Monte Carlos for LHC predictions

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SMU Dallas, 30 November 2015

Monte Carlos for LHC predictions

