$\text{Error} \sim \Lambda_{\text{QCD}}/Q$$

Red : Perturbatively Calculable

Blue : Non-Perturbative Input Needed

Collinear (Standard) Case

- Perturbative QCD factorization theorem:

$$\sigma \sim \int \mathcal{H}(Q, \mu/Q, \alpha_s(\mu)) \otimes f_{q/P}(x_1;\mu) \otimes f_{\bar{q}/\bar{P}}(x_2;\mu)$$

Small Coupling: Perturbation Theory

$$C_0 + C_1\alpha_s(\mu) + C_2\alpha_s(\mu)^2 + C_3\alpha_s(\mu)^3 + \cdots$$

Defined in terms of elementary fields

$$\int \frac{dw^-}{(2\pi)} e^{-i\xi P^+ w^-} \langle P | \bar{\psi}_0(0, w^-, \mathbf{0}_t) \frac{\gamma^+}{2} \psi_0(0, 0, \mathbf{0}_t) | P \rangle$$

$$\text{Error} \sim \Lambda_{\text{QCD}}/Q$$

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Small Coupling: Perturbation Theory

$$C_0 + C_1\alpha_s(\mu) + C_2\alpha_s(\mu)^2 + C_3\alpha_s(\mu)^3 + \cdots$$

Auxiliary parameter: Arbitrary

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Auxiliary parameter: Arbitrary

- DGLAP evolution

(Dokshitzer-Gribov-Lipatov-Altarelli-Parisi)

$$\frac{d}{d\ln\mu} f_{j/P}(x;\mu) = 2\int P_{jj'}(x') \otimes f_{j'/P}(x/x';\mu)$$

- *Factorization* + *Evolution*: Universal PDFs

"Portable"

Renormalization Group Equations for Collinear PDFs

- $$f_{j/p}(\xi;\mu) = \sum_i \int \frac{dz}{z}\, Z_{ji}(z, g_s(\mu))\, f_{0,i/p}(\xi/z)$$
  $$= Z_{ji} \otimes f_{0,i/p}$$

- RG invariance: $$\frac{d}{d\ln\mu}\, f_{0,i/p}(\xi/z) = 0$$

- RG equation: $$\frac{d}{d\ln\mu} f_{j/p}(\xi;\mu) = 2\sum_i \int \frac{dz}{z}\, P_{ji}(z, g(\mu))\, f_{i/p}(\xi/z;\mu)$$

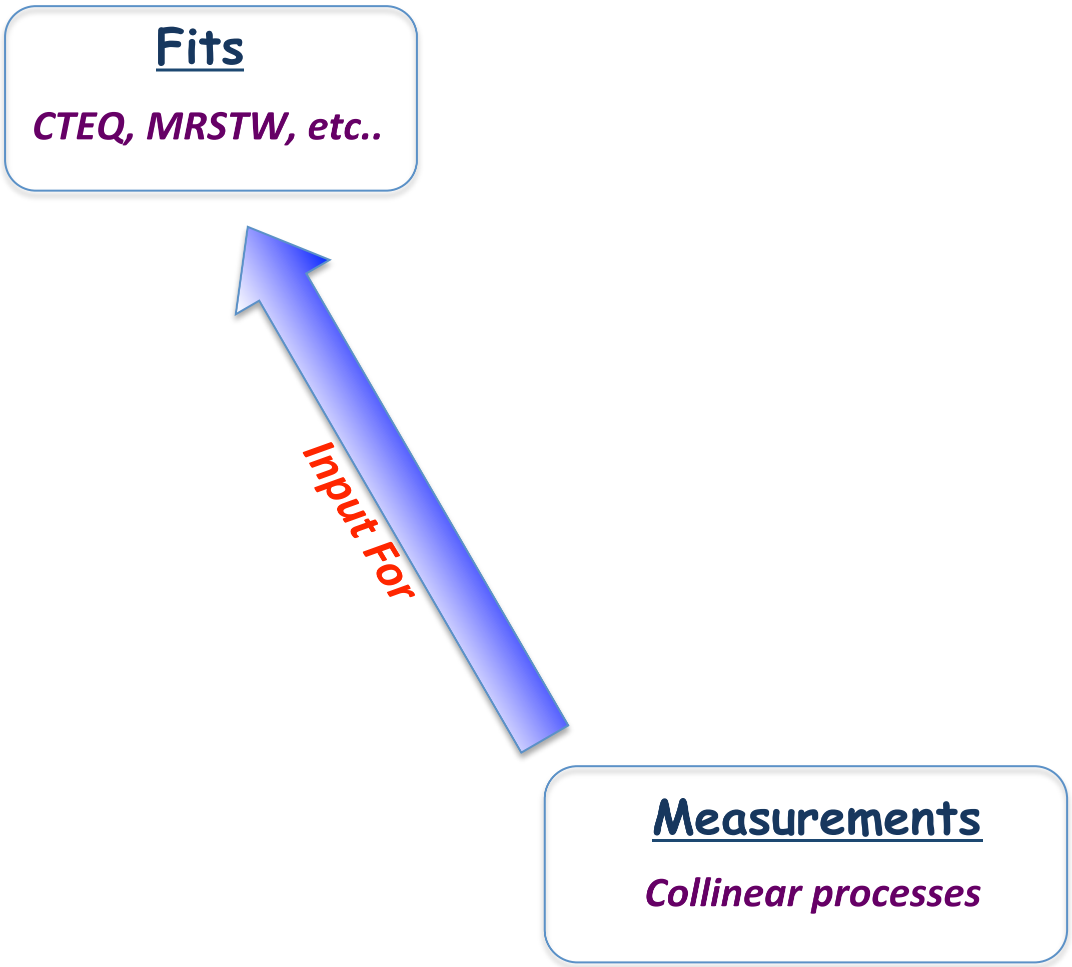
DGLAP *(Dokshitzer-Gribov-Lipatov-Altarelli-Parisi)*

Implementing Collinear Factorization

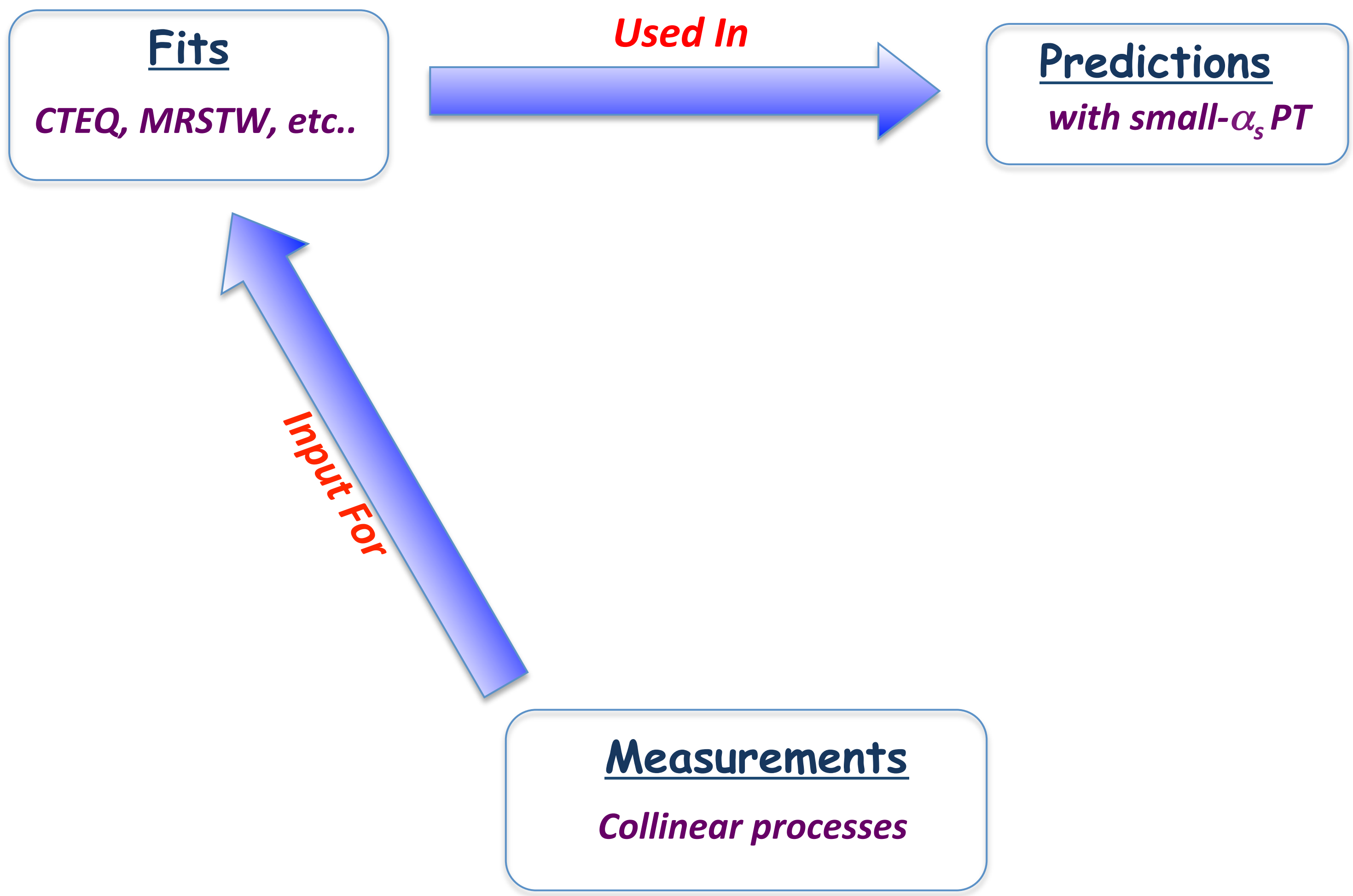
Measurements

Collinear processes

Implementing Collinear Factorization



Implementing Collinear Factorization



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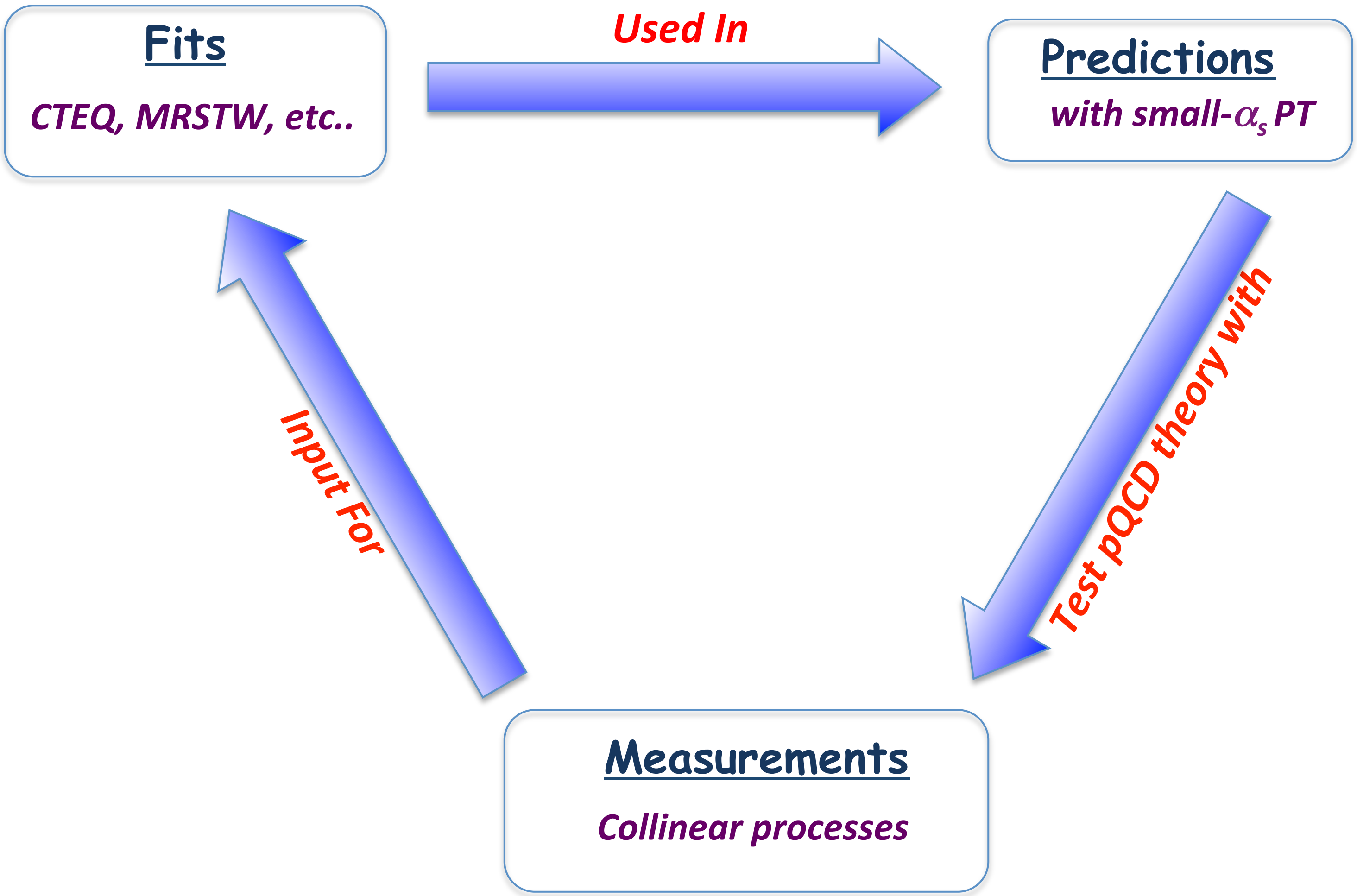

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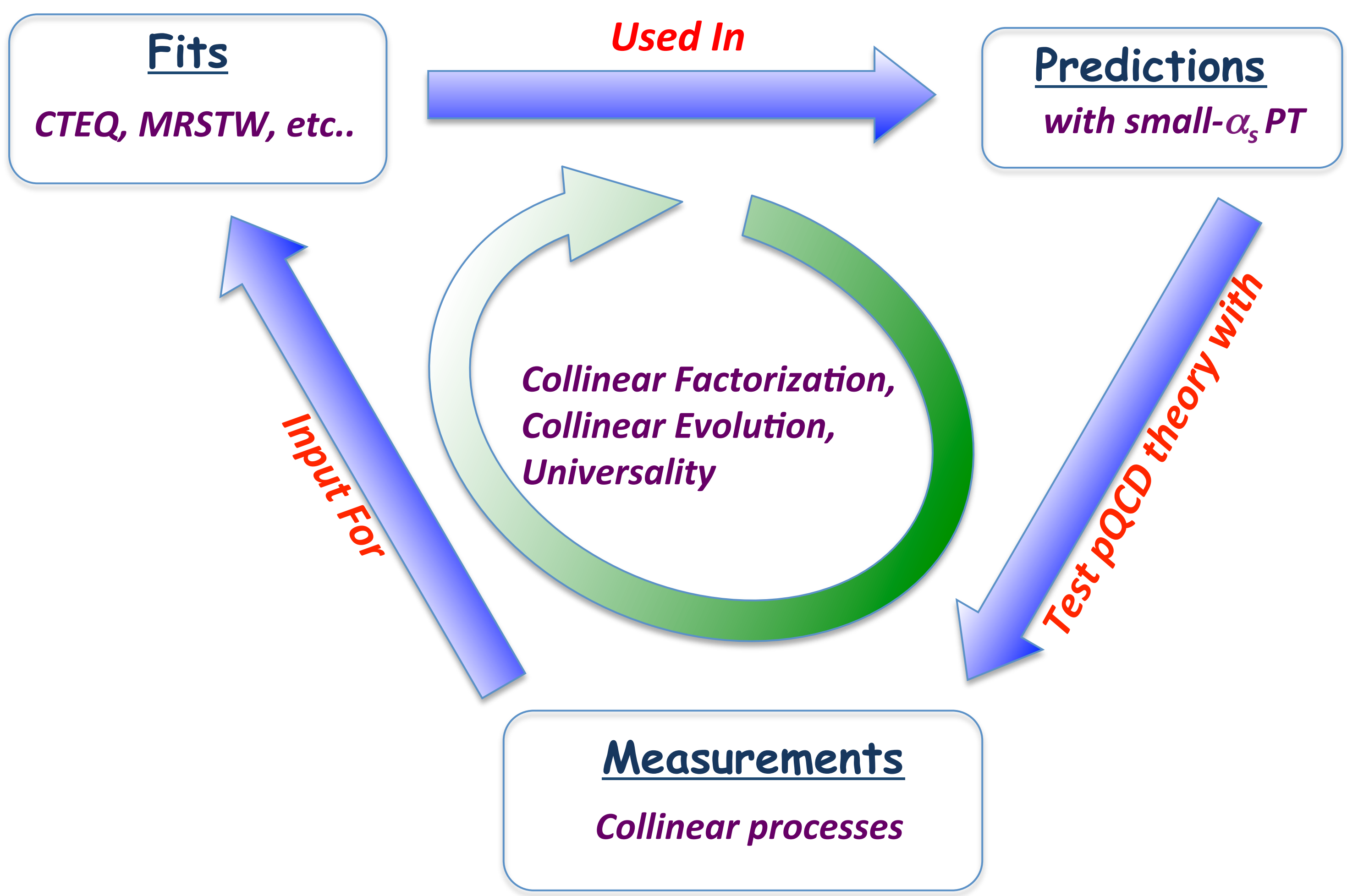

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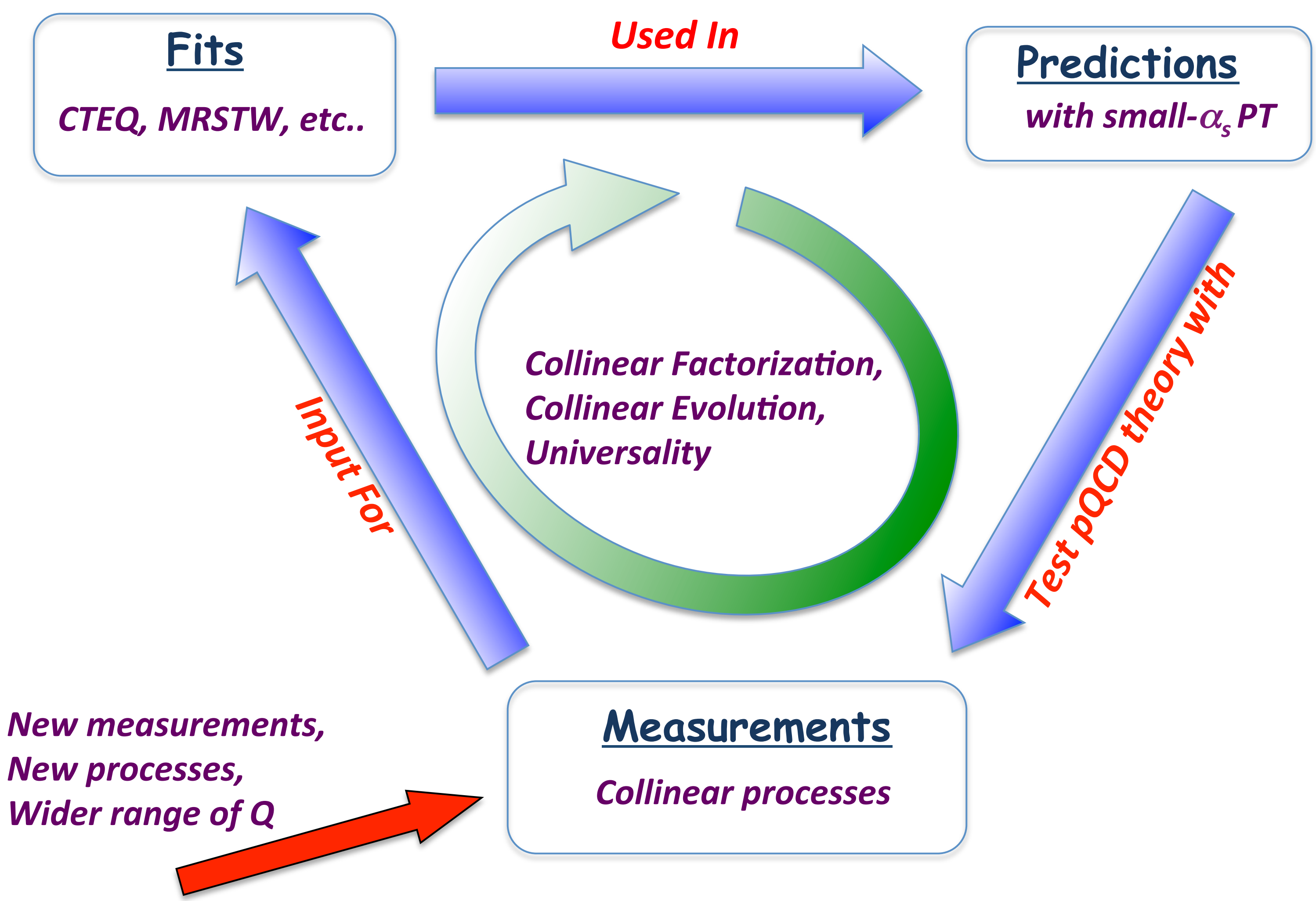

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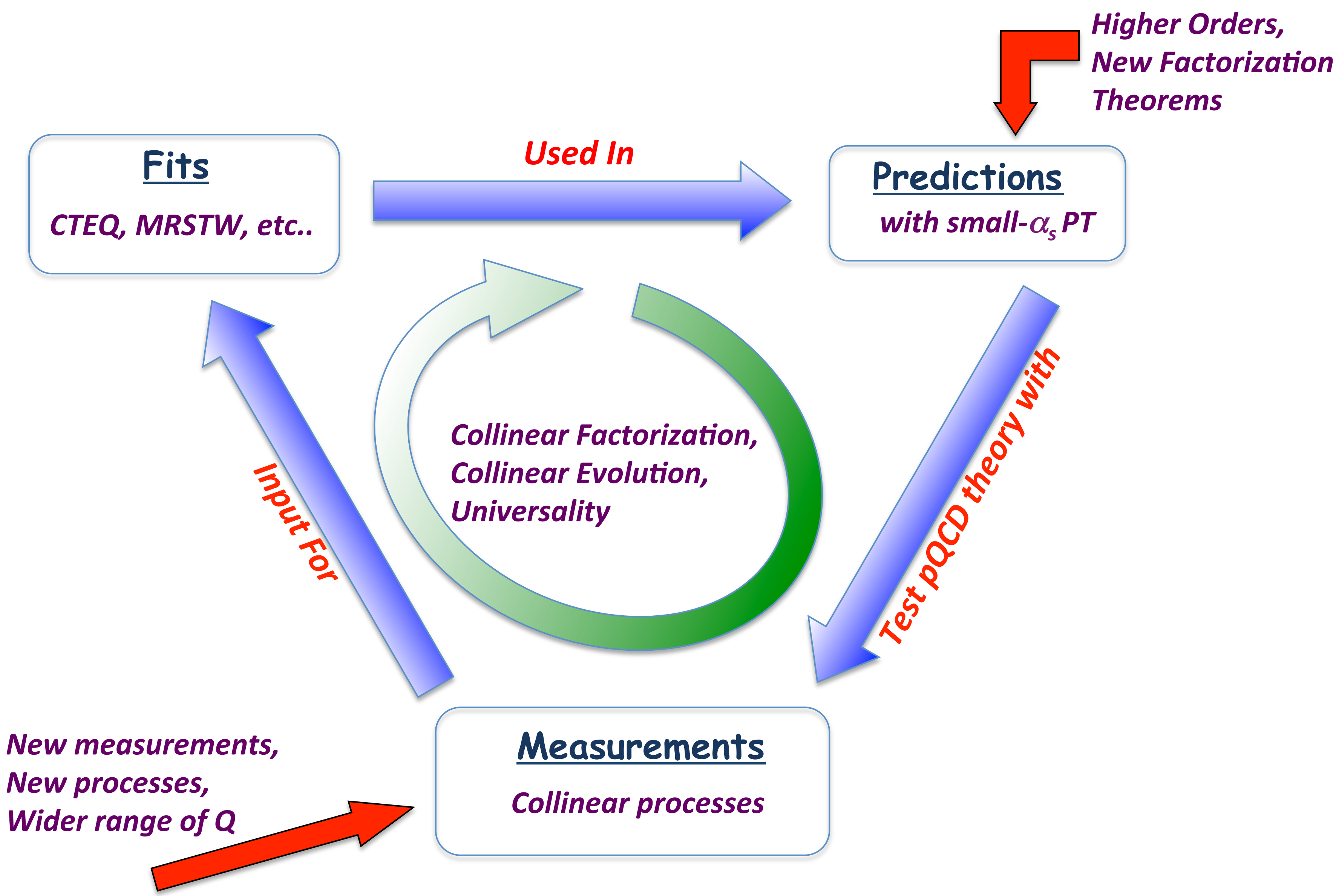
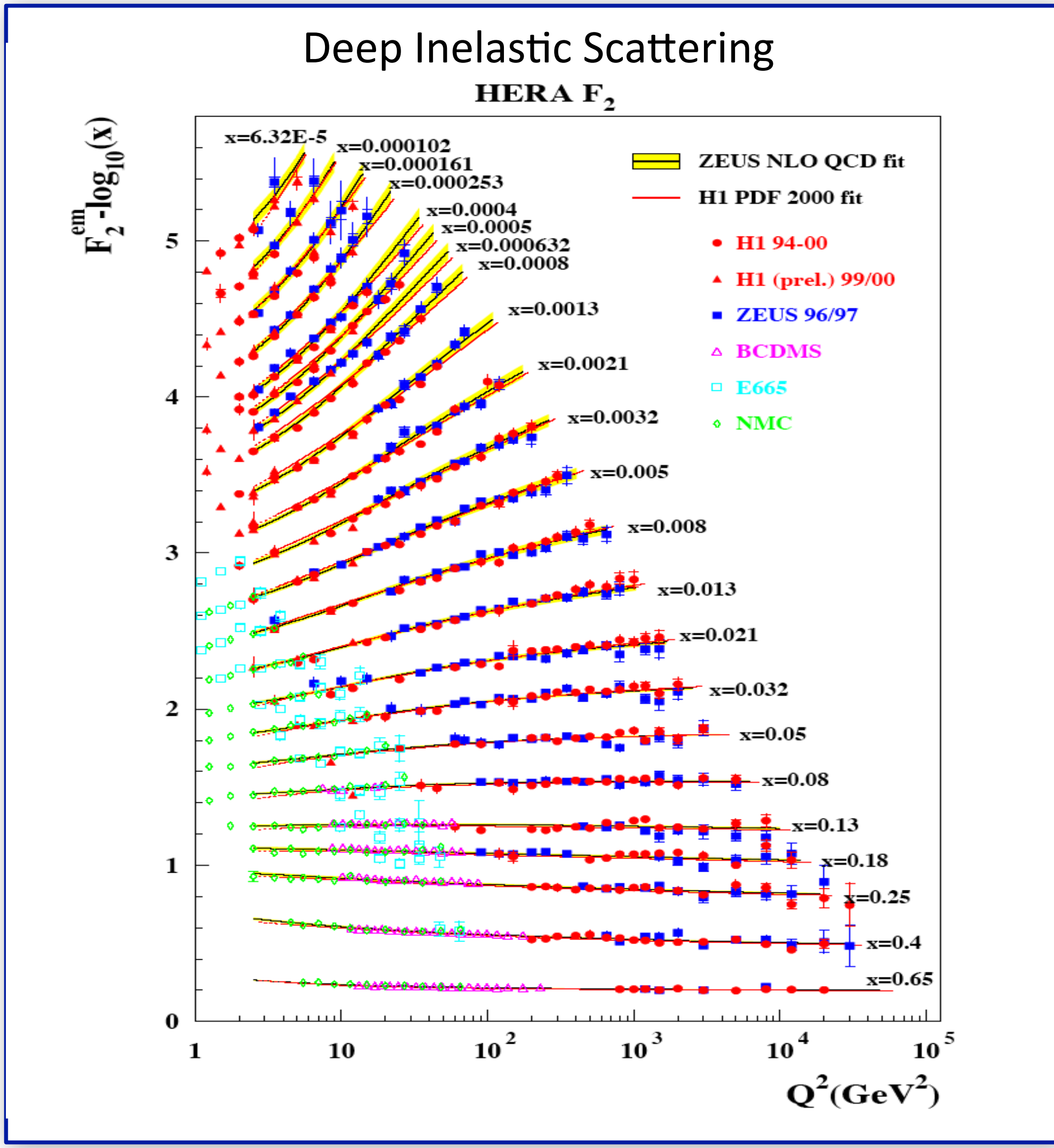
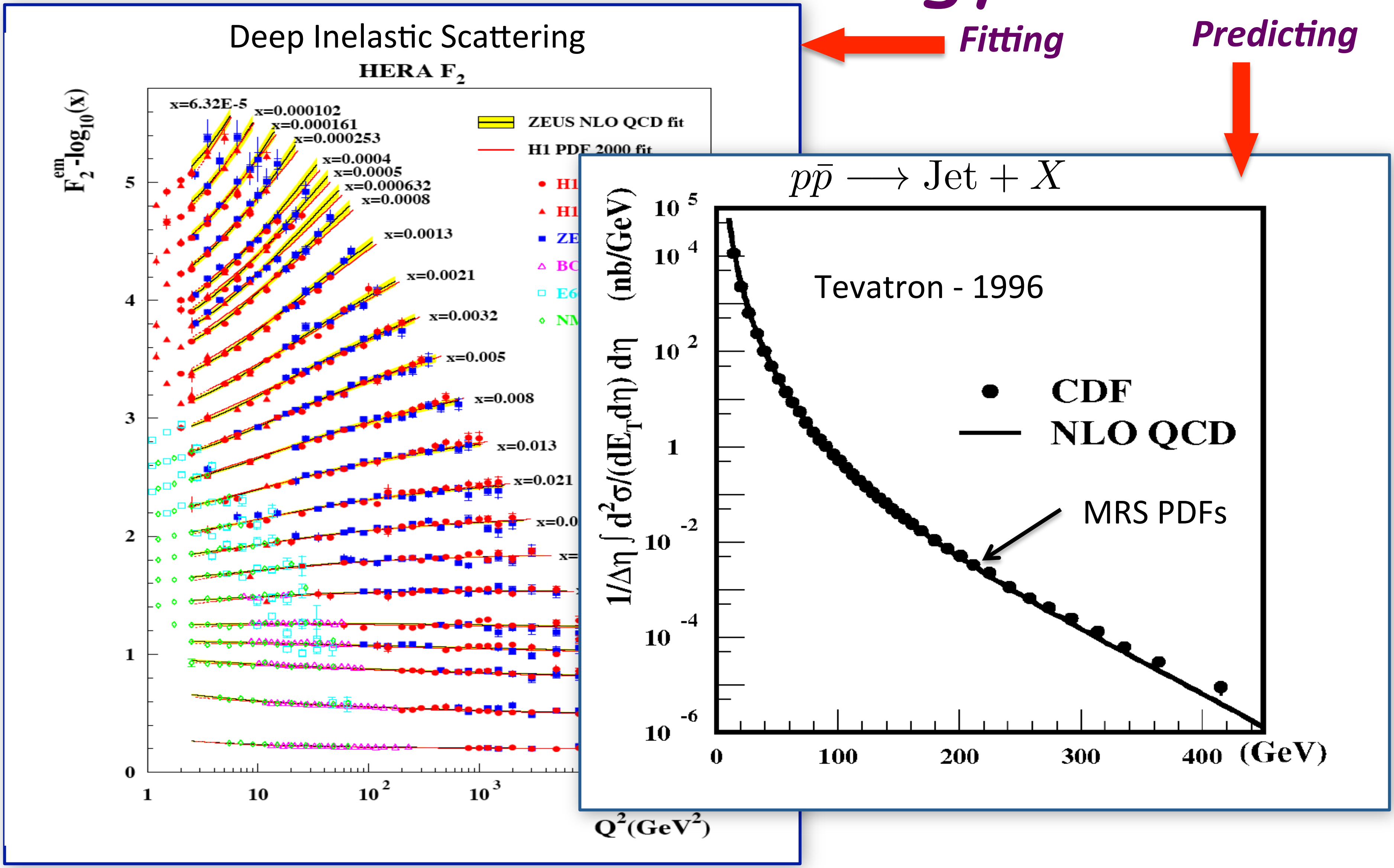


Phenomenology



Fitting

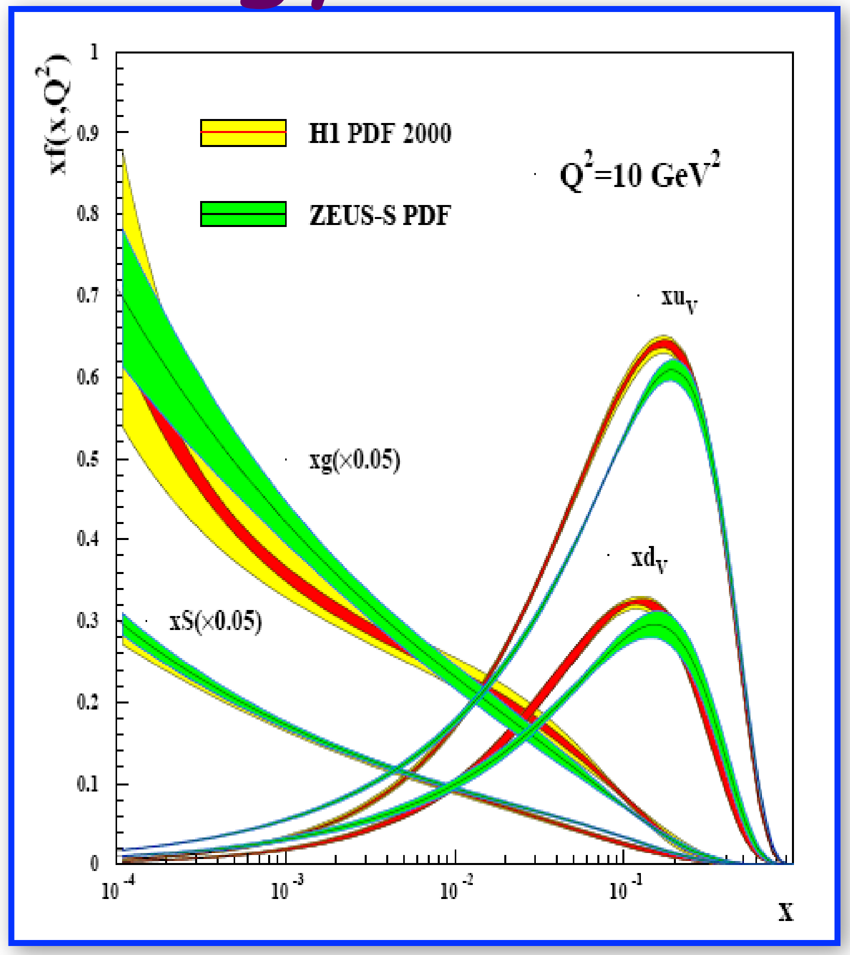
Phenomenology



Fitting

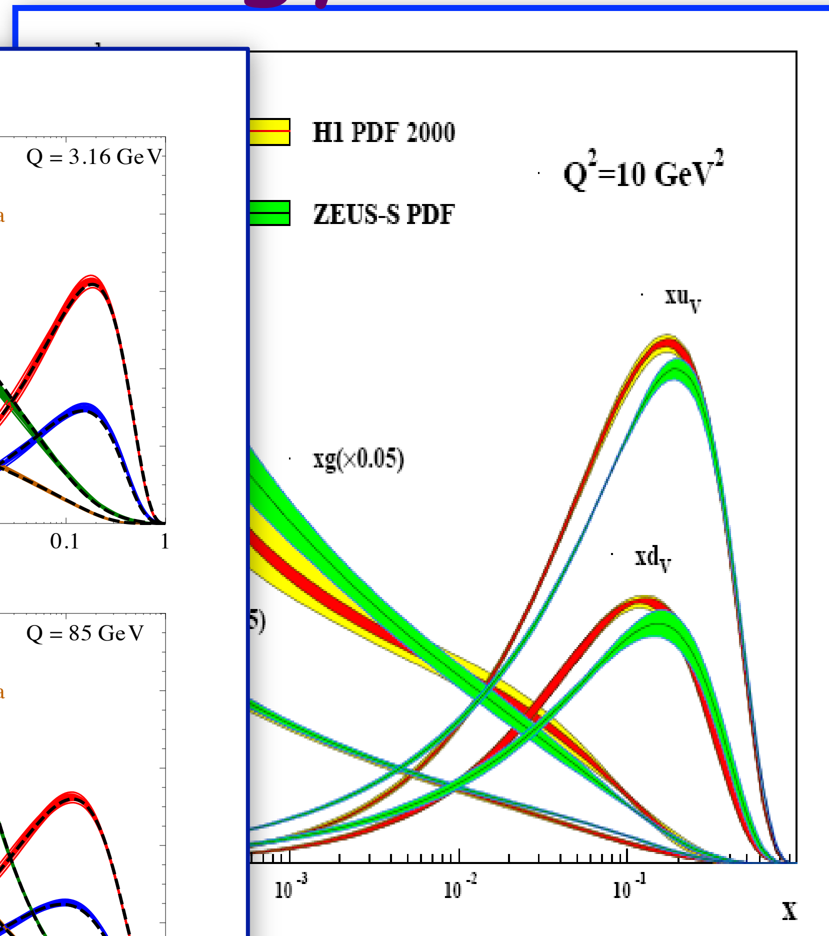
Predicting

Phenomenology

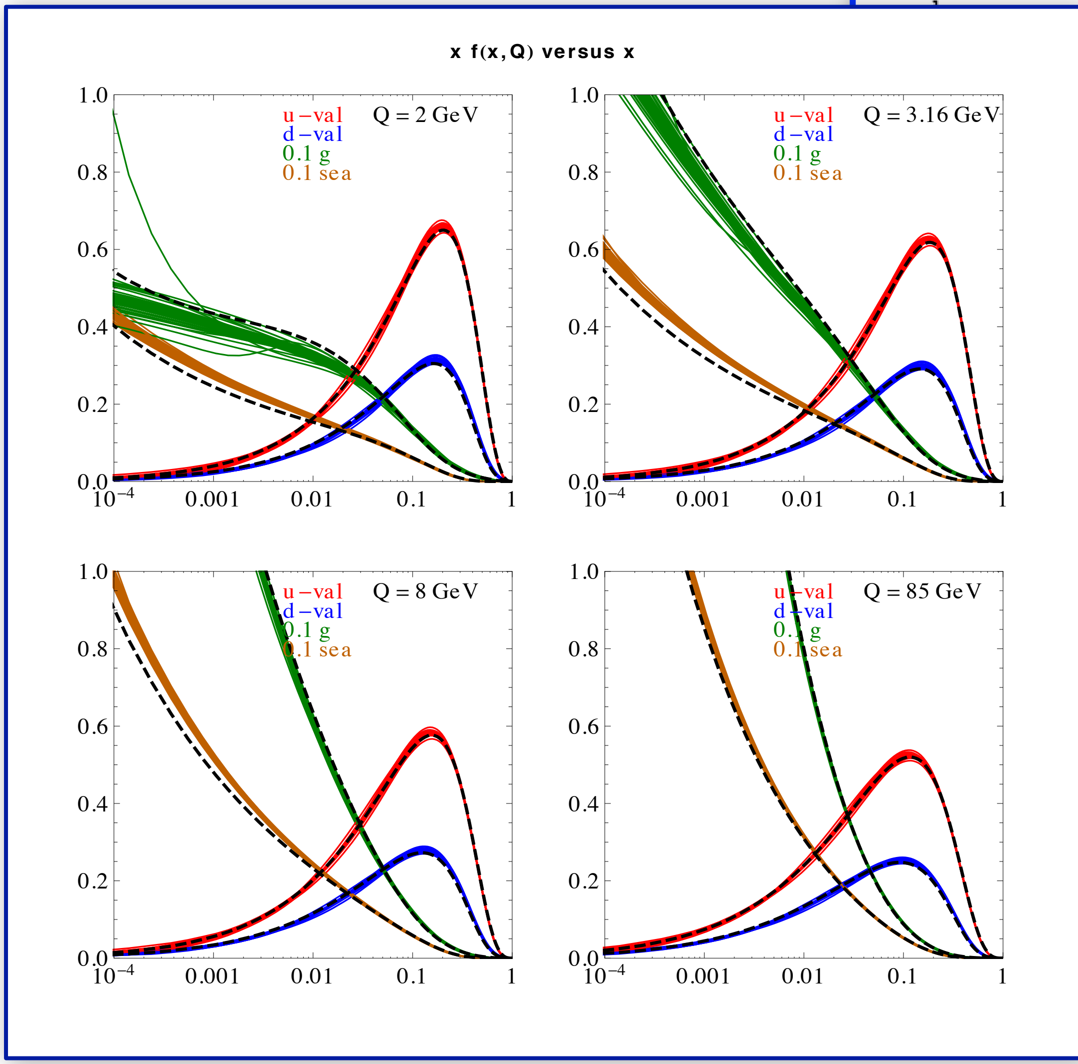


PRD67, 012007 (2003)

Phenomenology

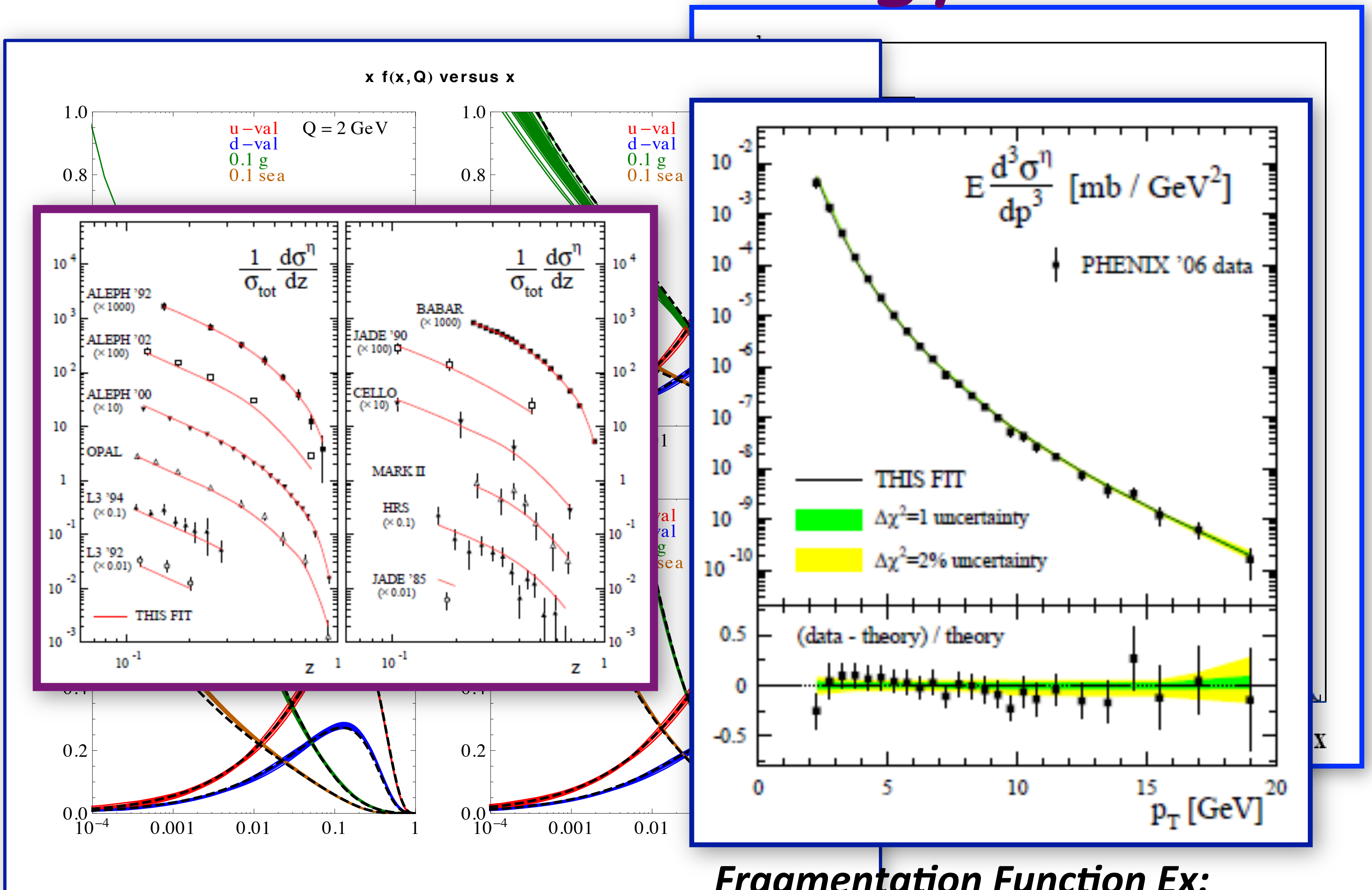


67, 012007 (2003)



CTEQ10 NNLO **arXiv:1302.6246**

Phenomenology



CTEQ10 NNLO **arXiv:1302.6246**

Fragmentation Function Ex:

PRD83, 034002 (2011)

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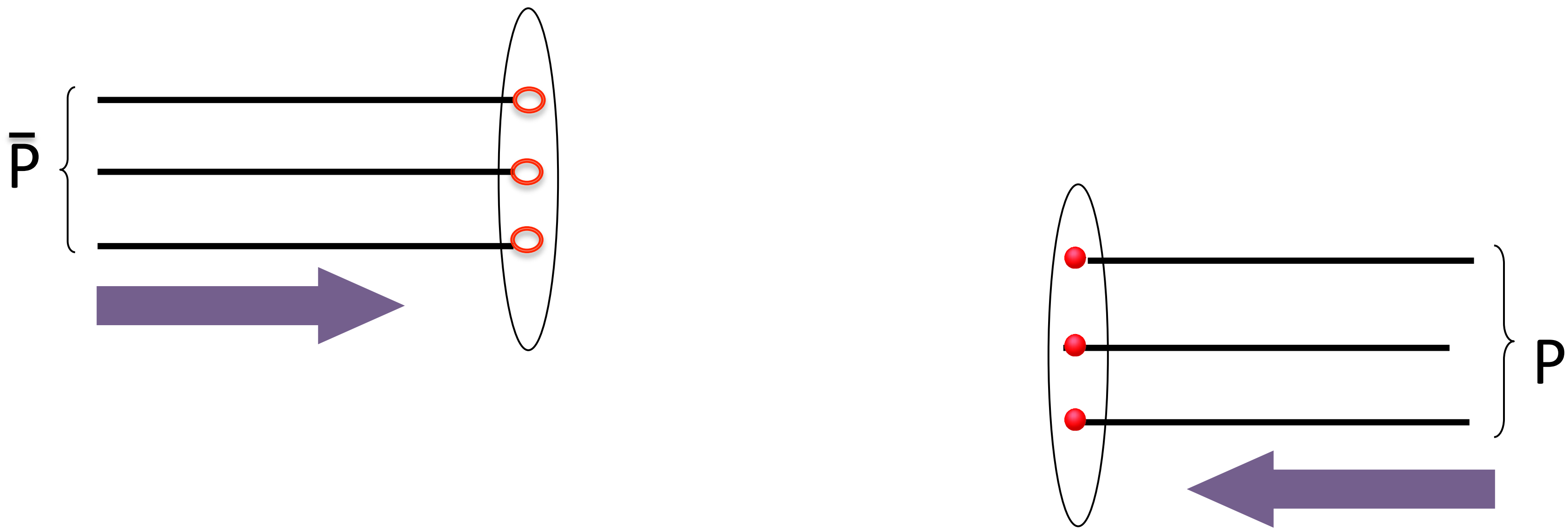

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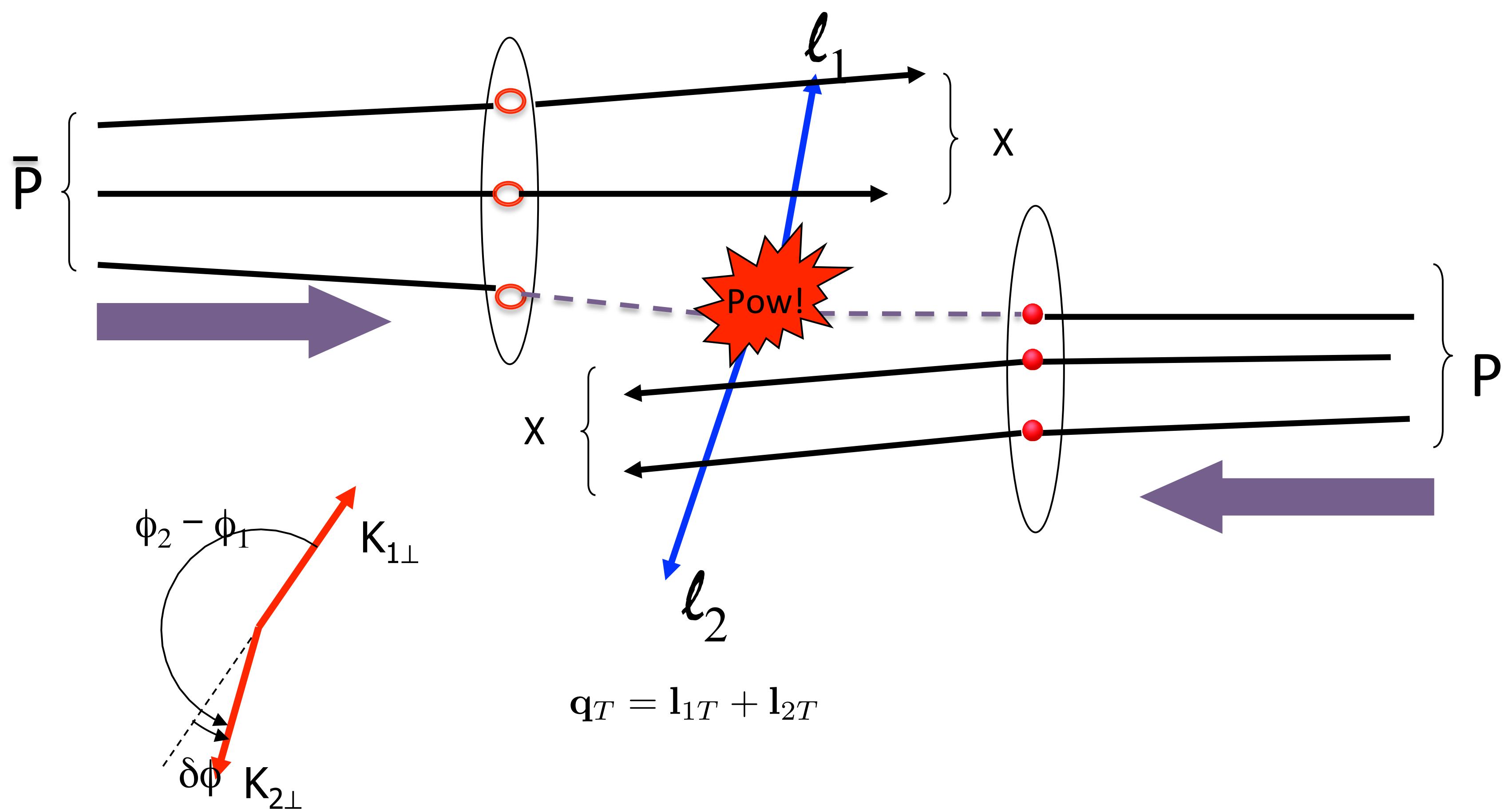
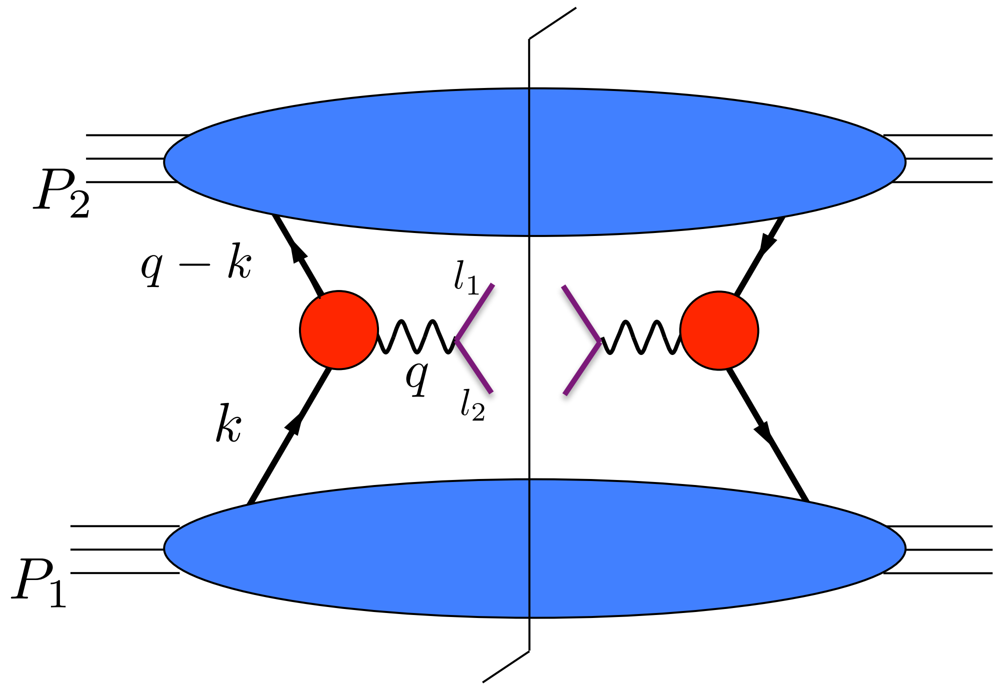


$$\mathbf{q}_T = \mathbf{l}_{1T} + \mathbf{l}_{2T}$$

Example 2:

- Drell-Yan:



Collinear case

Get: $\displaystyle\int d\mathbf{q}_T \, \frac{d\sigma}{d\mathbf{q}_T \cdots}$

(Or large q_T)

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$$\mathbf{k}_{1T} + \mathbf{k}_{2T} = \mathbf{q}_T$$

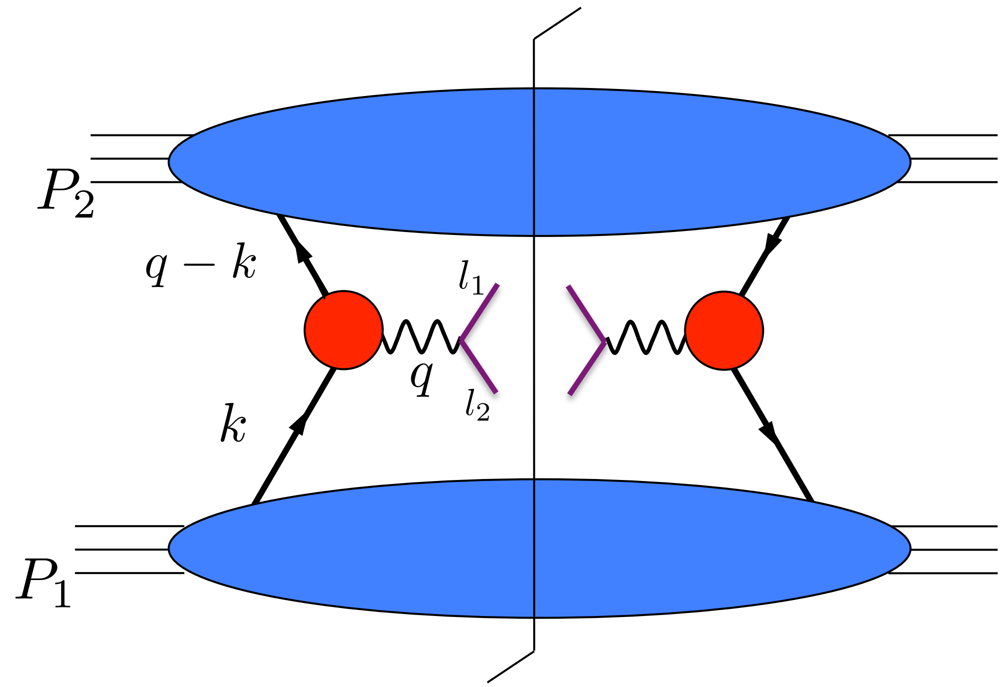
Example 2:

- Drell-Yan:

TMD *case*

Get: $\dfrac{d\sigma}{d\mathbf{q}_T \cdots}$

For all q_T.



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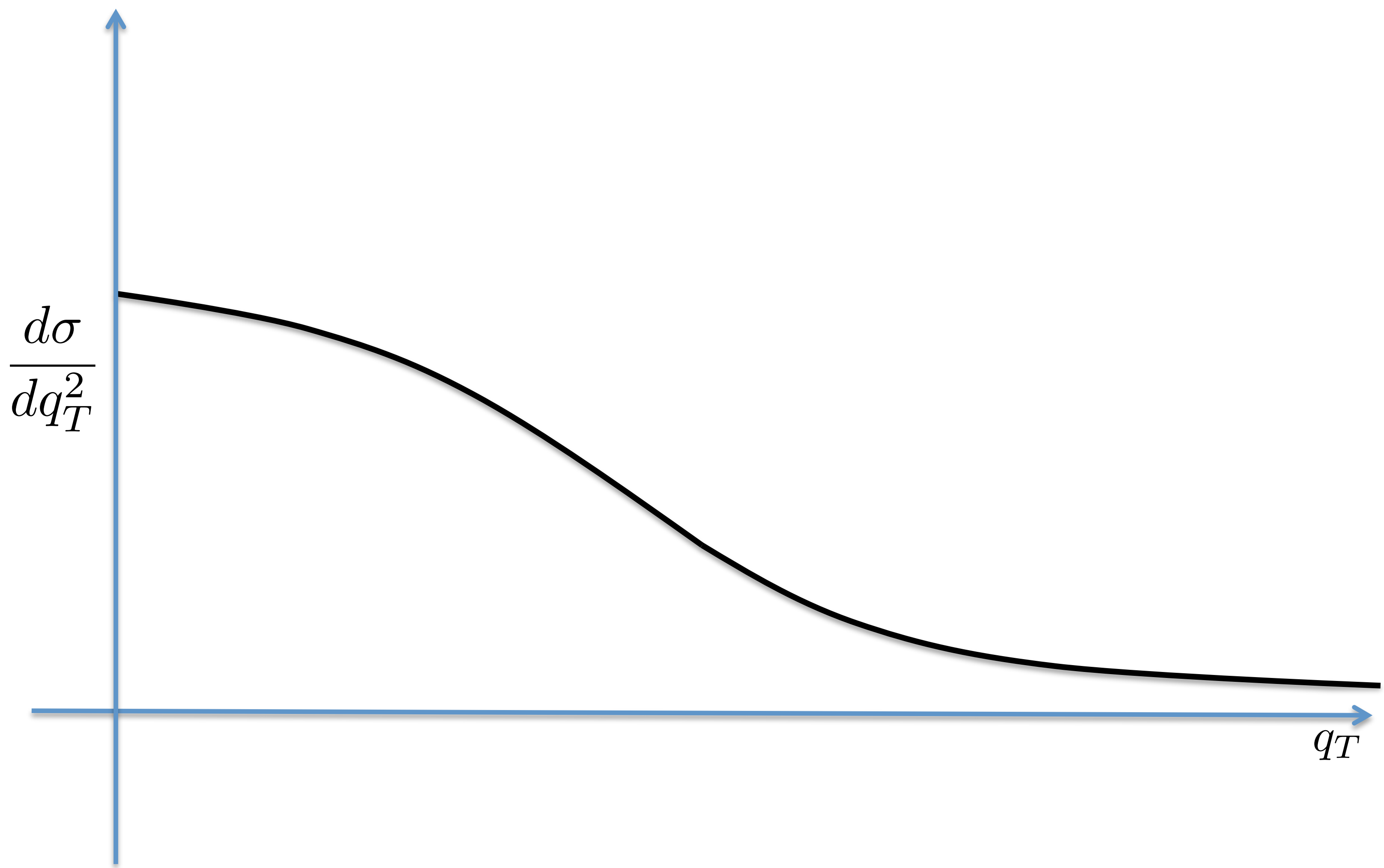
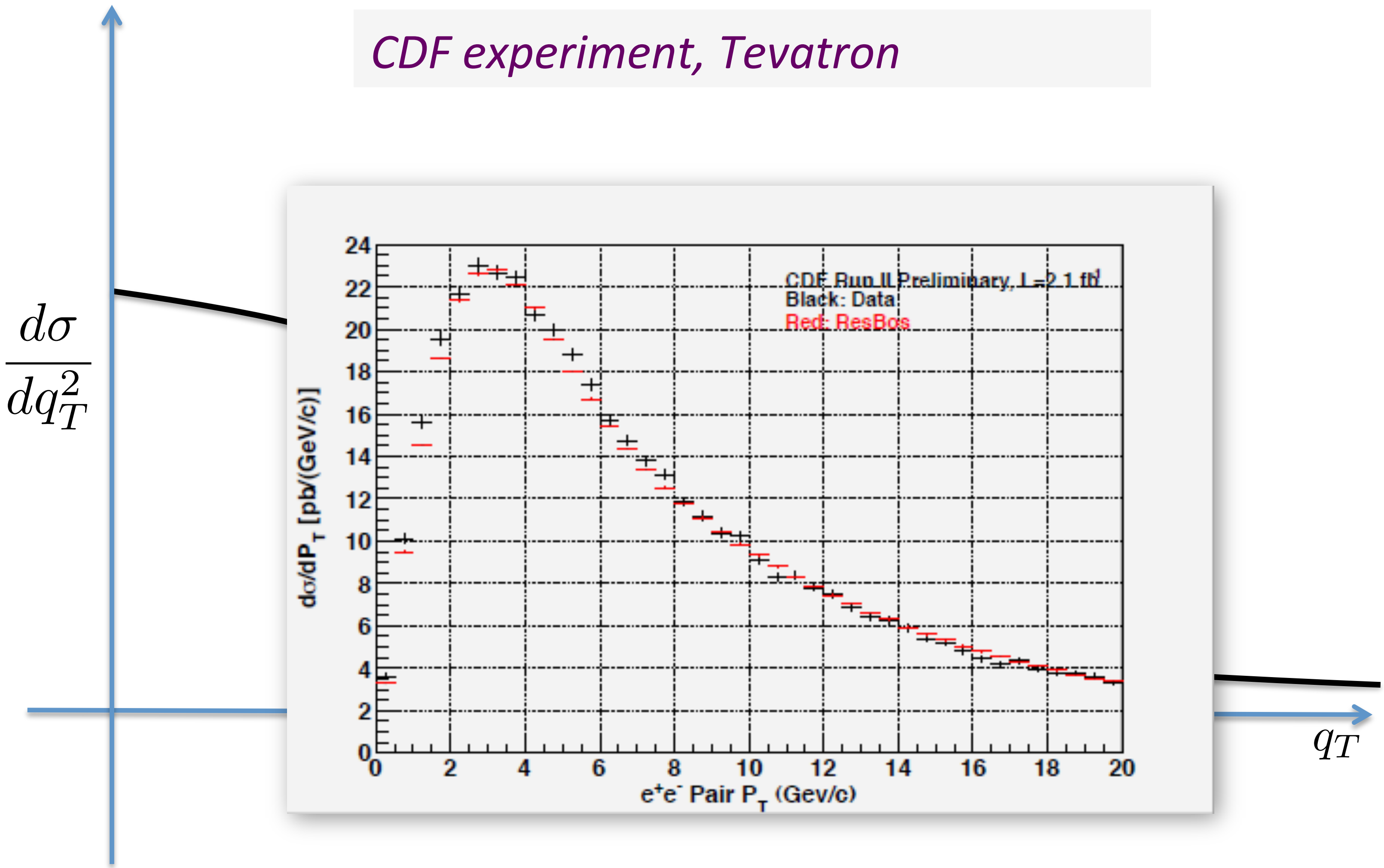
Transverse Momentum:

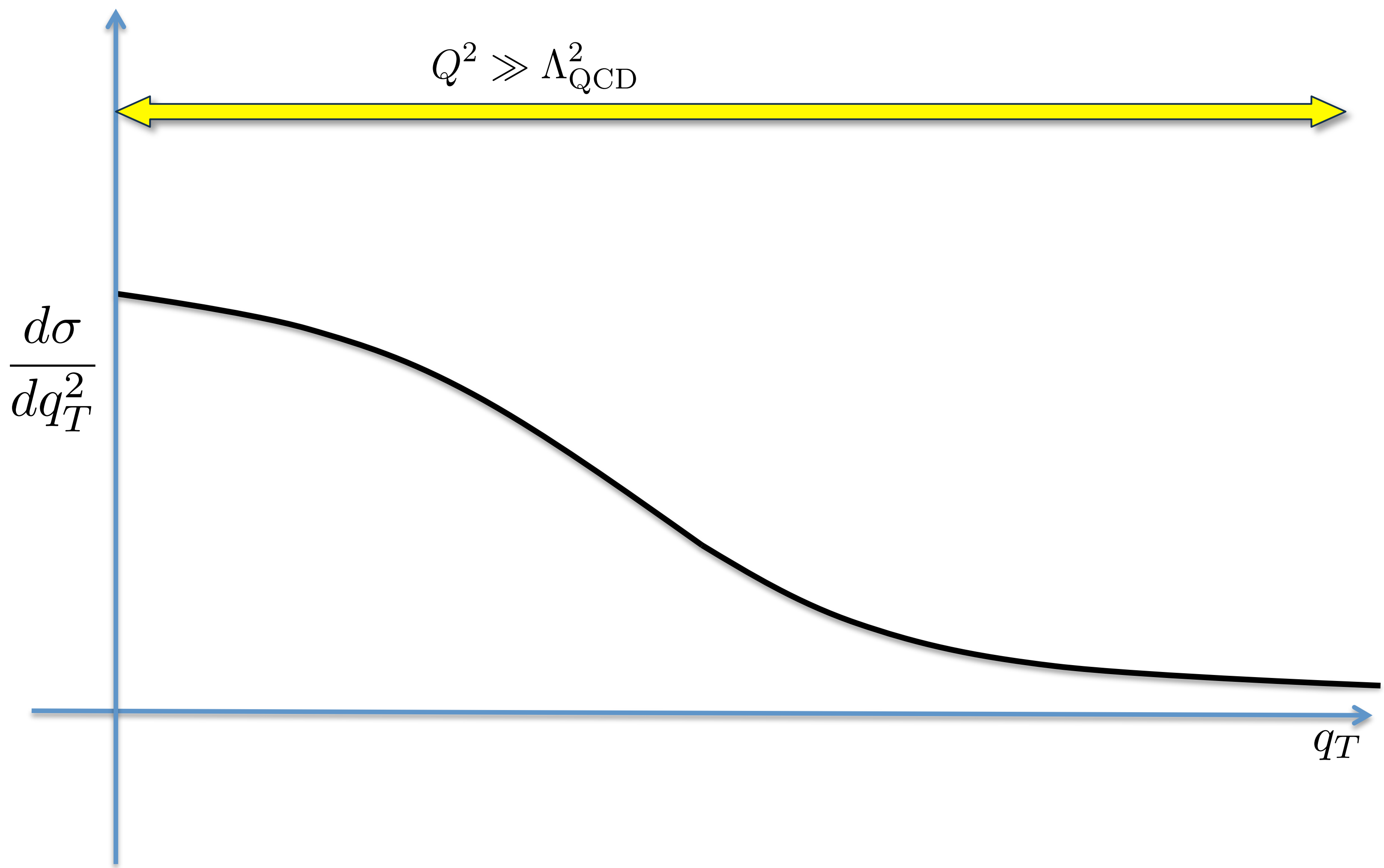


Transverse Momentum:

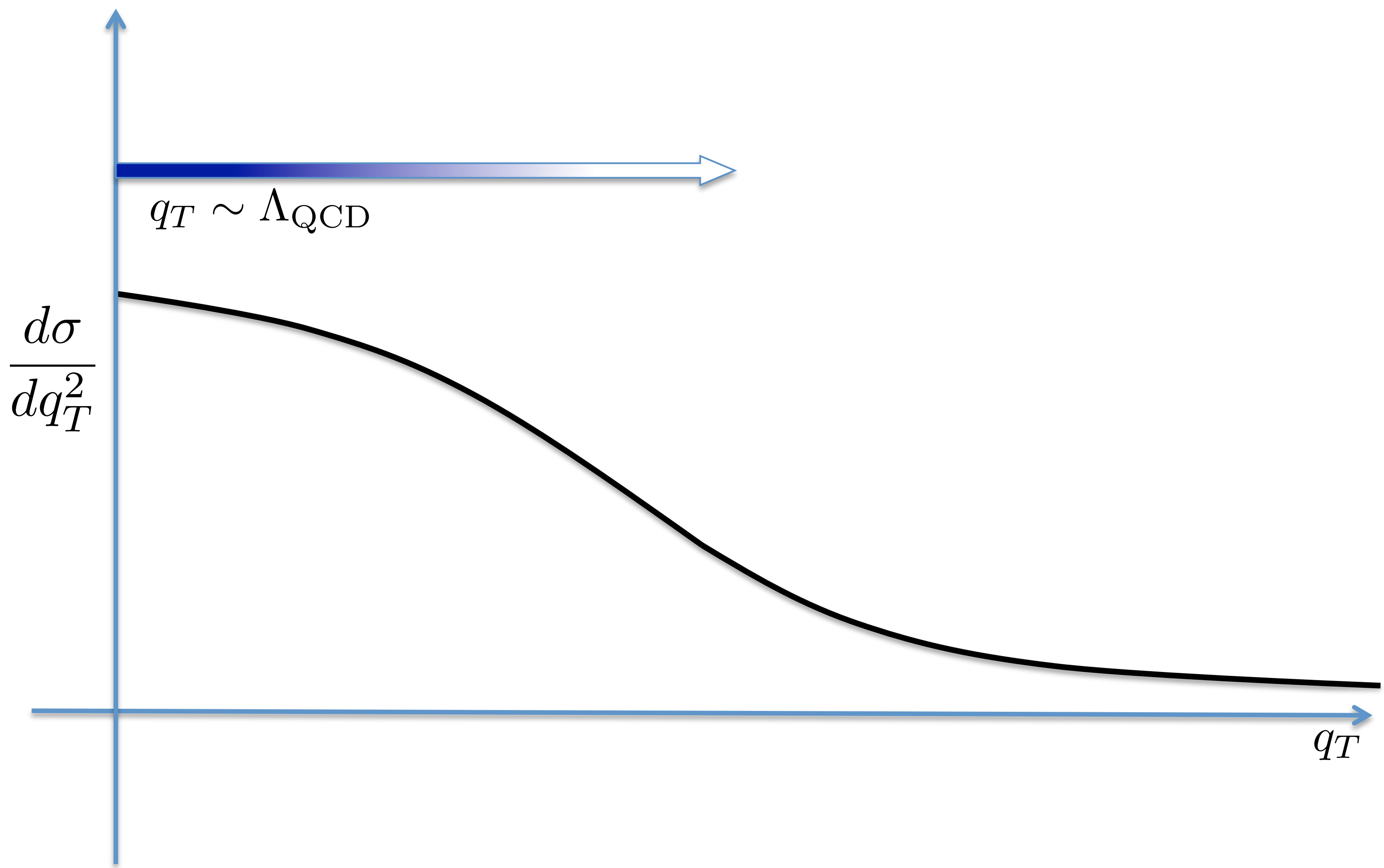
CDF experiment, Tevatron



Transverse Momentum:



Small Transverse Momentum:



TMD-Factorization

- Collinear factorization theorem relied on *collinear* approximations.

TMD-Factorization

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- Accounting for intrinsic transverse momentum requires *new factorization theorems*.

Recall Collinear Case:

- Parton Model

$$\sigma \sim \int \underbrace{\mathcal{H}(Q)} \otimes \underbrace{f_{q/P}(x_1)} \otimes \underbrace{f_{\bar{q}/\bar{P}}(x_2)}$$

Elementary collision
Short distance scales

Hadron Structure: large distance scales

- Perturbative QCD factorization theorem

$$\sigma \sim \int \underbrace{\mathcal{H}(Q, \mu/Q, \alpha_s(\mu))} \otimes f_{q/P}(x_1;\mu) \otimes f_{\bar{q}/\bar{P}}(x_2;\mu)$$

Small Coupling:
Perturbation Theory

Auxiliary parameter: Arbitrary
Defined in terms of QFT operators

TMD-Factorization

- TMD Parton Model

$$\sigma \sim \int \mathcal{H}(Q) \otimes F_{q/P}(x_1, \mathbf{k}_{1T}) \otimes F_{\bar{q}/\bar{P}}(x_2, \mathbf{q}_T - \mathbf{k}_{1T})$$

Parton Model

TMD-Factorization

- TMD Parton Model

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Parton Model

Elementary collision

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$\sim 1/Q$

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Number densities

"Transverse Momentum Dependent Parton Distribution Functions" (TMD PDFs)

TMD-Factorization

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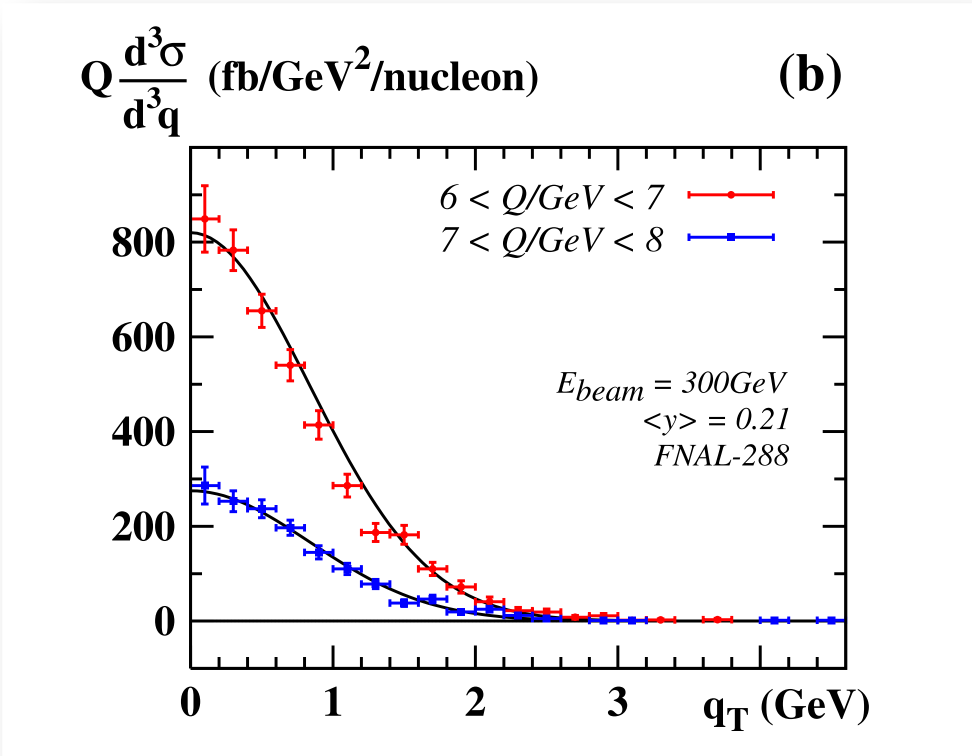
"Transverse Momentum Dependent Parton Distribution Functions" (TMD PDFs)

- Past Approaches:
  - Non-perturbative TMD parton model descriptions:
  - Transverse Momentum Resummation

Guassian Fits

(Schweitzer, Teckentrup, Metz (2010))



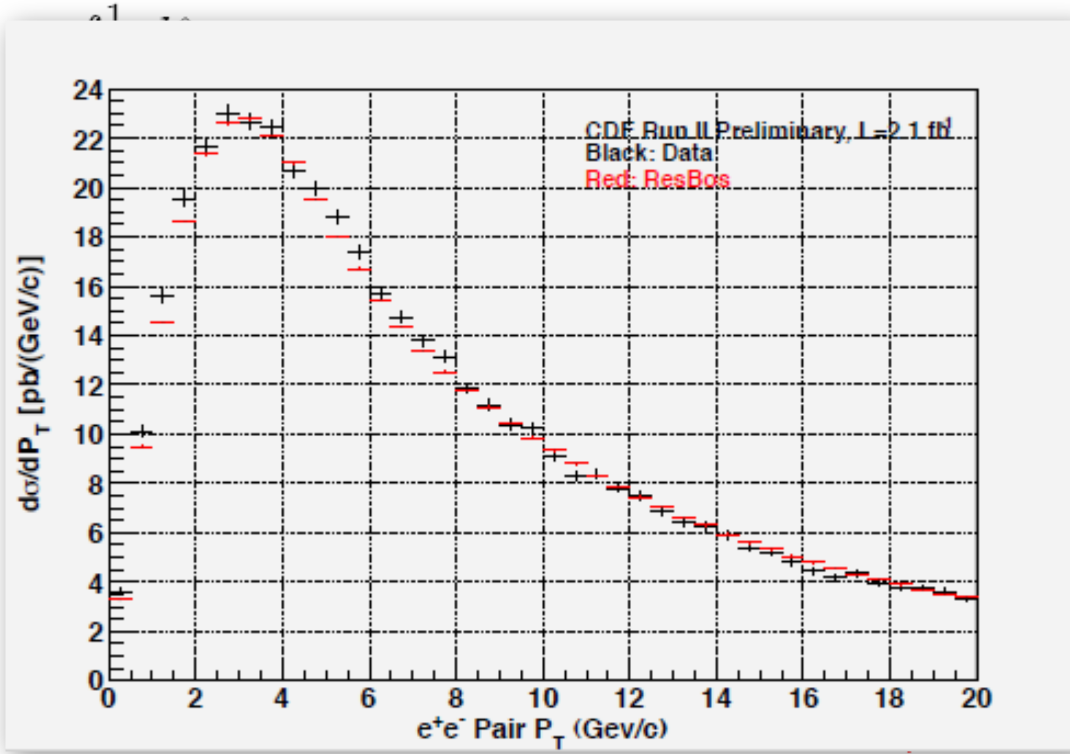
Resummation

$$d\sigma \sim \int d^2\mathbf{b}\, e^{-i\mathbf{b}\cdot\mathbf{q}_\perp} \ldots$$

CDF experiment, Tevatron ...re.)

$\ldots\mu_b)$

$\ldots\mu_b)$

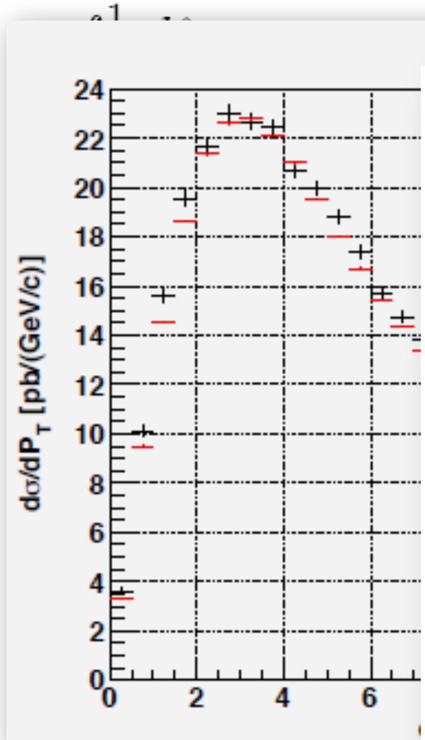


(Collins, Soper, Sterman (CSS) formalism (1982,1983))

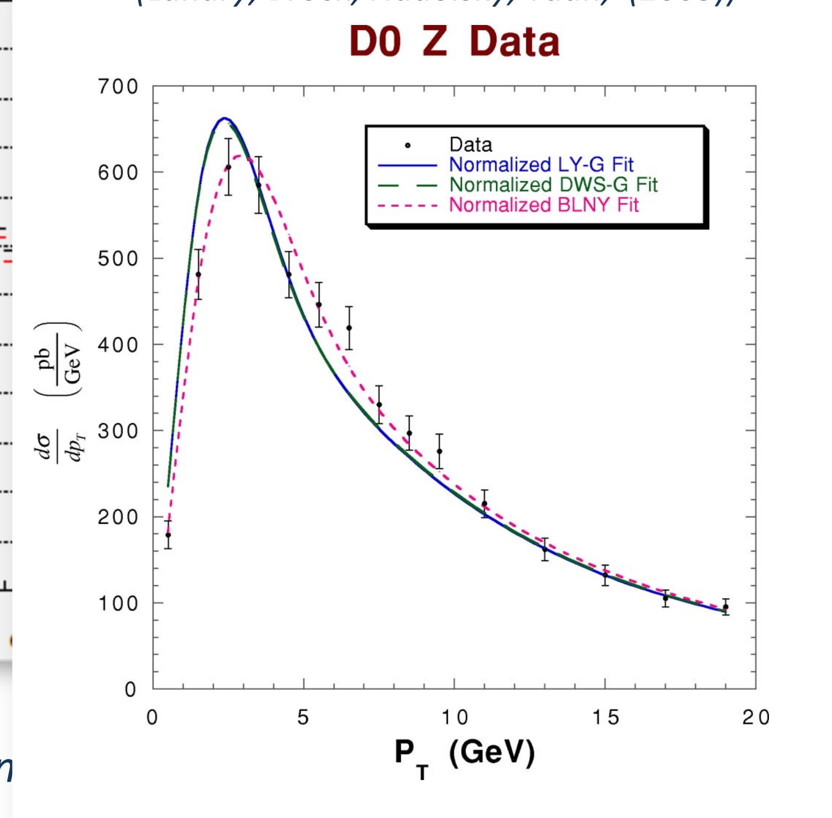
Resummation

$$d\sigma \sim \int d^2\mathbf{b}\, e^{-i\mathbf{b}\cdot\mathbf{q}}$$

CDF experiment, Tevatron ure.)



(Landry, Brock, Nadolsky, Yuan, (2003))



(Collins, Soper, Sterman (CSS) formalism

TMD-Factorization: QCD

- Unified Formalism

$$\sigma \sim \int \mathcal{H}(Q, \mu/Q, \alpha_s(\mu)) \otimes F_{q/P}(x_1, \mathbf{k}_{1T}, \mu, \zeta_1) \otimes F_{\bar{q}/\bar{P}}(x_2, \mathbf{q}_T - \mathbf{k}_{1T}, \mu, \zeta_2)$$

Pert. QCD

Small Coupling: Perturbation Theory

$$C_0 + C_1\alpha_s(\mu) + C_2\alpha_s(\mu)^2 + C_3\alpha_s(\mu)^3 + \cdots$$

+ Y-term

2 Auxiliary parameters: Arbitrary

$$\zeta_1\zeta_2 \sim Q^4$$

(Collins, Soper, Sterman (CSS) formalism (1982,1983))

(Collins Extension: (2011), Chapts. 10,13,14)

TMD-Evolution

- Recall Collinear / DGLAP:

$$\frac{d}{d\ln\mu} f_{j/P}(x;\mu) = 2\int P_{jj'}(x') \otimes f_{j'/P}(x/x';\mu)$$

TMD-Evolution

- Recall Collinear / DGLAP:

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- TMD Case:

$$\frac{\partial \ln \tilde{F}(x,b_T;\mu,\zeta)}{\partial \ln\sqrt{\zeta}} = \tilde{K}(b_T;\mu)$$

$$\frac{d\tilde{K}(b_T;\mu)}{d\ln\mu} = -\gamma_K(g(\mu))$$

$$\frac{d\ln\tilde{F}(x,b_T;\mu,\zeta)}{d\ln\mu} = \gamma_F(g(\mu);\zeta/\mu^2)$$

((CSS) formalism (1982,1983))

(Collins Extension: (2011), Chapts. 10,13,14)

TMD PDF Definitions

- Defined in terms of elementary field operators.

(Collins, POS (2003)), (Worked out in Collins, Book (2011))

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  - Sivers Function: Non-zero, reverses sign in Drell-Yan vs. SIDIS
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    (Brodsky, Hwang, Schmidt (2002)), (Collins, (2002))

- Constrained by factorization derivation.

- Factorization breaking.

  (Collins, Qiu (2007)), (TCR, Mulders (2010)), (TCR, (2013))

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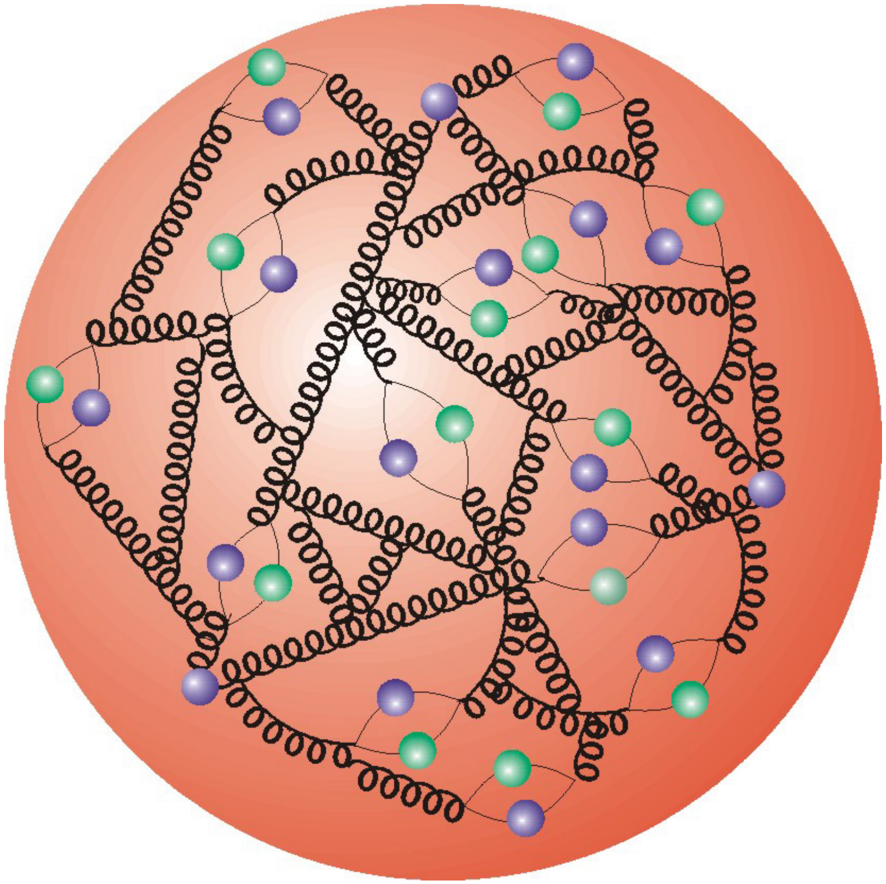
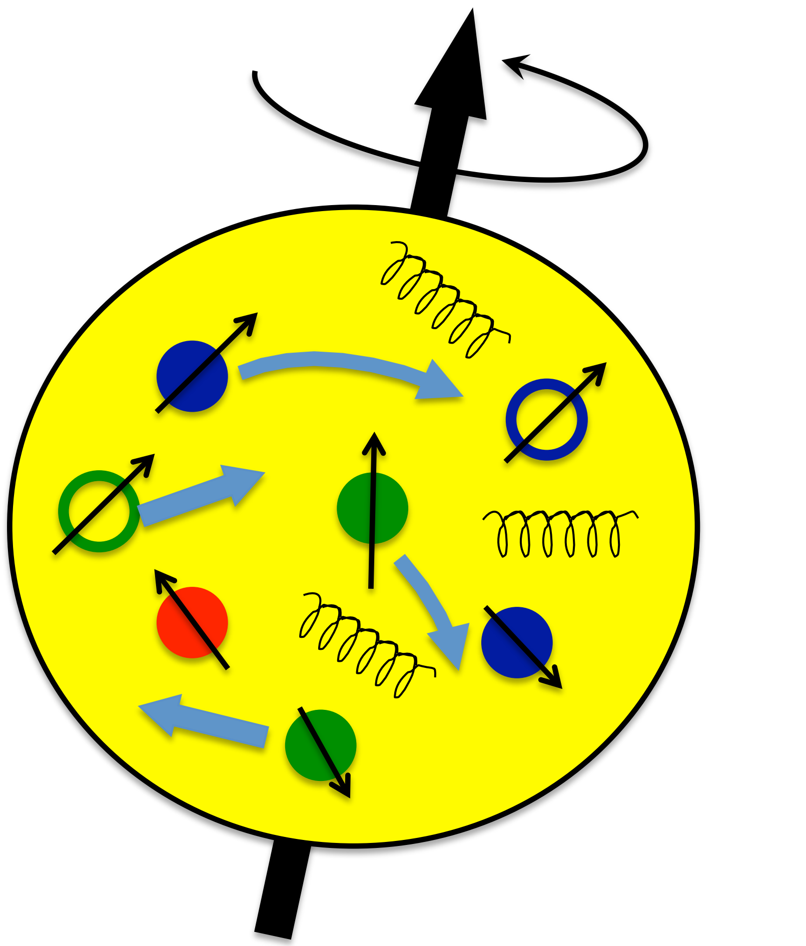
TMD vs. Collinear

- TMDs: Rich source of information about hadron structure.
  - *TMD Zoo*

TMD vs. Collinear



TMD vs. Collinear



TMD Taxonomy

Also Fragmentation Functions

Parton Model

Quark \ Proton	Unpolarized	Longitudinally polarized	Transversely polarized
Unpolarized	$f_1(x, k_T)$	✗	$f_{1T}^{\perp}(x, k_T)$
Longitudinally polarized	✗	$g_{1L}(x, k_T)$	$g_{1T}(x, k_T)$
Transversely polarized	$h_1^{\perp}(x, k_T)$	$h_{1L}(x, k_T)$	$h_{1T}(x, k_T)$ $h_{1T}^{\perp}(x, k_T)$

TMD Taxonomy

Also Fragmentation Functions

Parton Model

Sivers

Quark \ Proton	Unpolarized	Longitudinally polarized	Transversely polarized
Unpolarized	$f_1(x, k_T)$	✗	$f_{1T}^{\perp}(x, k_T)$
Longitudinally polarized	✗	$g_{1L}(x, k_T)$	$g_{1T}(x, k_T)$
Transversely polarized	$h_1^{\perp}(x, k_T)$	$h_{1L}(x, k_T)$	$h_{1T}(x, k_T)$ $h_{1T}^{\perp}(x, k_T)$

Boer-Mulders

"Worm Gear"

"Pretzelocity"

TMD Taxonomy

Also Fragmentation Functions

Parton Model

Sivers

Zero in parton model by TP

Quark \ Proton	Unpolarized	Longitudinally polarized	Transversely polarized
Unpolarized	$f_1(x, k_T)$	✗	$f_{1T}^{\perp}(x, k_T)$
Longitudinally polarized	✗	$g_{1L}(x, k_T)$	$g_{1T}(x, k_T)$
Transversely polarized	$h_1^{\perp}(x, k_T)$	$h_{1L}(x, k_T)$	$h_{1T}(x, k_T)$ $h_{1T}^{\perp}(x, k_T)$

Boer-Mulders

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$$\sigma \sim \int \mathcal{H}(Q) \otimes F_{q/P}(x_1, \mathbf{k}_{1T}, S_1) \otimes F_{\bar{q}/\bar{P}}(x_2, \mathbf{q}_T - \mathbf{k}_{1T}, S_2)$$

$$\sigma \sim \int \mathcal{H}(Q) \otimes F_{q/P}(x_1, \mathbf{k}_{1T}, S_1) \otimes D_{H/q}(z, \mathbf{q}_T + \mathbf{k}_{1T})$$

Quark \ Proton	Unpolarized	Longitudinally polarized	Transversely polarized
Unpolarized	$f_1(x, k_T)$	✗	$- f_{1T}^{\perp}(x, k_T)$ $+ f_{1T}^{\perp}(x, k_T)$
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Non-Zero!

TMD vs. Collinear

- TMDs: Rich source of information about hadron structure.
  - *TMD Zoo*

- More complicated fitting:

$$f_{f/P}(x) \longrightarrow F_{f/P}(x, \mathbf{k}_T)$$

Need non-perturbative descriptions

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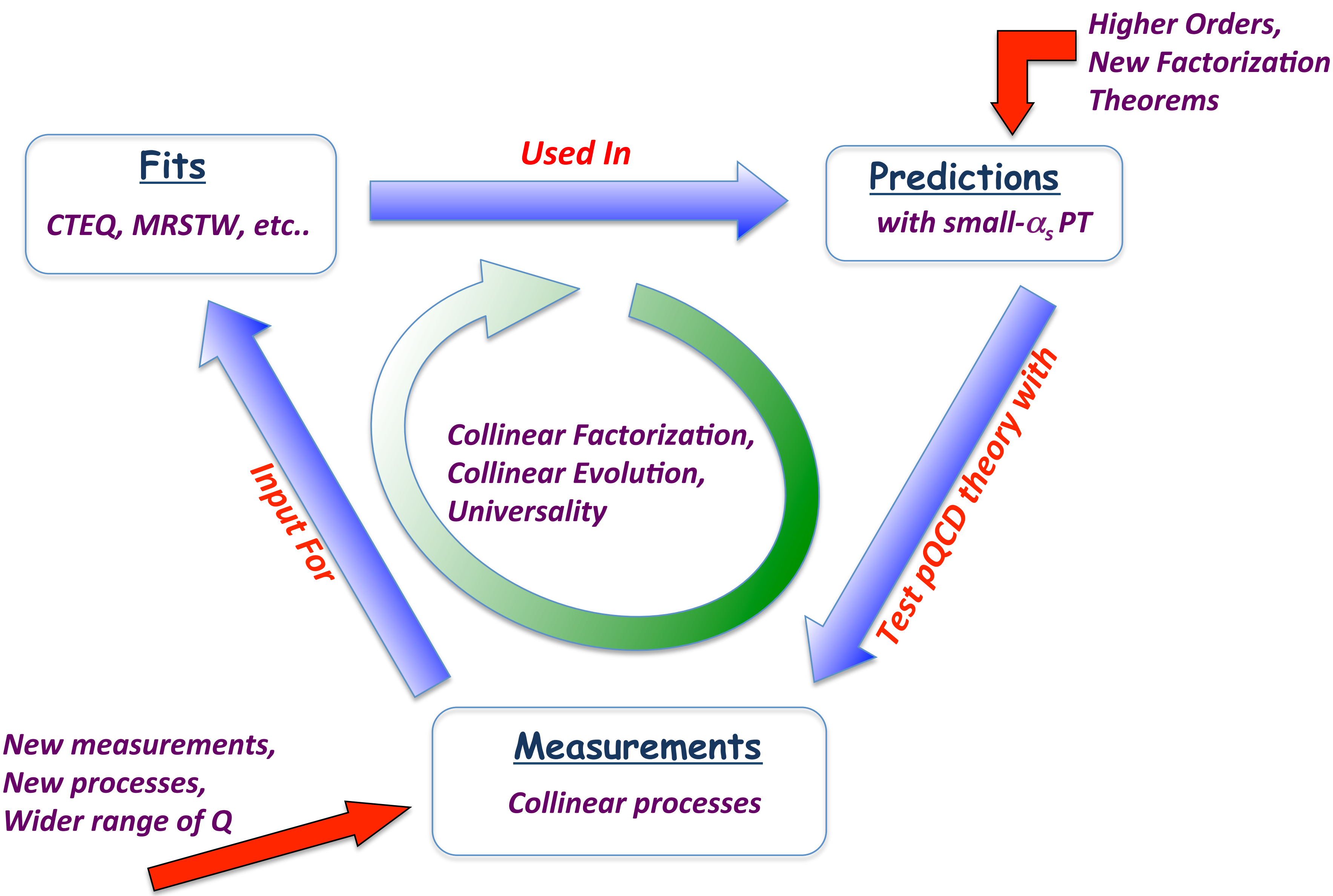
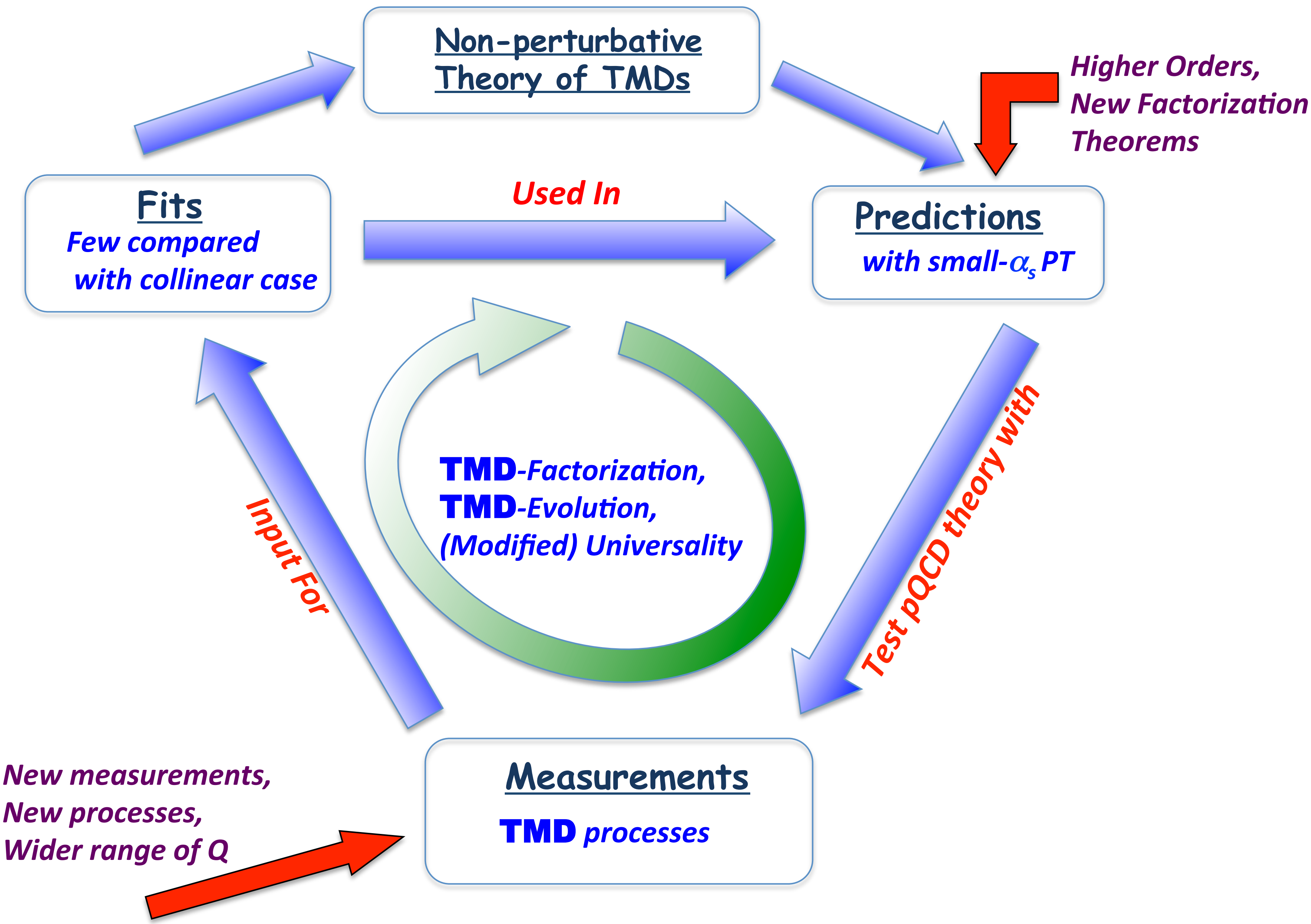
Need non-perturbative descriptions

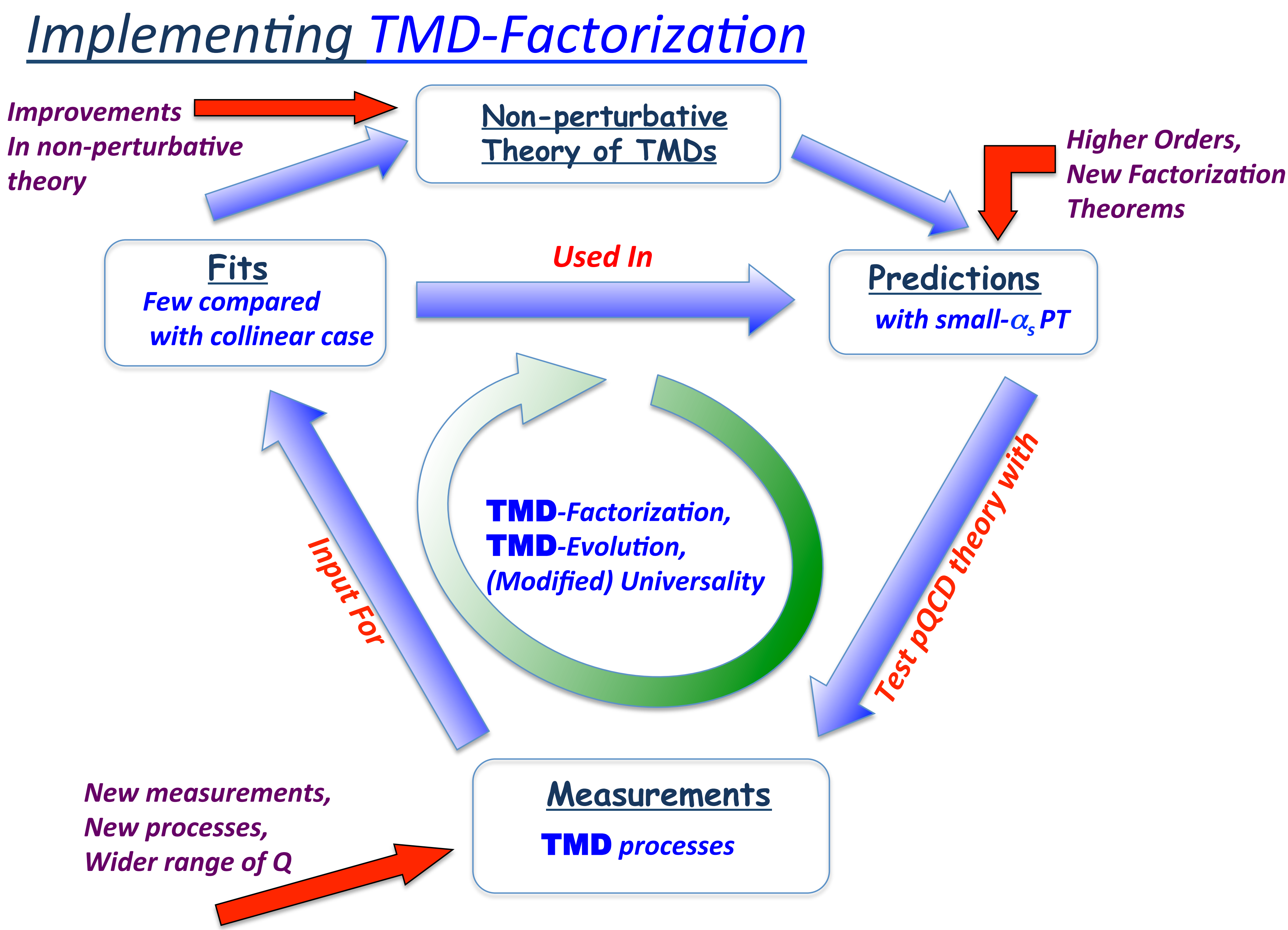
- Cases of non-universality / TMD-factorization breaking.

Implementing Collinear Factorization

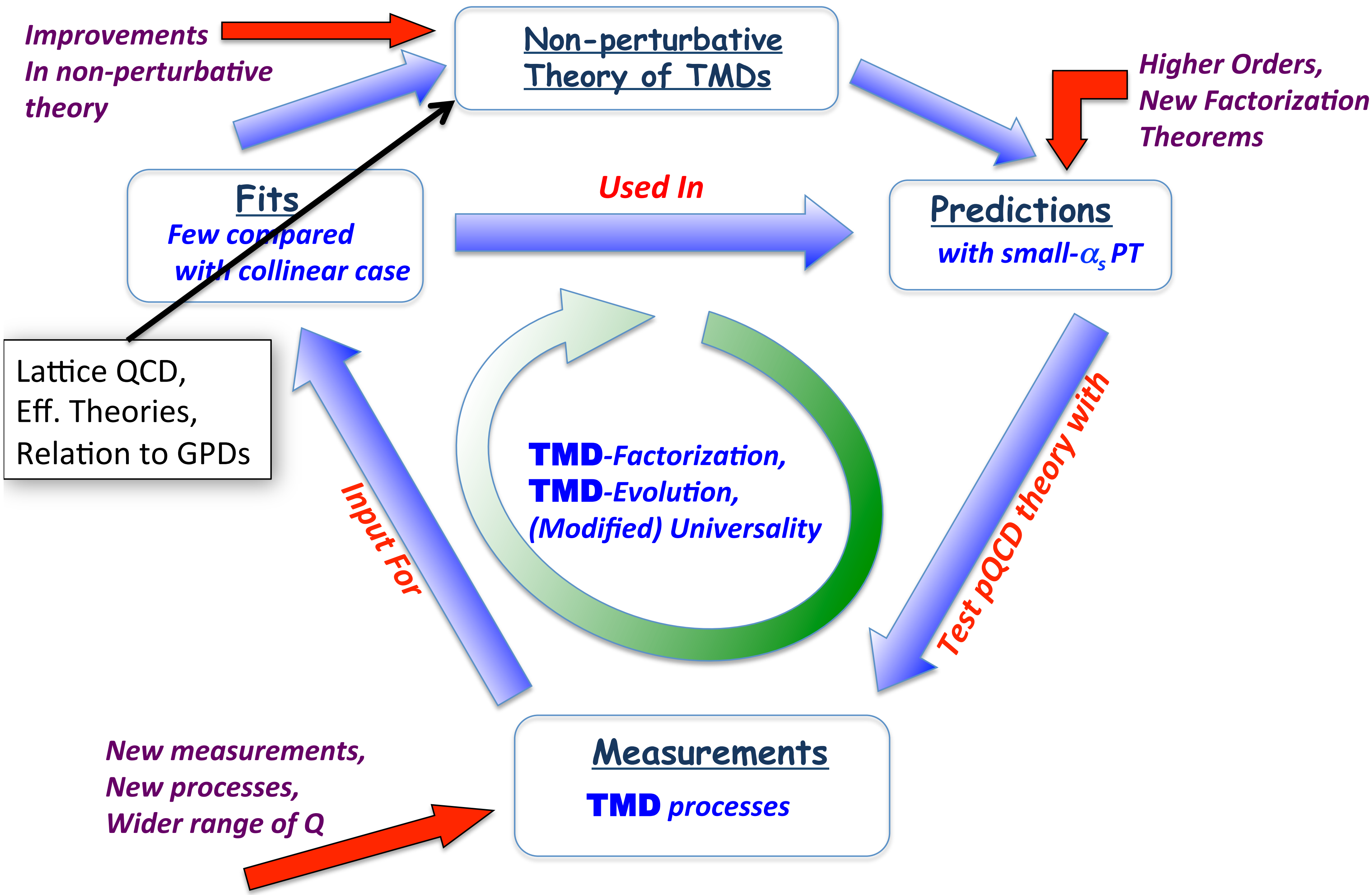


Implementing TMD-Factorization




Implementing TMD-Factorization
Improvements
In non-perturbative
theory
Non-perturbative
Theory of TMDs
Higher Orders,
New Factorization
Theorems
Fits
Few compared
with collinear case
Used In
Predictions
with small-α_s PT
TMD-Factorization,
TMD-Evolution,
(Modified) Universality
Input For
Test pQCD theory with
New measurements,
New processes,
Wider range of Q
Measurements
TMD processes

Implementing TMD-Factorization



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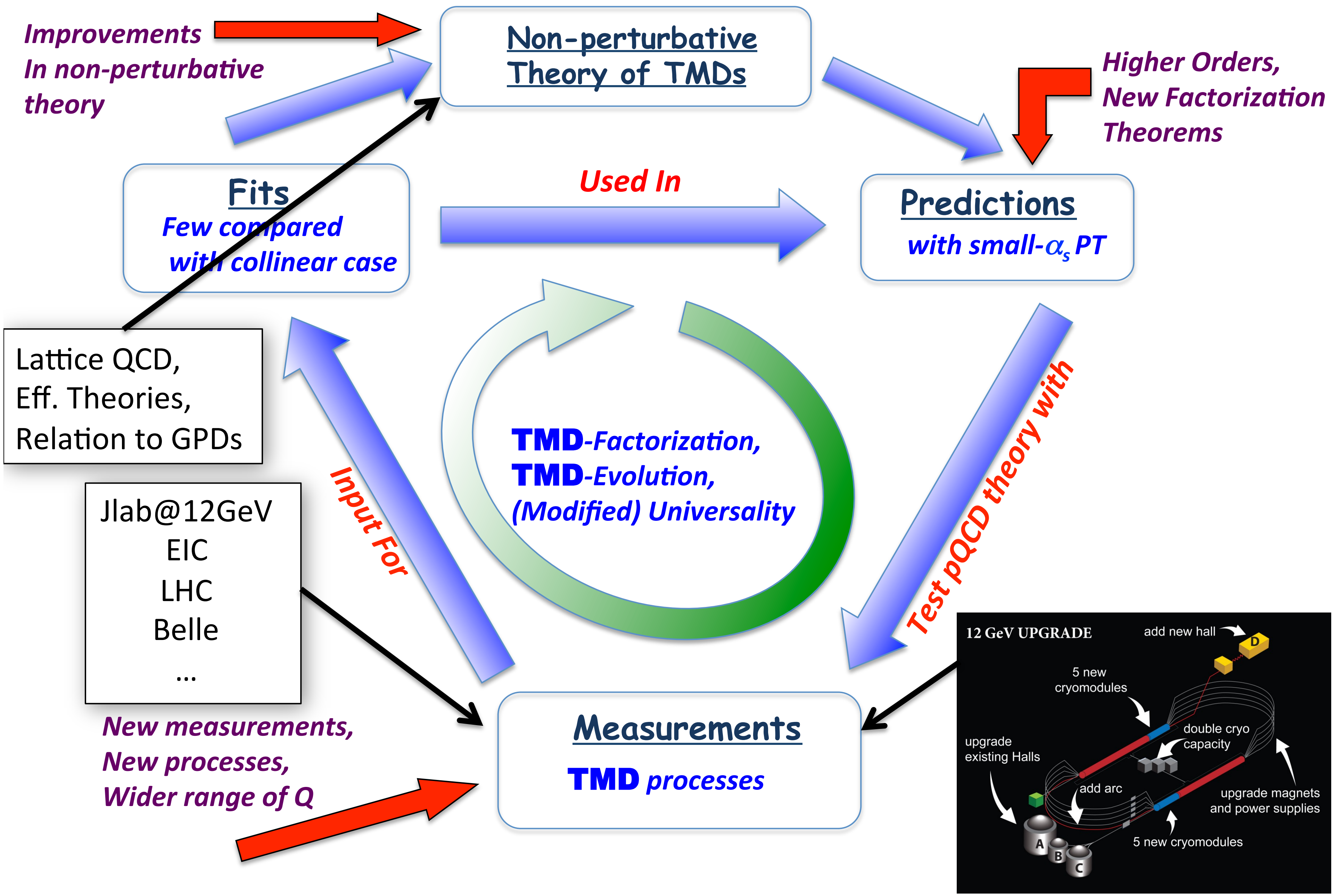

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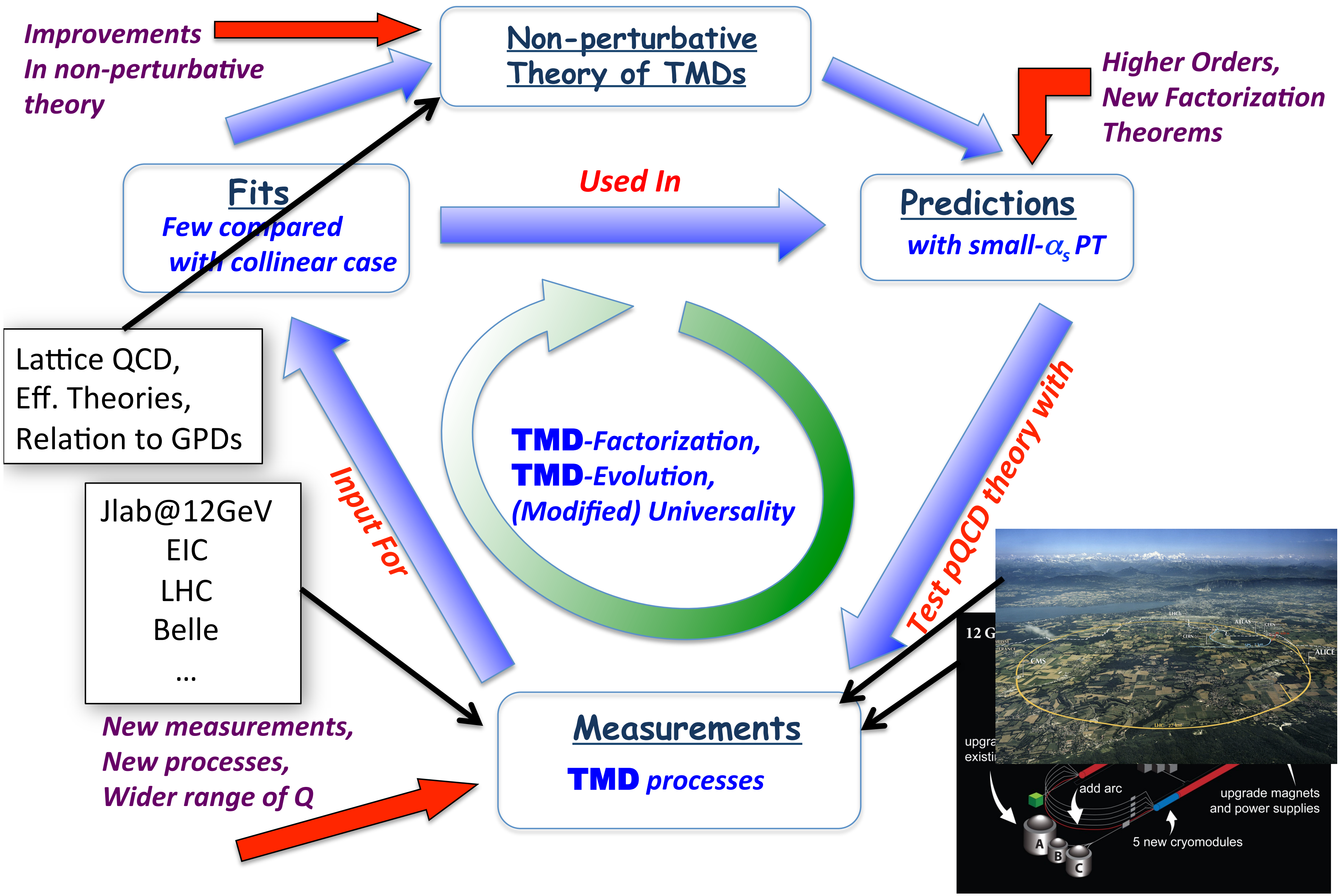

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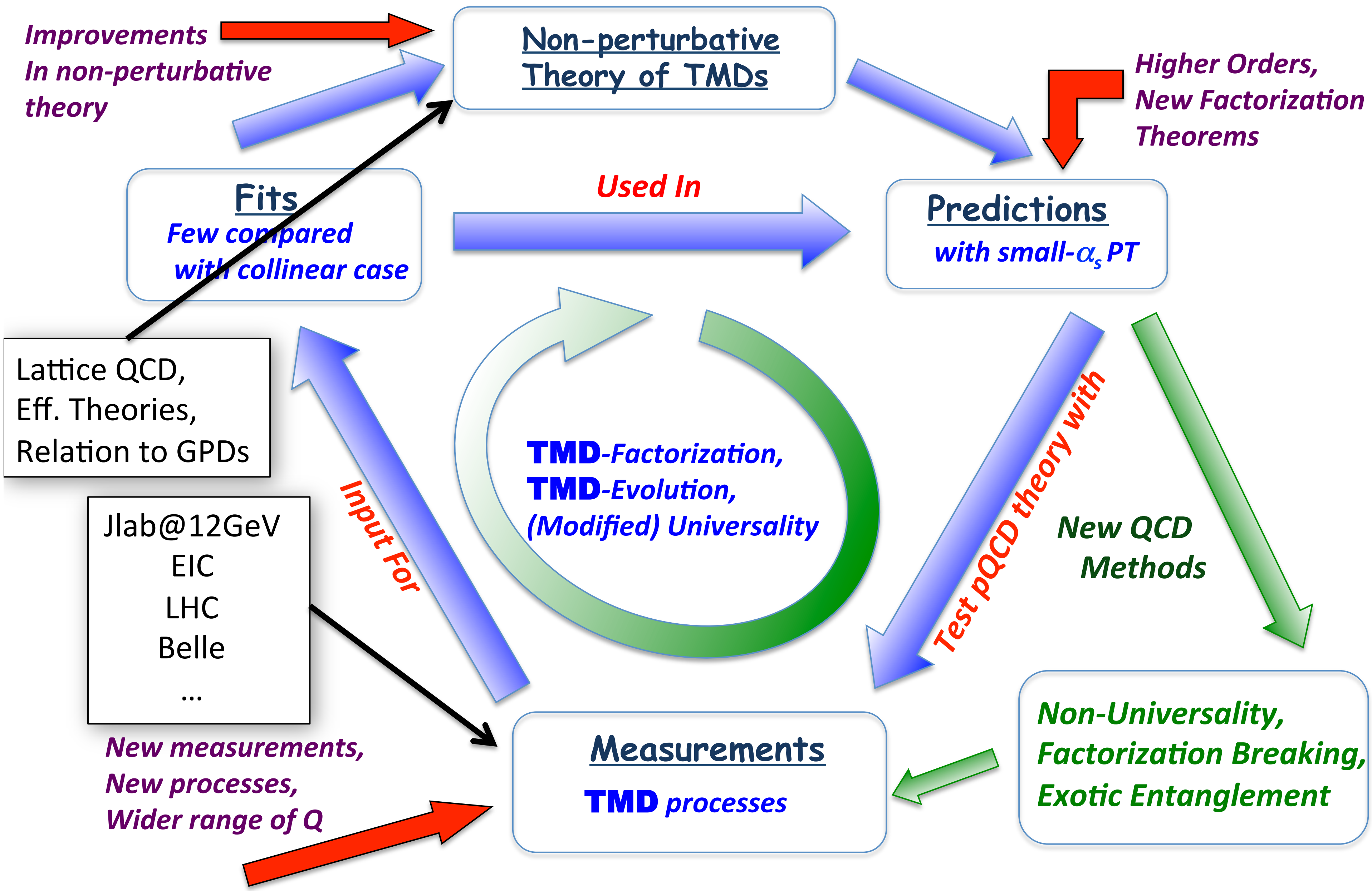
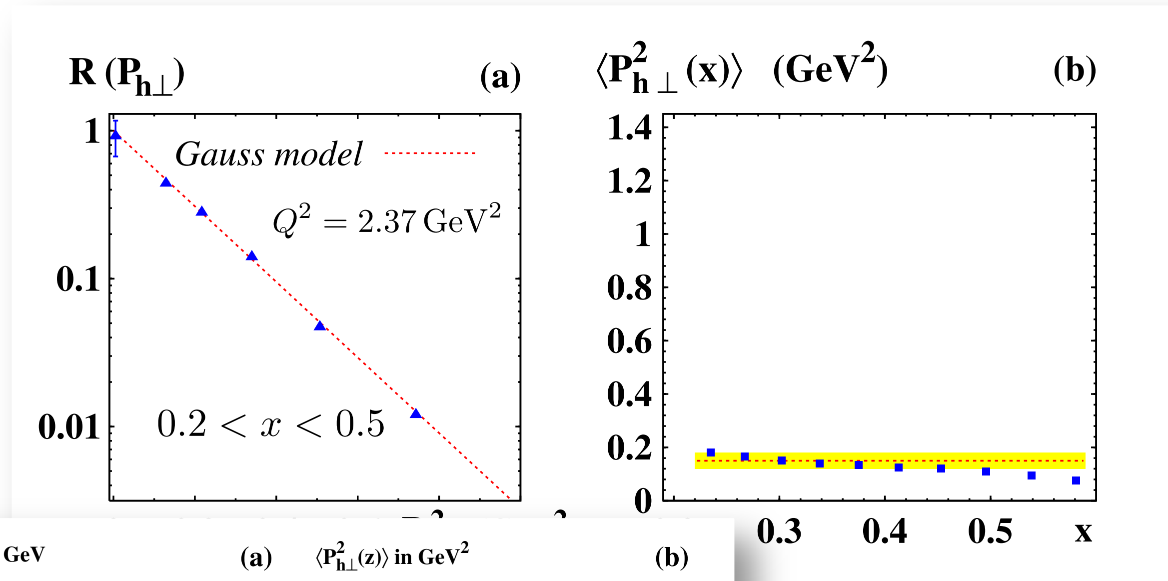
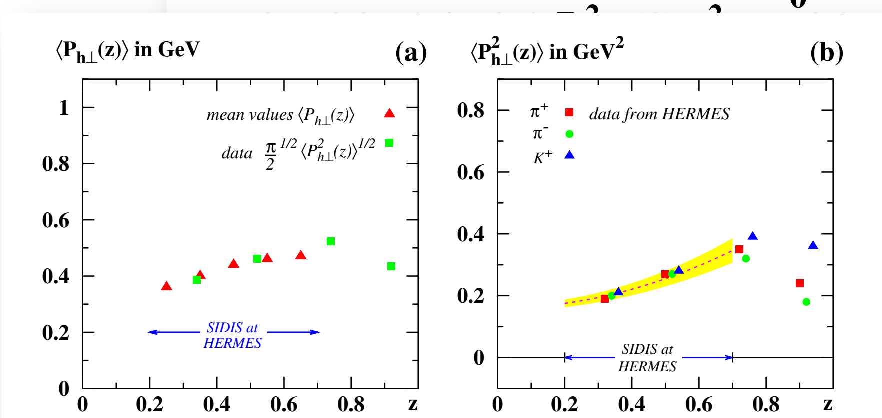


Extractions of TMD PDFs

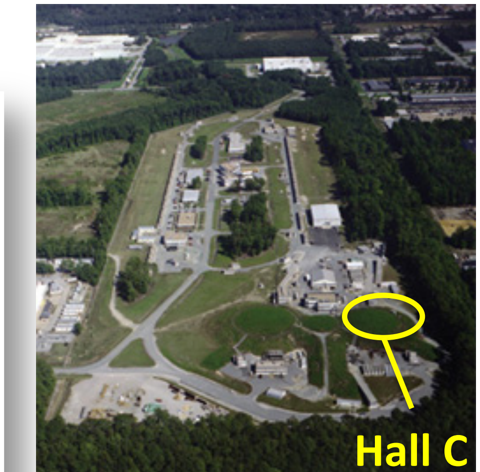
- Fixed Scale Fits.

(Schweitzer, Teckentrup, Metz (2010))





JLab



Good evidence for
Gaussian shape at small Q, P_T

$$\langle x \rangle = 0.09 \quad Q^2 = 2.4\,\mathrm{GeV}^2$$

HERMES $\langle p_T^2 \rangle = (0.38 \pm 0.06)\ \mathrm{GeV}^2$

$$\langle K_T^2 \rangle = (0.16 \pm 0.01)\ \mathrm{GeV}^2$$

Extractions of TMD PDFs

- ResBos: CSS formalism

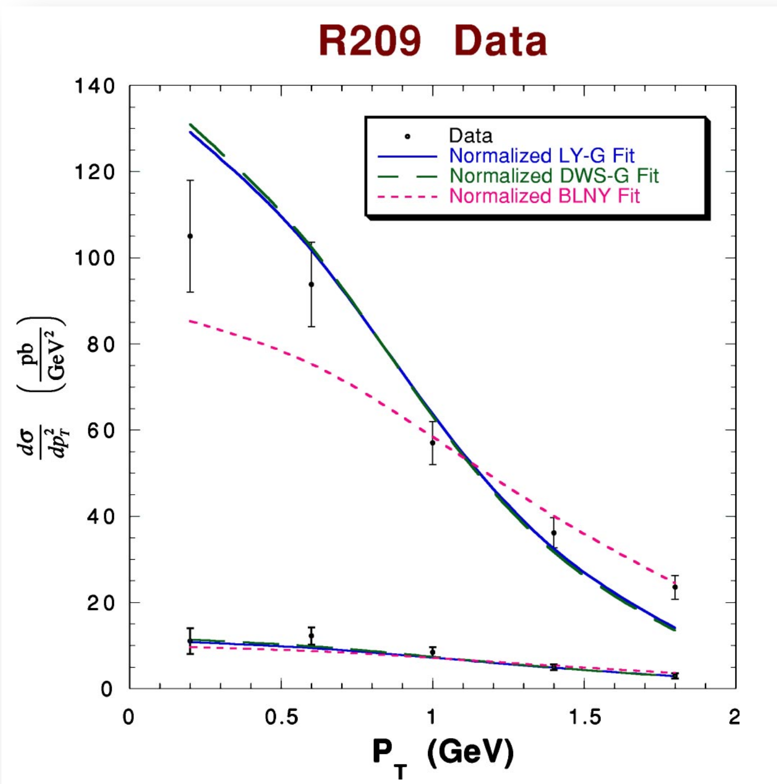
http://hep.pa.msu.edu/resum/

(Landry, Brock, Nadolsky, Yuan, (2003))

$$g_K(b_T)\ln\left(\frac{Q}{Q_0}\right) = -g_2\frac{1}{2}b_T^2\ln\left(\frac{Q}{Q_0}\right)$$

Gaussian ansatz

$$g_2 = .68\ \mathrm{GeV}^2 \qquad b_{\max} = .5\ \mathrm{GeV}^{-1}$$



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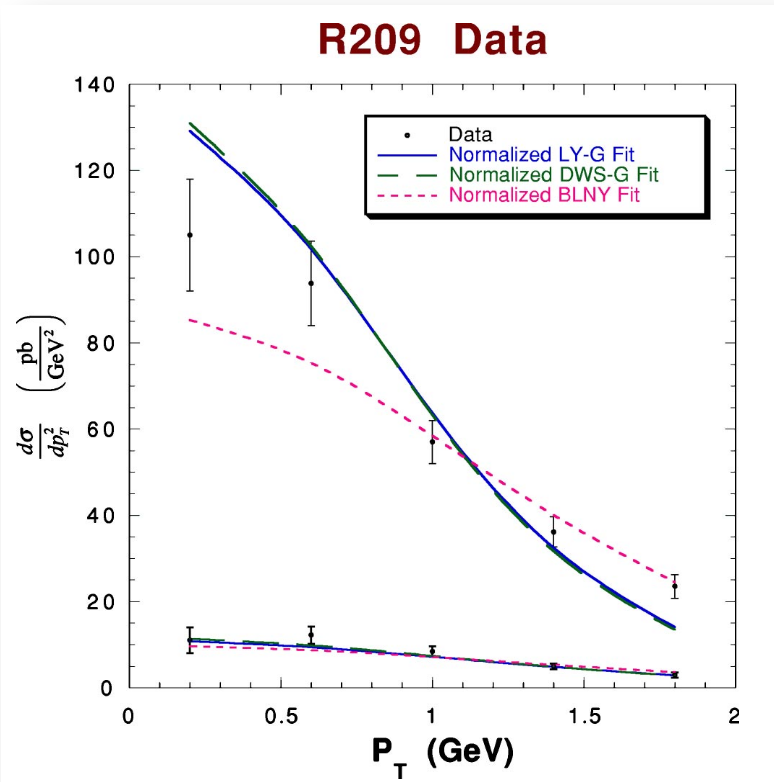

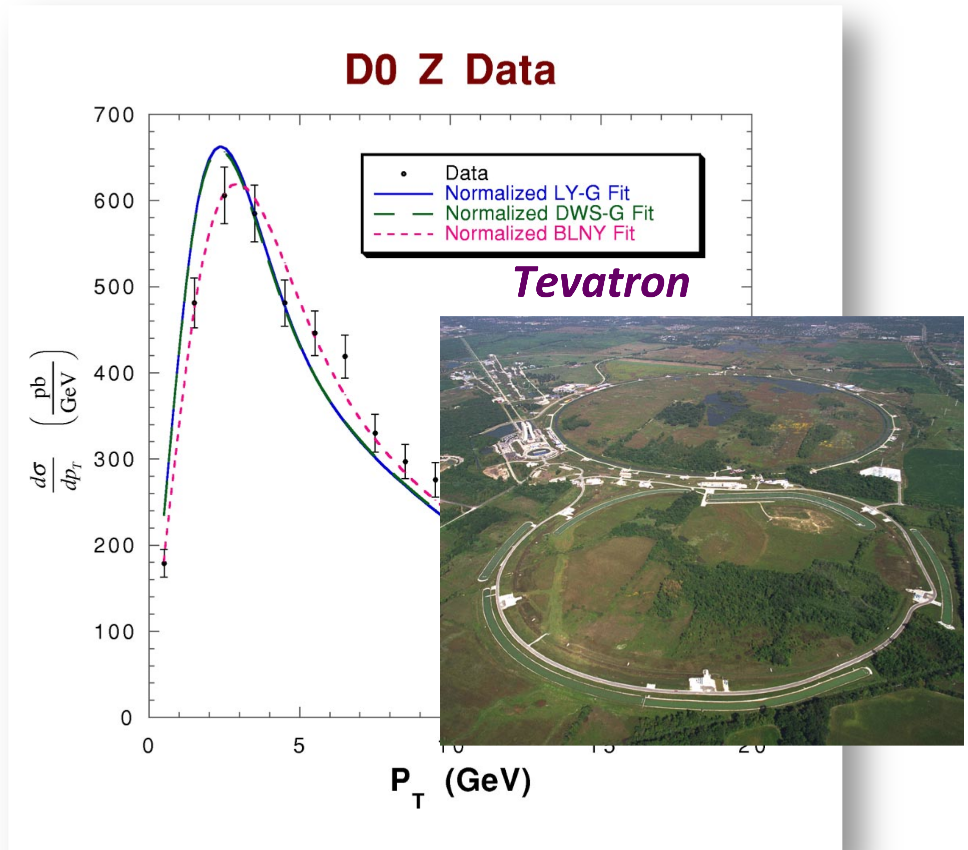

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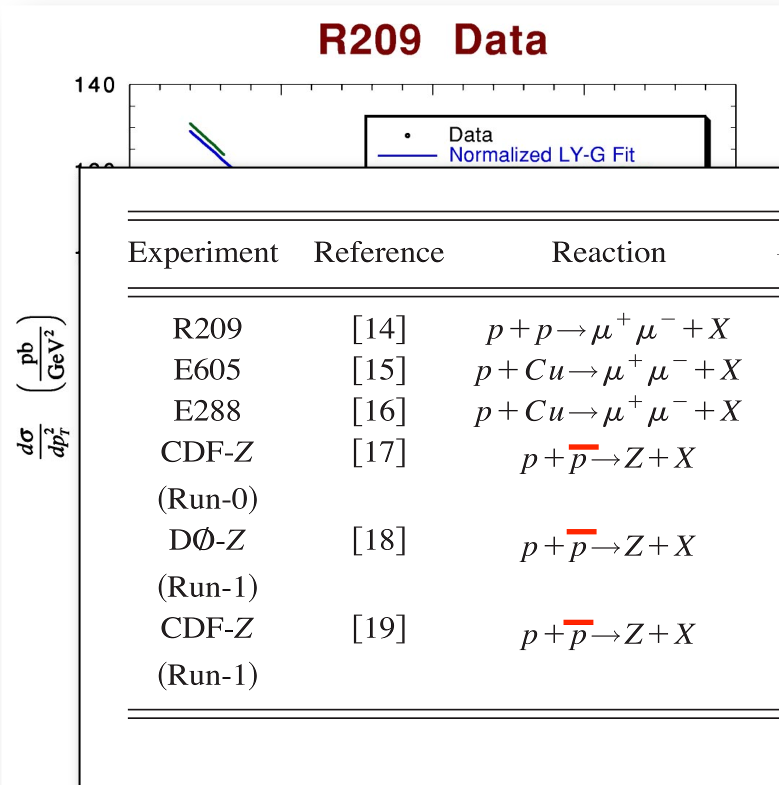
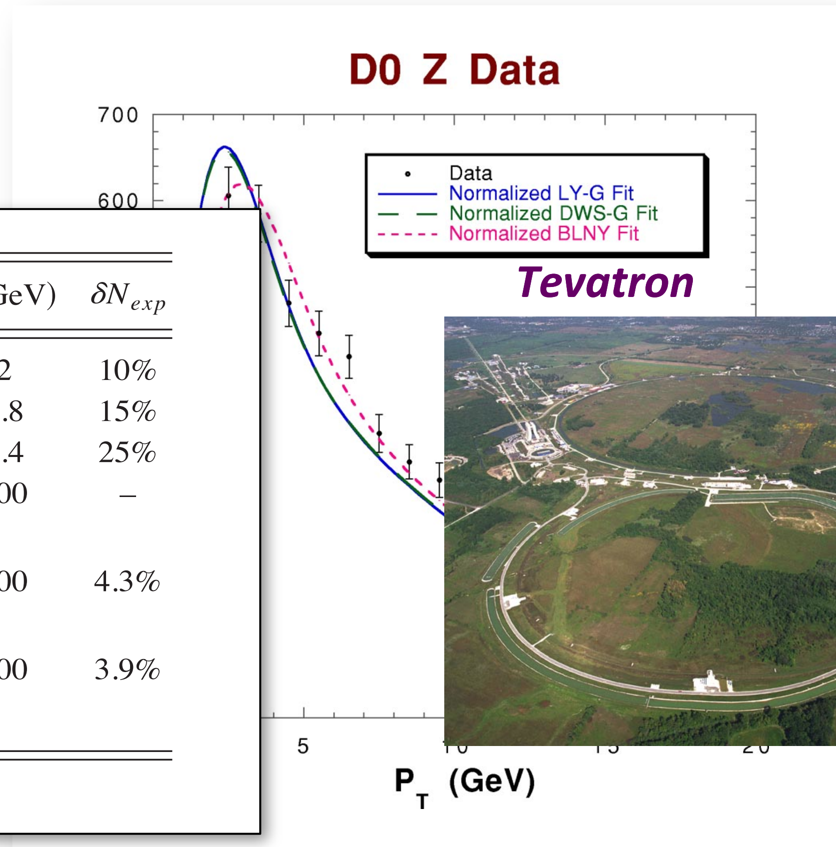


Experiment	Reference	Reaction	$\sqrt{S}$(GeV)	δN_{exp}
R209	[14]	$p+p\to\mu^+\mu^-+X$	62	10%
E605	[15]	$p+Cu\to\mu^+\mu^-+X$	38.8	15%
E288	[16]	$p+Cu\to\mu^+\mu^-+X$	27.4	25%
CDF-Z (Run-0)	[17]	$p+\bar{p}\to Z+X$	1800	–
DØ-Z (Run-1)	[18]	$p+\bar{p}\to Z+X$	1800	4.3%
CDF-Z (Run-1)	[19]	$p+\bar{p}\to Z+X$	1800	3.9%



Look at one TMD PDF

One physical scale for evolution, dictated by requirements of PT:

$$\mu \sim \sqrt{\zeta_1} \sim \sqrt{\zeta_2} \sim Q$$
$$\zeta_1 \zeta_2 \sim Q^4$$

Ex: Matching Prescription:

$$\mathbf{b}_*(\mathbf{b}_\mathrm{T}) \equiv \frac{\mathbf{b}_\mathrm{T}}{\sqrt{1 + b_T^2/b_{\max}^2}}$$

$$\mu_b \equiv C_1/|\mathbf{b}_*(b_T)|$$

$$\tilde{F}_{f/P}(x, \mathbf{b}_\mathrm{T}; Q, Q^2) =$$

Collinear PDFs

$$\mathrm{A}\left\{ \quad \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P}(\hat{x}, \mu_b) \times \right.$$

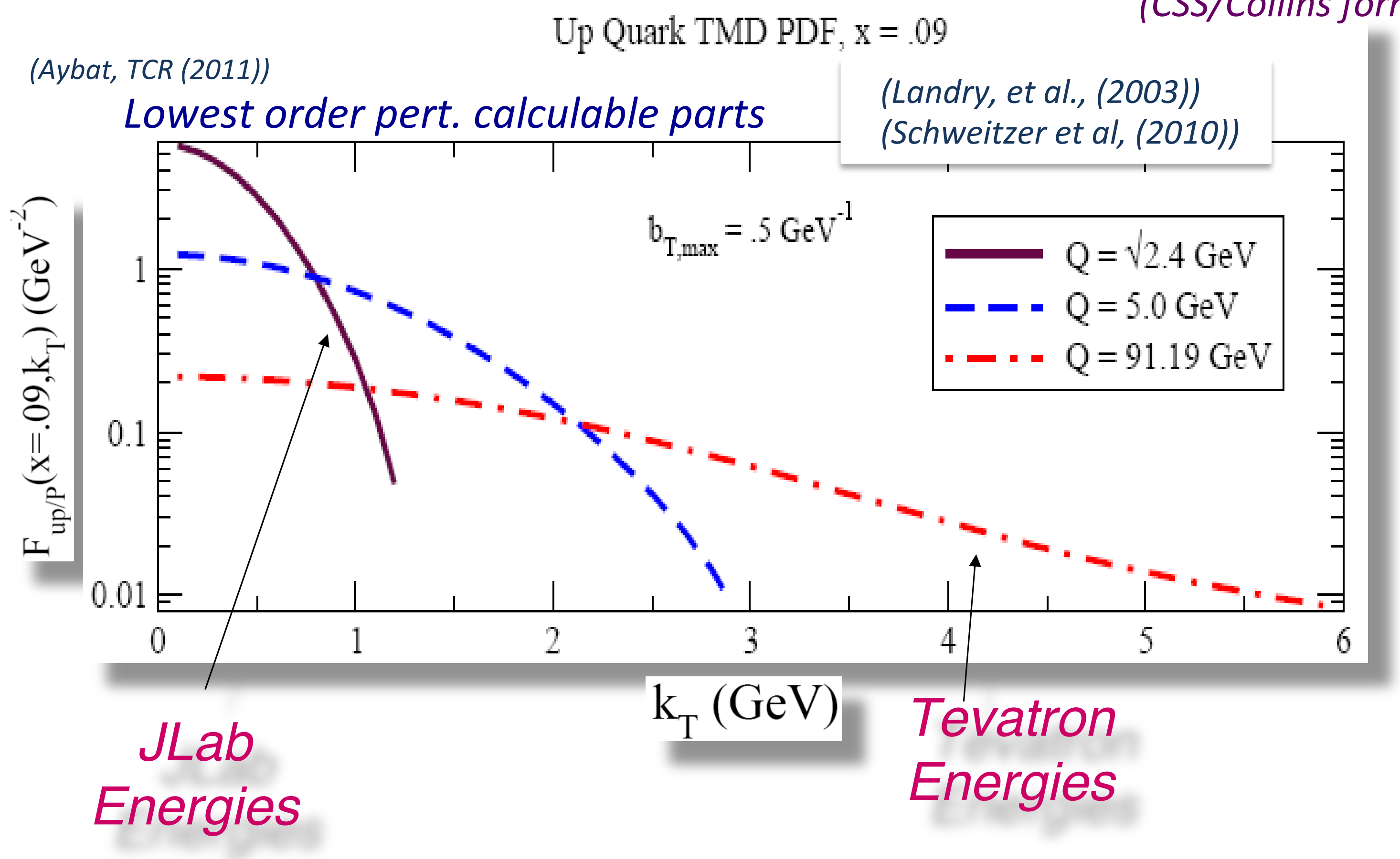
$$\mathrm{B}\left\{ \quad \times \exp\left\{ \ln \frac{Q}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^Q \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{Q}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times \right.$$

$$\mathrm{C}\left\{ \quad \times \exp\left\{ \underline{g_{f/P}(x, b_T)} + \underline{g_K(b_T)} \ln \frac{Q}{Q_0} \right\} \right.$$

Nonperturbative large b_T behavior

Evolved TMD PDFs: constructed from old fits

(CSS/Collins formalism.)



https://projects.hepforge.org/tmd/

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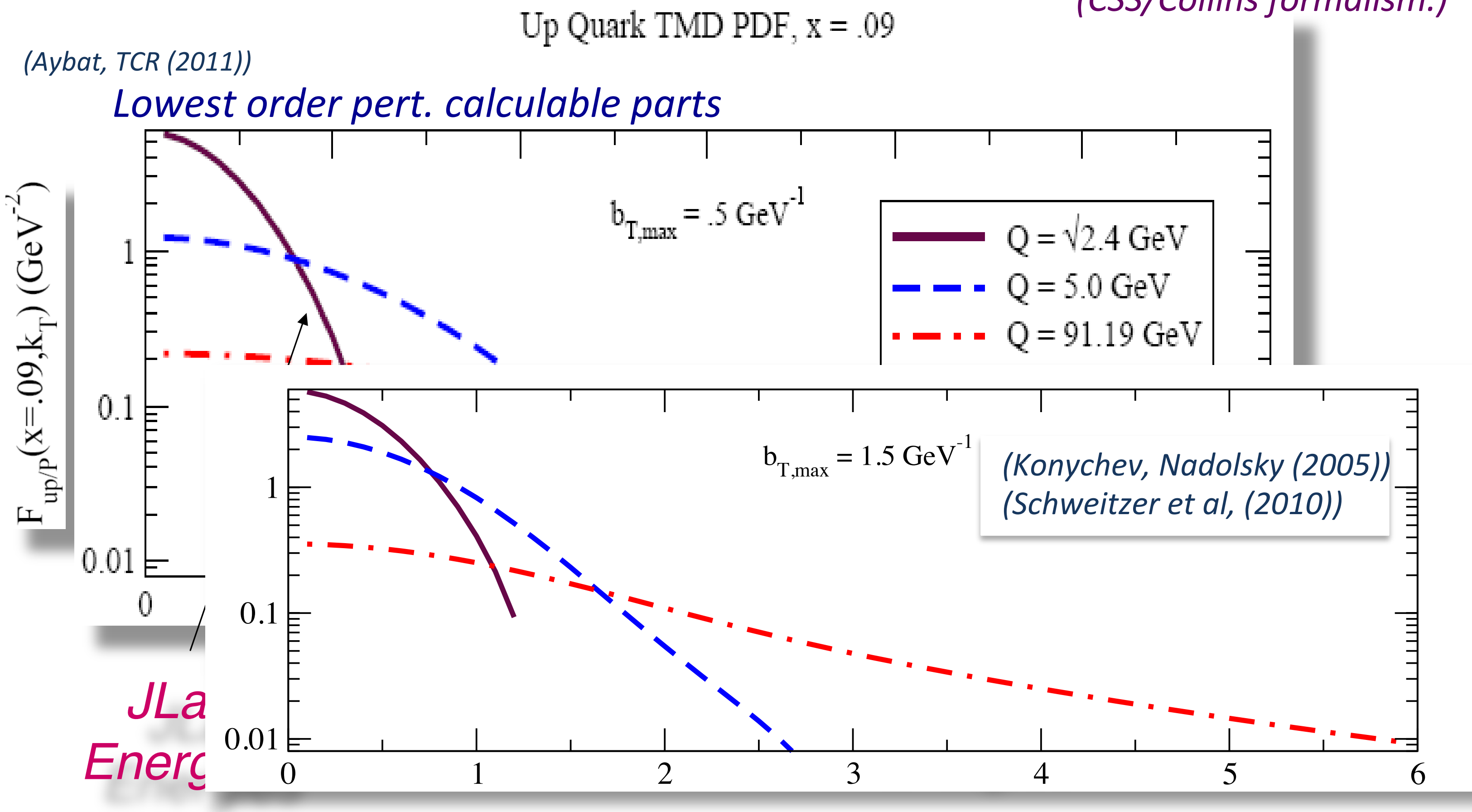

https://projects.hepforge.org/tmd/

Polarized TMD PDFs:

- Same definition, same evolution equations

$$F_{f/P^{\uparrow}}(x, k_T, S; \mu, \zeta_F)$$
$$= F_{f/P}(x, k_T; \mu, \zeta_F) - F_{1T}^{\perp f}(x, k_T; \mu, \zeta_F) \frac{\epsilon_{ij} k_T^i S^j}{M_p}$$

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- Derivative evolves in b_T-space

$$\tilde{F}_{1T}^{\prime \perp f}(x, b_T; \mu, \zeta_F)$$
$$= -2\pi \int_0^{\infty} dk_T k_T^2 J_1(k_T b_T) F_{1T}^{\perp f}(x, k_T; \mu, \zeta_F).$$

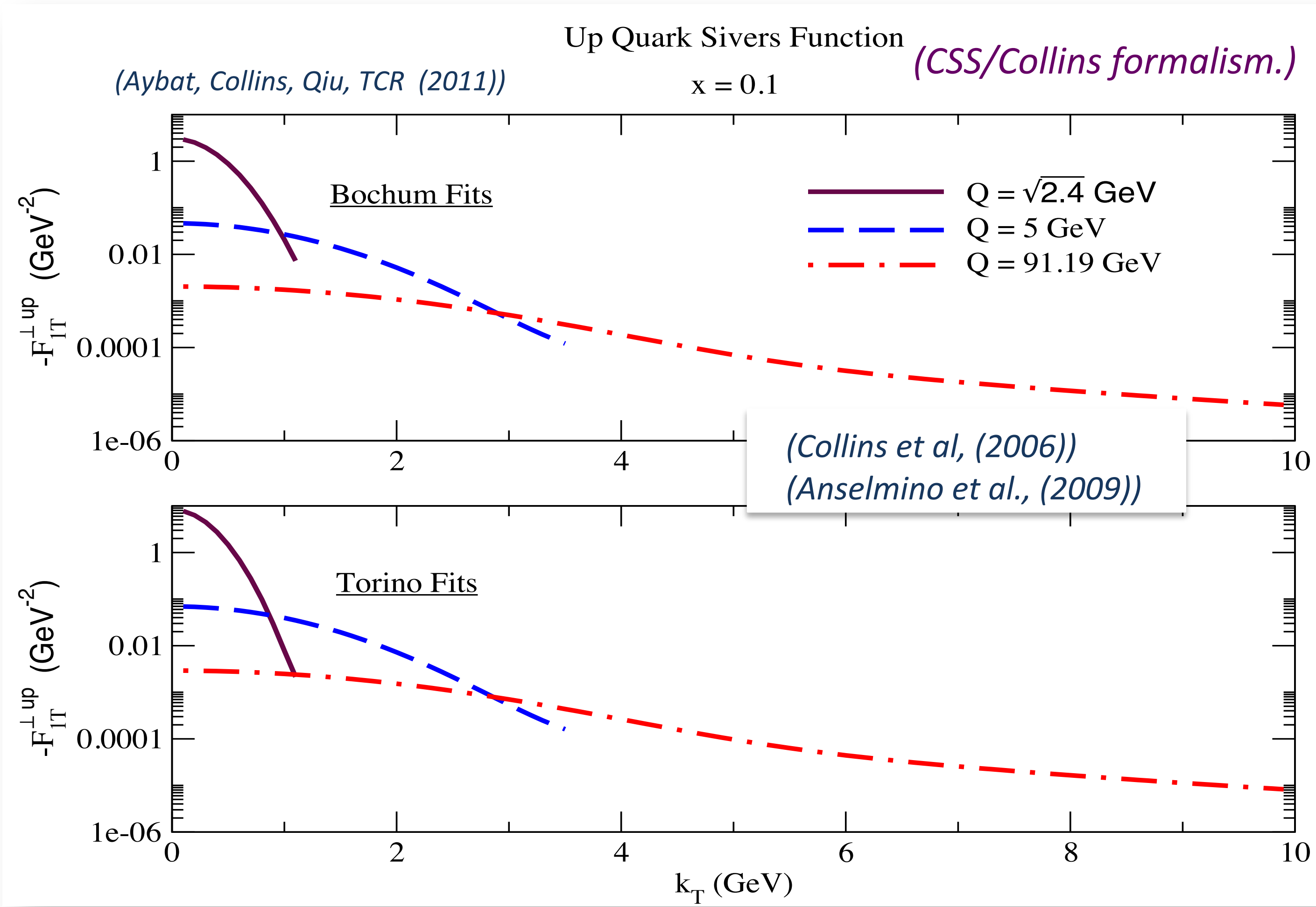
Polarized TMD PDFs:

- Same definition, same evolution equations

(Aybat, Collins, Qiu, TCR (2011))

$$
\begin{aligned}
&\tilde{F}'^{\perp f}_{1T}(x, b_T; \mu, \zeta_F) \\
&\quad = \sum_j \frac{M_p b_T}{2} \int_x^1 \frac{d\hat{x}_1 d\hat{x}_2}{\hat{x}_1 \hat{x}_2} \tilde{C}^{\text{Sivers}}_{f/j}(\hat{x}_1, \hat{x}_2, b_*; \mu_b^2, \mu_b, g(\mu_b)) \\
&\qquad \times T_{Fj/P}(\hat{x}_1, \hat{x}_2, \mu_b) \exp\left\{ \ln\frac{\sqrt{\zeta_F}}{\mu_b} \tilde{K}(b_*; \mu_b) \right. \\
&\qquad \left. + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln\frac{\sqrt{\zeta_F}}{\mu'} \gamma_K(g(\mu')) \right] \right\} \\
&\qquad \times \exp\left\{ -g^{\text{Sivers}}_{f/P}(x, b_T) - g_K(b_T) \ln\frac{\sqrt{\zeta_F}}{Q_0} \right\}.
\end{aligned}
$$

Evolved TMD PDFs: constructed from old fits



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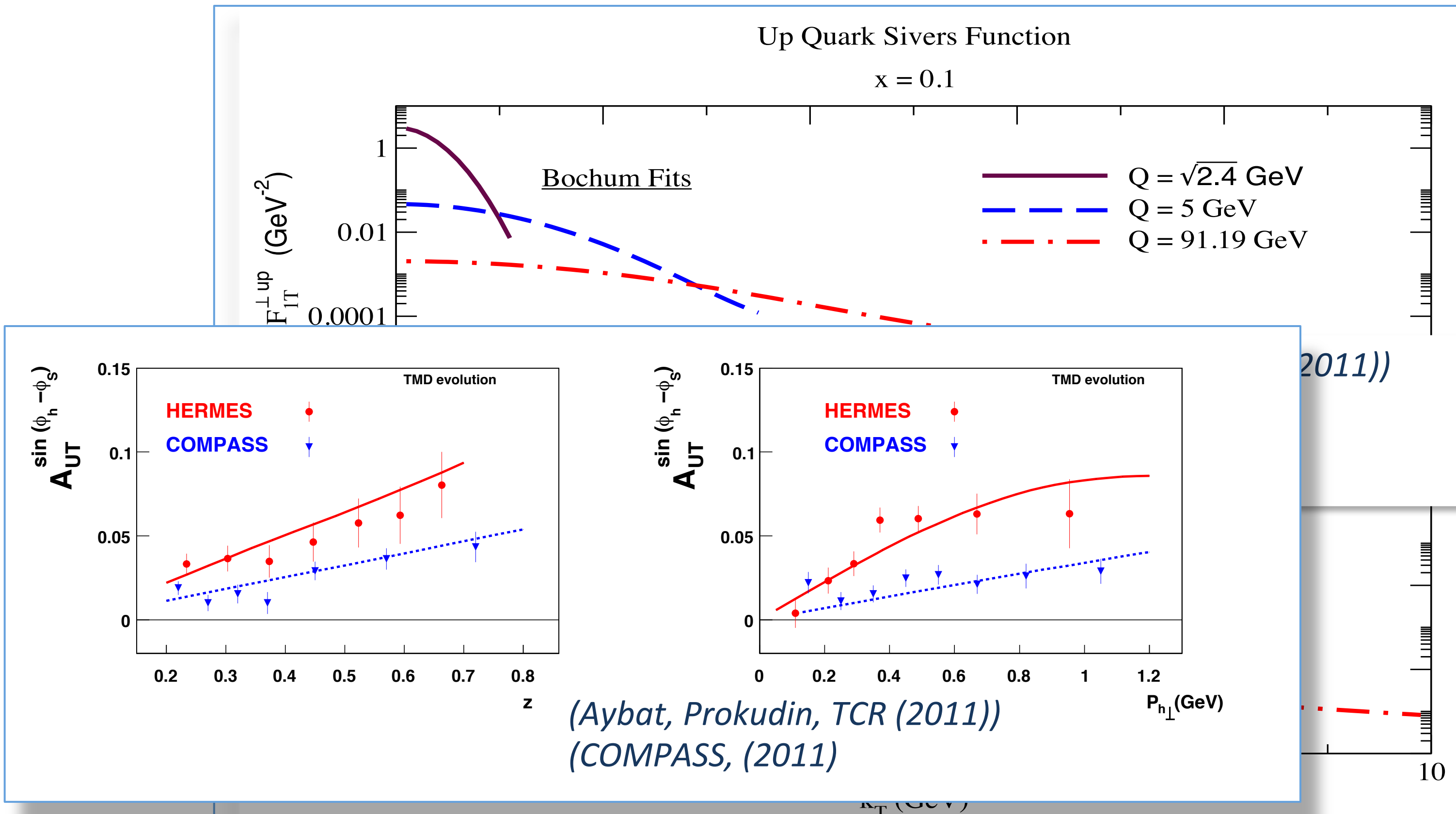

(Aybat, Prokudin, TCR (2011))
(COMPASS, (2011))

https://projects.hepforge.org/tmd/

Questions

- High Q fits extrapolated to low Q (≈ 1 GeV) gives extremely rapid evolution.
  - Is this realistic??
  - Recently questioned.

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  - Collins, Soper, Sterman (1985): non-perturbative evolution of k_T-dependence becomes negligible at Q ≈ 10^8 GeV.
  - Global fits find important non-perturbative evolution of k_T-dependence.
    (Recent: (Guzzi, Nadolksy, Wang (2013))
  - Other approaches claim perturbative evolution down to Q ≈ 1 GeV .
    (Sun-Yuan (2013)) (Echevarria, Idilbi, Schafer, Scimemi (2012))

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- Important for studies of Sivers SIDIS/DY sign flip.

Why Study Perturbative QCD?

- Test QCD as theory of Strong Interaction.

- Support for other HEP (high energy QCD, BSM, etc…)
  - Resummation, Jets, showers etc…
  - LHC / Higgs studies
  - Processes with new physics.
  - Backgrounds, Jet Vetos…

Transverse Momentum Dependent (TMD) Factorization

<u>Main theme of talk.</u>

- Hadronic/Nuclear Structure with quark/gluon degrees of freedom.
  - **<u>Hadron Structure</u>**
  - Confinement
  - Lattice QCD
  - Chiral Symmetry Breaking
  - Non-perturbative QCD

New fits: *To do*

- Incorporate data from all types of processes.
  - SIDIS, DY, e^+e^-, different targets….

- Incorporate all types of observables.
  - Unpolarized cross sections, spin asymmetries…

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$$d\sigma_{\mathrm{SIDIS}} = \sum_f \mathcal{H}_{f,\mathrm{SIDIS}}(Q) \otimes F_{f/H_1}(x, k_{1T}, Q) \otimes D_{H_2/f}(z, k_{2T}, Q) \quad + Y_{\mathrm{SIDIS}}$$

$$d\sigma_{\mathrm{DY}} = \sum_f \mathcal{H}_{f,\mathrm{DY}}(Q) \otimes F_{f/H_1}(x_1, k_{1T}, Q) \otimes F_{\bar{f}/H_2}(x_2, k_{2T}, Q) \quad + Y_{\mathrm{Drell-Yan}}$$

$$d\sigma_{\mathrm{e^+e^-}} = \sum_f \mathcal{H}_{f,\mathrm{e^+e^-}}(Q) \otimes D_{H_1/\bar{f}}(z_1, k_{1T}, Q) \otimes D_{H_2/f}(z_2, k_{2T}, Q) \quad + Y_{\mathrm{e^+e^-}}$$

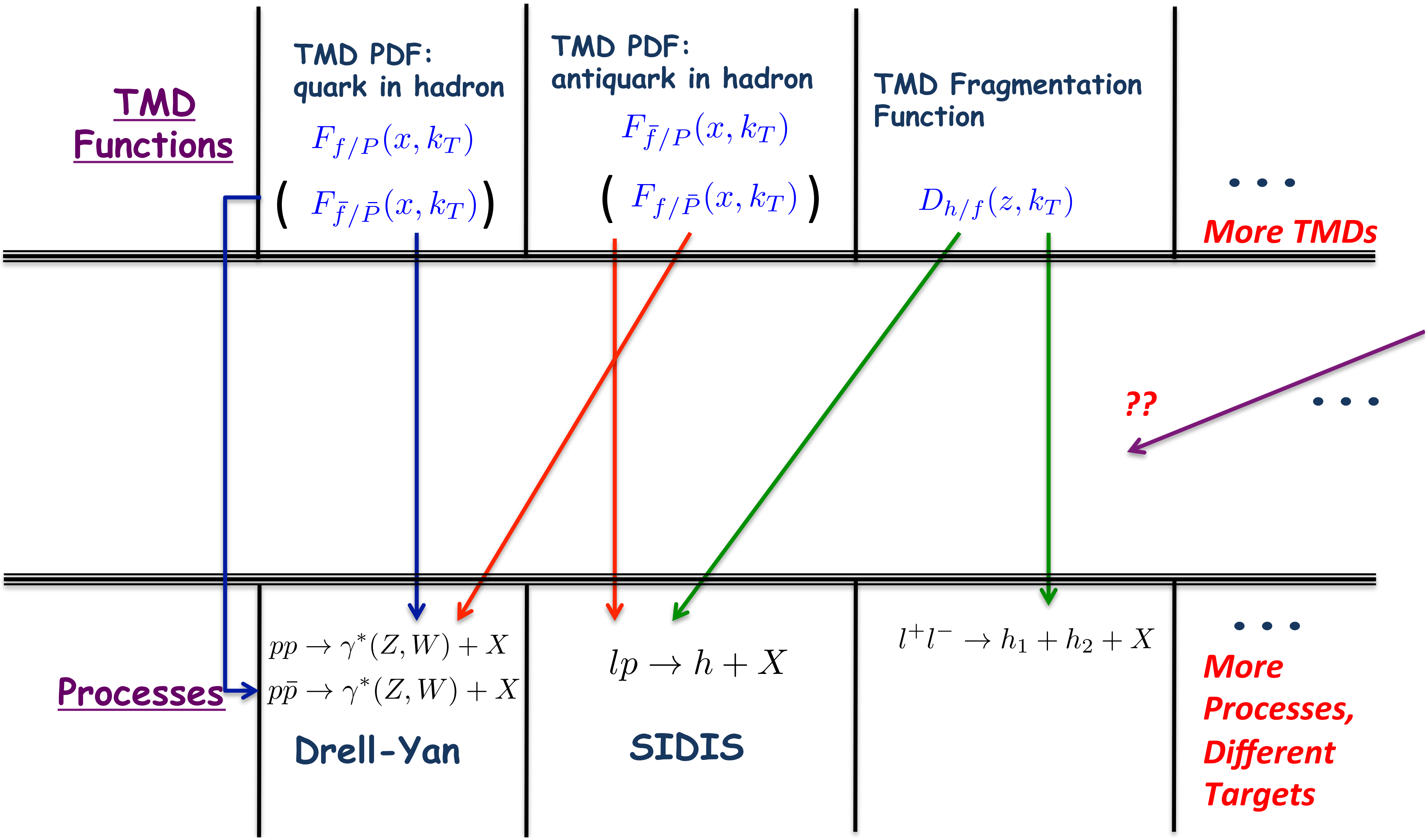
Constraining Non-Perturbative Parts

<u>**TMD Functions**</u>	TMD PDF: quark in hadron	TMD PDF: antiquark in hadron	TMD Fragmentation Function	...
	$F_{f/P}(x, k_T)$	$F_{\bar{f}/P}(x, k_T)$		
	$\left(F_{\bar{f}/\bar{P}}(x, k_T) \right)$	$\left(F_{f/\bar{P}}(x, k_T) \right)$	$D_{h/f}(z, k_T)$	More TMDs

Constraining Non-Perturbative Parts

TMD Functions	TMD PDF: quark in hadron $F_{f/P}(x, k_T)$ $\left(F_{\bar{f}/\bar{P}}(x, k_T) \right)$	TMD PDF: antiquark in hadron $F_{\bar{f}/P}(x, k_T)$ $\left(F_{f/\bar{P}}(x, k_T) \right)$	TMD Fragmentation Function $D_{h/f}(z, k_T)$	. . . *More TMDs*
Processes	$pp \to \gamma^*(Z, W) + X$ $p\bar{p} \to \gamma^*(Z, W) + X$ Drell-Yan	$lp \to h + X$ SIDIS	$l^+l^- \to h_1 + h_2 + X$	. . . *More Processes, Different Targets*

Constraining Non-Perturbative Parts



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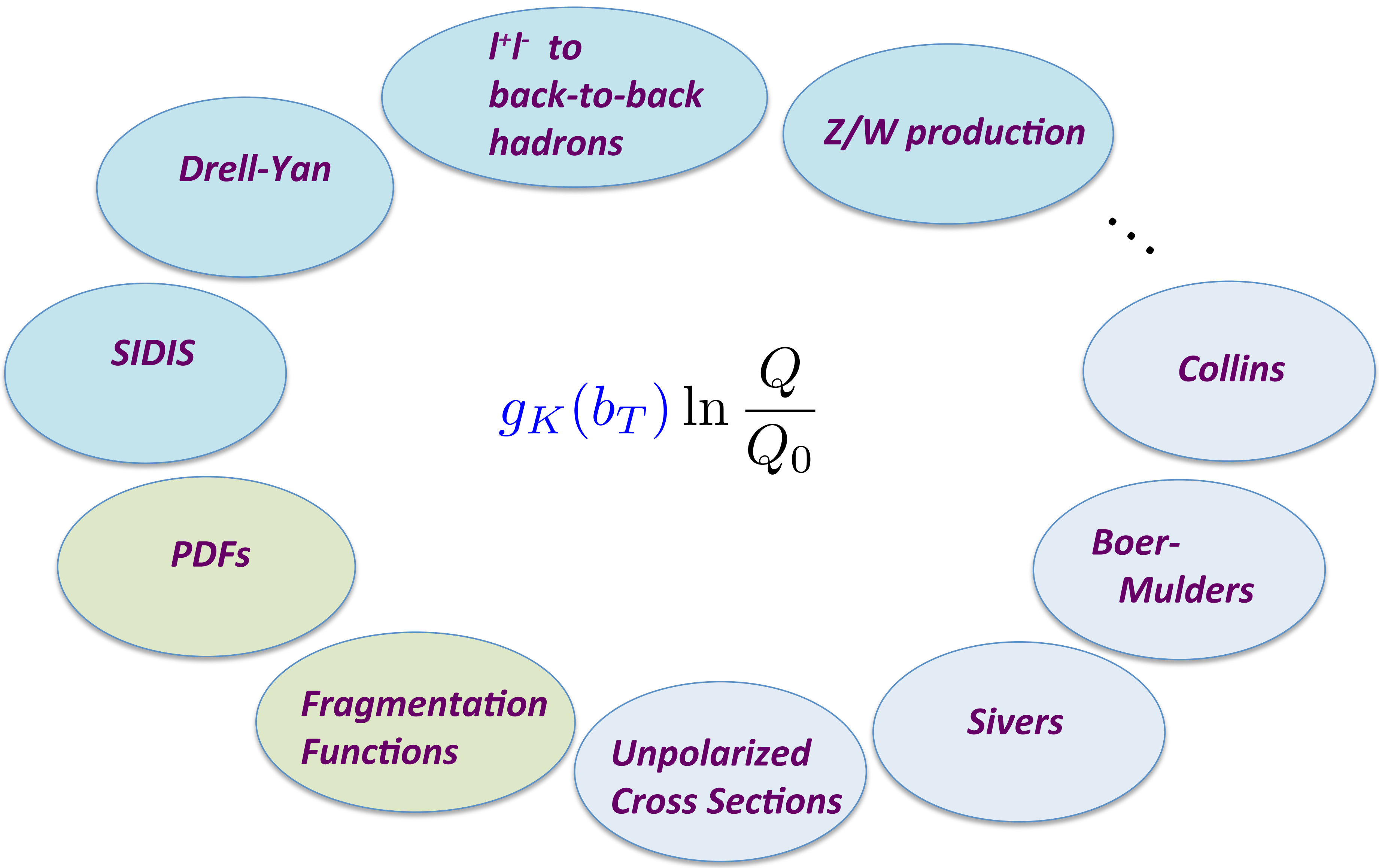
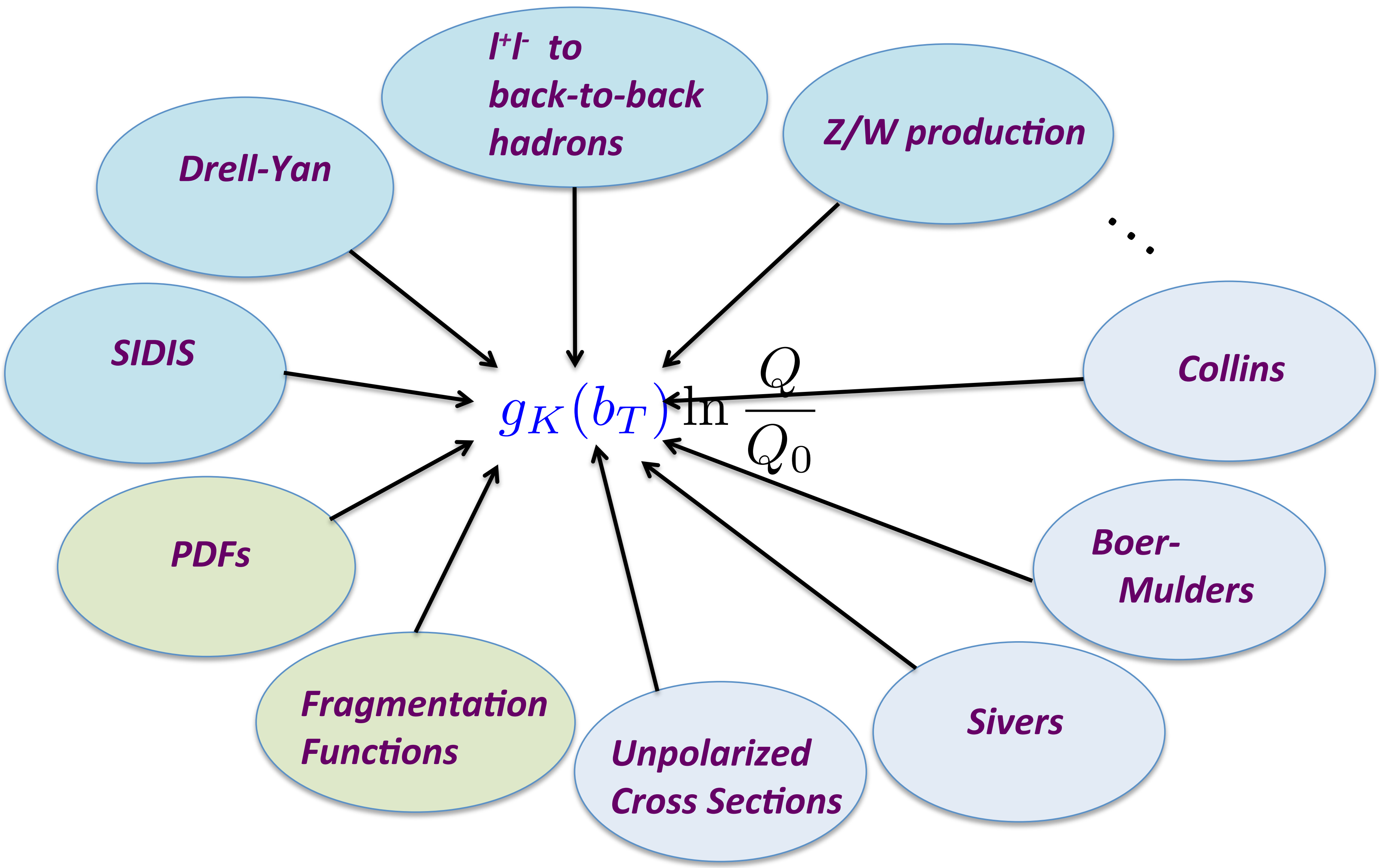


$g_K(b_T) \ln \frac{Q}{Q_0}$
l^+l^- to back-to-back hadrons
Z/W production
Drell-Yan
SIDIS
PDFs
Fragmentation Functions
Unpolarized Cross Sections
Sivers
Boer-Mulders
Collins
...



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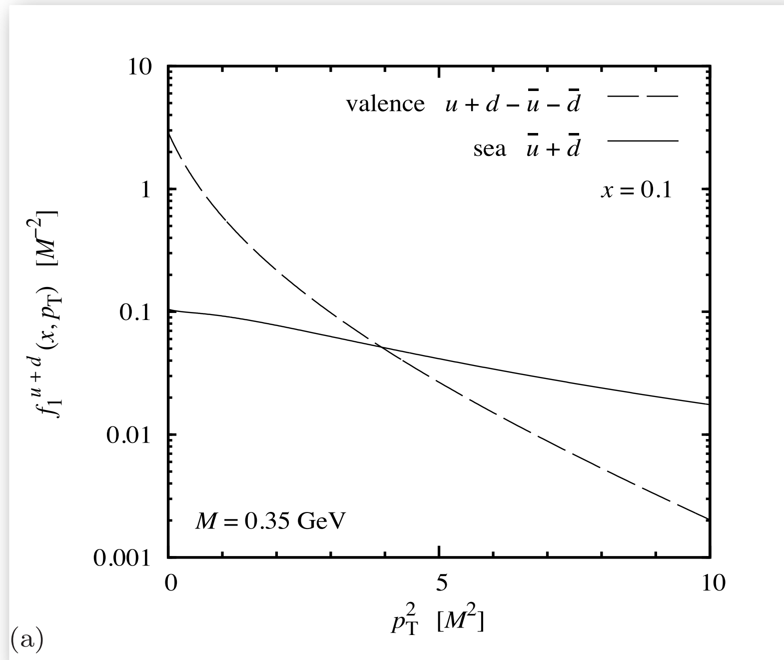
- Non-perturbative physics

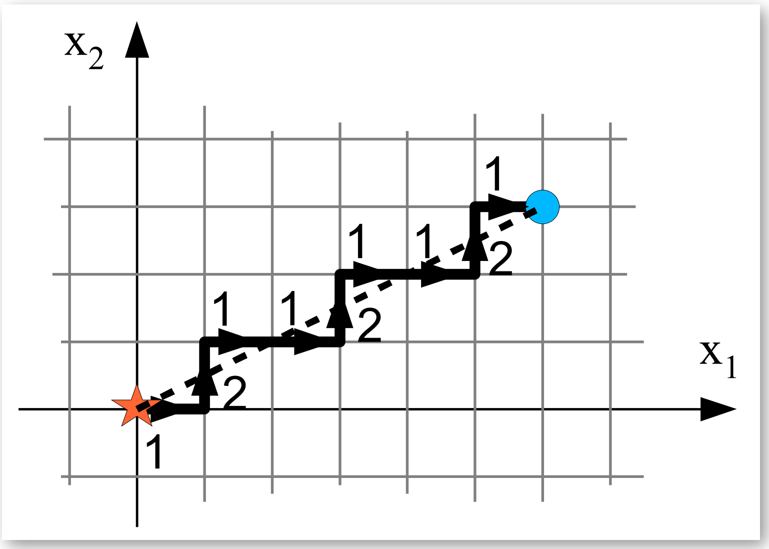
- Chiral symmetry breaking:

(Schweitzer, Strikman, Weiss, (2013))



- Chiral symmetry breaking:

- Lattice QCD



(M. Engelhardt et al., (2012))
(B. Musch et al., (2011))

- Chiral symmetry breaking:

- Lattice QCD

- Non-perturbative models
  - Bag models
    (A. Courtoy et al., (2008,2009))
  - Light-cone wave function
    (B. Pasquini, F. Yuan (2010))
  - Others…

Example:

- Sea quark TMDs vs. valence quark TMDs:
  - $\bar{p}A \to l^+ + l^- + X$
    - $F_{\bar{q}/\bar{p}} \otimes F_{q/A}$ → **Valence Quarks**
  - $pA \to l^+ + l^- + X$
    - $F_{q/p} \otimes F_{\bar{q}/A}$ → **Sea Quarks**

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    - $F_{q/p} \otimes F_{\bar{q}/A}$

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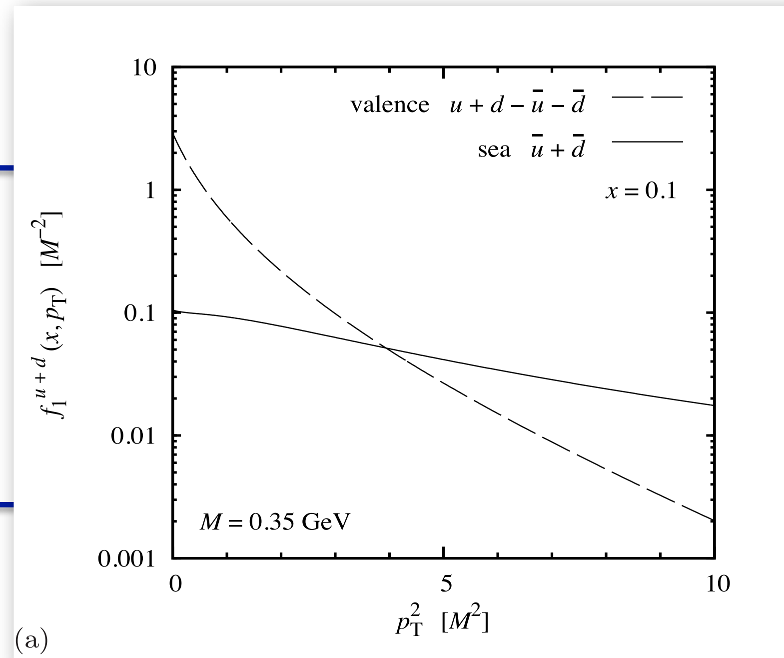

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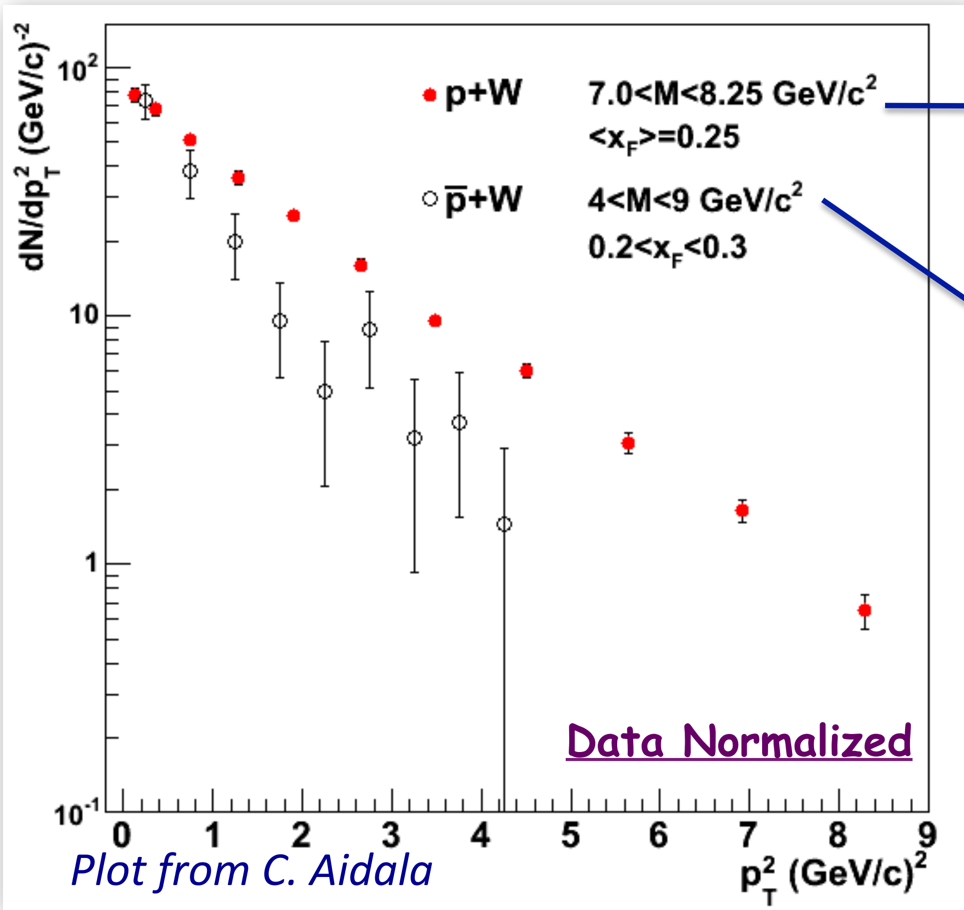

Fermilab, Oliver et al.,
AIP Conf.Proc. 45 (1978) 93-102
400 GeV proton

Anassontzis et al.,
Phys.Rev. D38 (1988) 1377
125 GeV antiprotons

Is there a difference?

Need better control
of evolution for moderate Q

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- Incorporate all types of observables.
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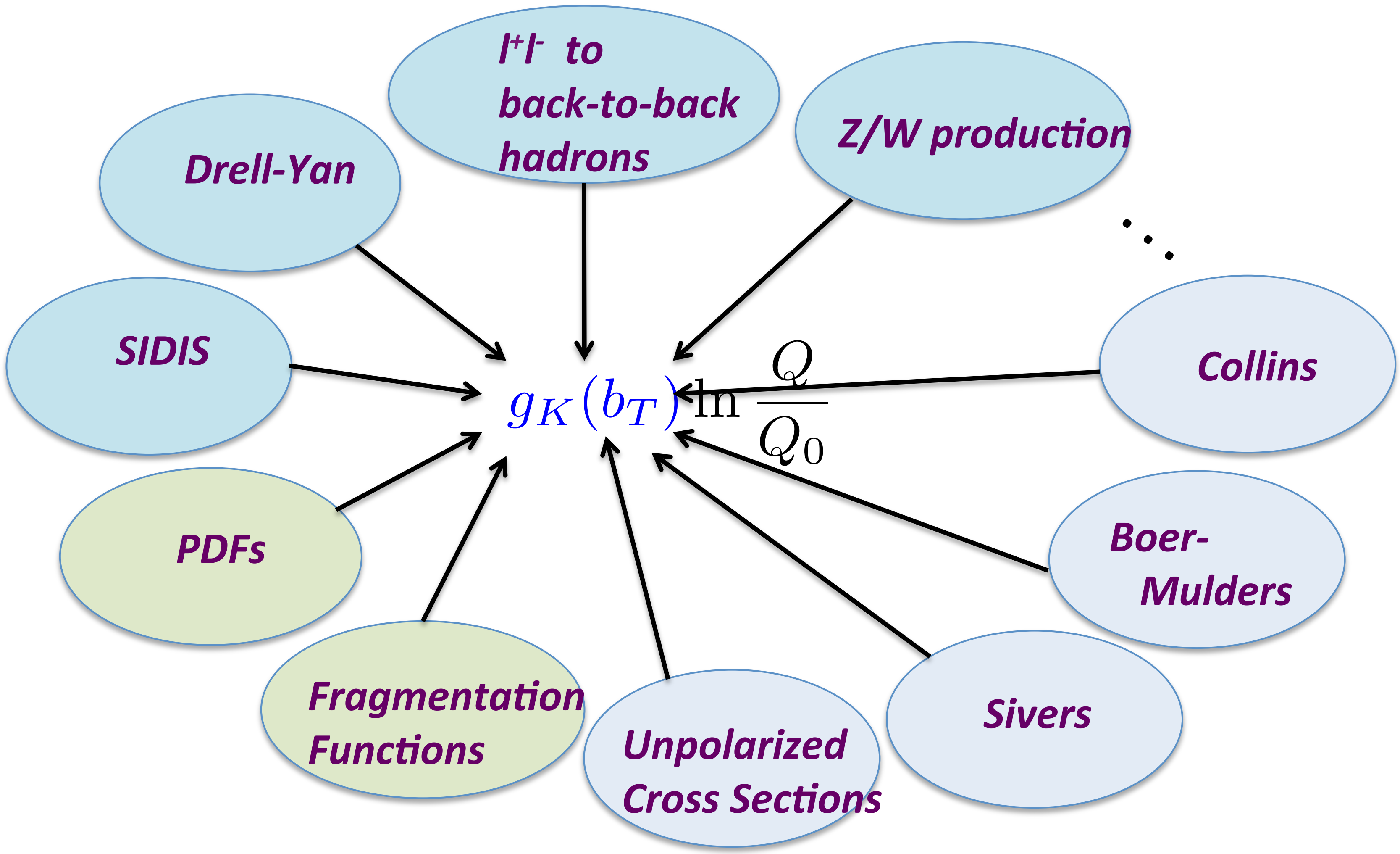
- Utilize/Test Strong Universality of Non-Perturbative Evolution.
- Non-perturbative physics
- Account for x, z, hadron species dependence. Take TMD picture/interpretation seriously.

Test non-perturbative evolution in unpolarized SIDIS



Test non-perturbative evolution in unpolarized SIDIS

Current work with C. Aidala and L. Gamberg

$$\mathbf{b}_*(\mathbf{b}_\mathrm{T}) \equiv \frac{\mathbf{b}_\mathrm{T}}{\sqrt{1 + b_T^2/b_\mathrm{max}^2}}$$

$$\mu_b \equiv C_1/|\mathbf{b}_*(b_T)|$$

$$g_K(b_T) \ln \frac{Q}{Q_0}$$

COMPASS, *C. Adolph et al., arXiv:1305.7317*

$$Q = 1 \sim 2 \ \ \mathrm{GeV}$$

Approx. fixed x,z bins

Fit: $$\frac{d\sigma}{dP_T^2} \propto \exp\left\{-\frac{P_T^2}{\langle P_T^2 \rangle}\right\}$$

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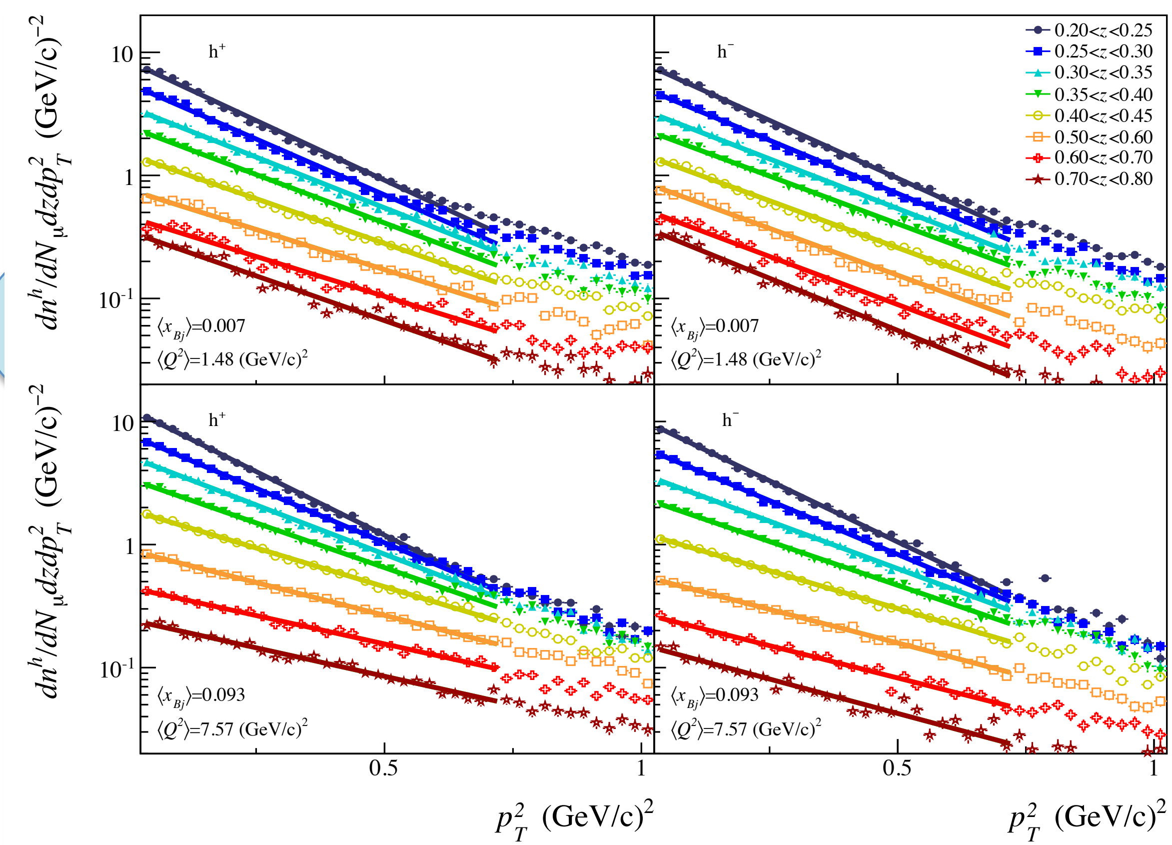

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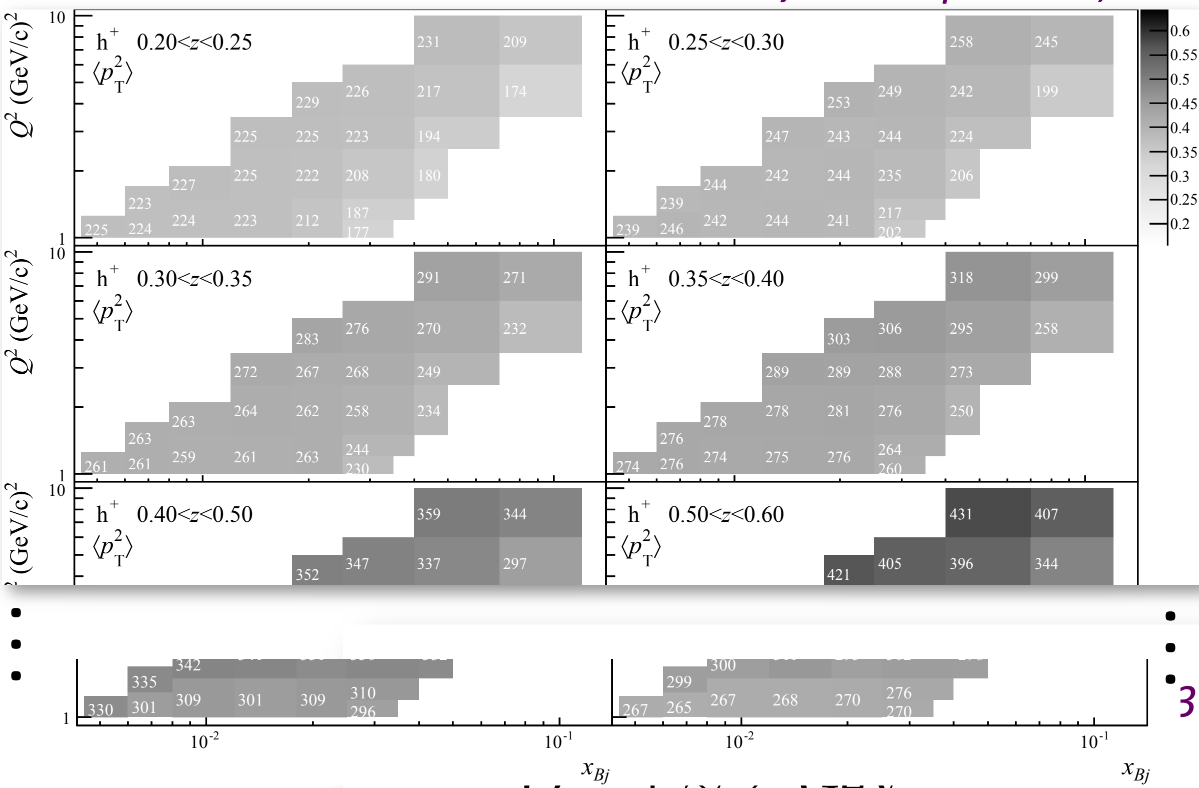

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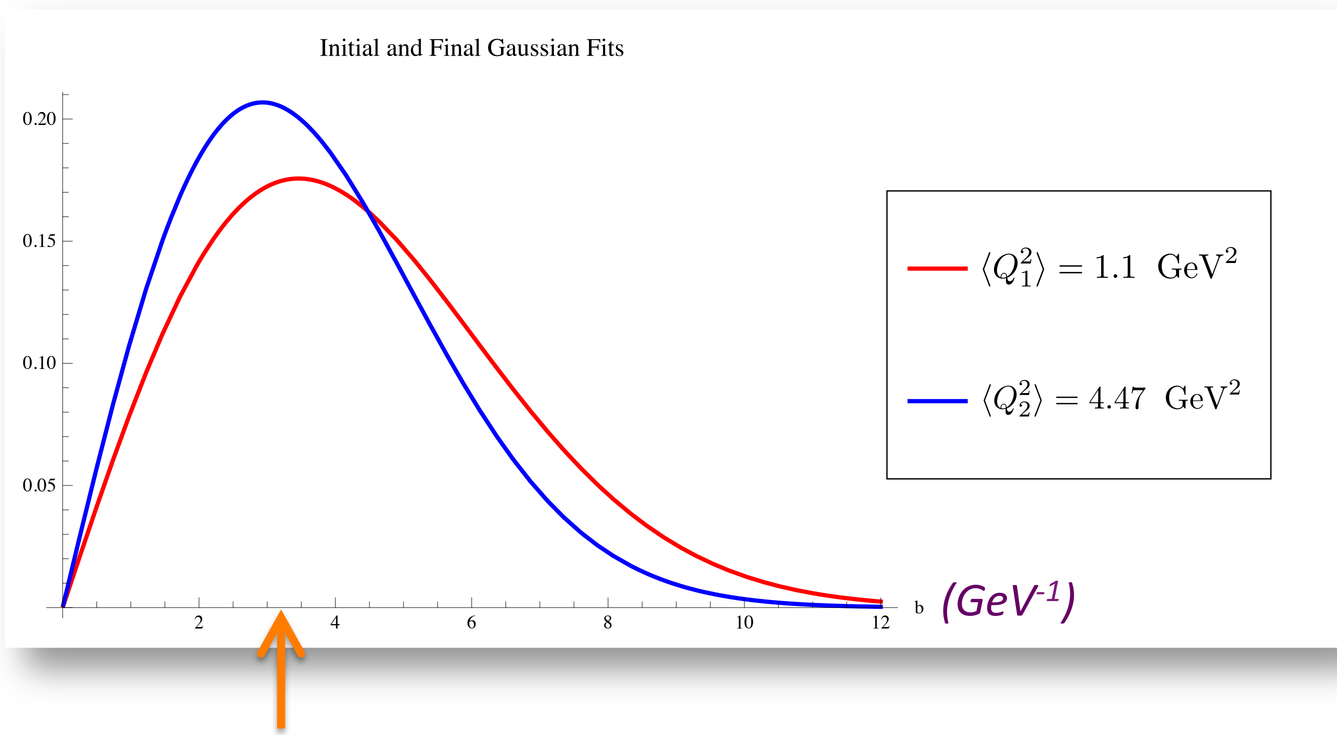
Test non-perturbative evolution in unpolarized SIDIS

Current work with C. Aidala and L. Gamberg

- Fastest evolution (b_T-space):



≈ Proton Radius

What is needed

- New global fits to semi-inclusive deep inelastic scattering over wide range of Q.

  (Nadolsky, Stump, Yuan (2000,2001))

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- TMD Pert. QCD is reaching a stage where two traditionally separate QCD styles are rapidly merging.
  - New, very labor intensive projects need to be pushed through.
  - Collaboration badly needed.

Hadron Structure in Transverse Momentum Dependent Resummation and Evolution

T. C. Rogers

C.N. Yang Institute for Theoretical Physics, SUNY Stony Brook

- ~~Perturbative QCD and Collinear Factorization.~~ ✓
- ~~Transverse Momentum Dependent (TMD) Factorization.~~ ✓
- ~~TMD project: Implementing TMD-factorization.~~ ✓

Seminar: Southern Methodist University – October 7, 2013

Hadron Structure in Transverse Momentum Dependent Resummation and Evolution

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For more details:

https://www.jlab.org/hugs/program.html

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Thank You!

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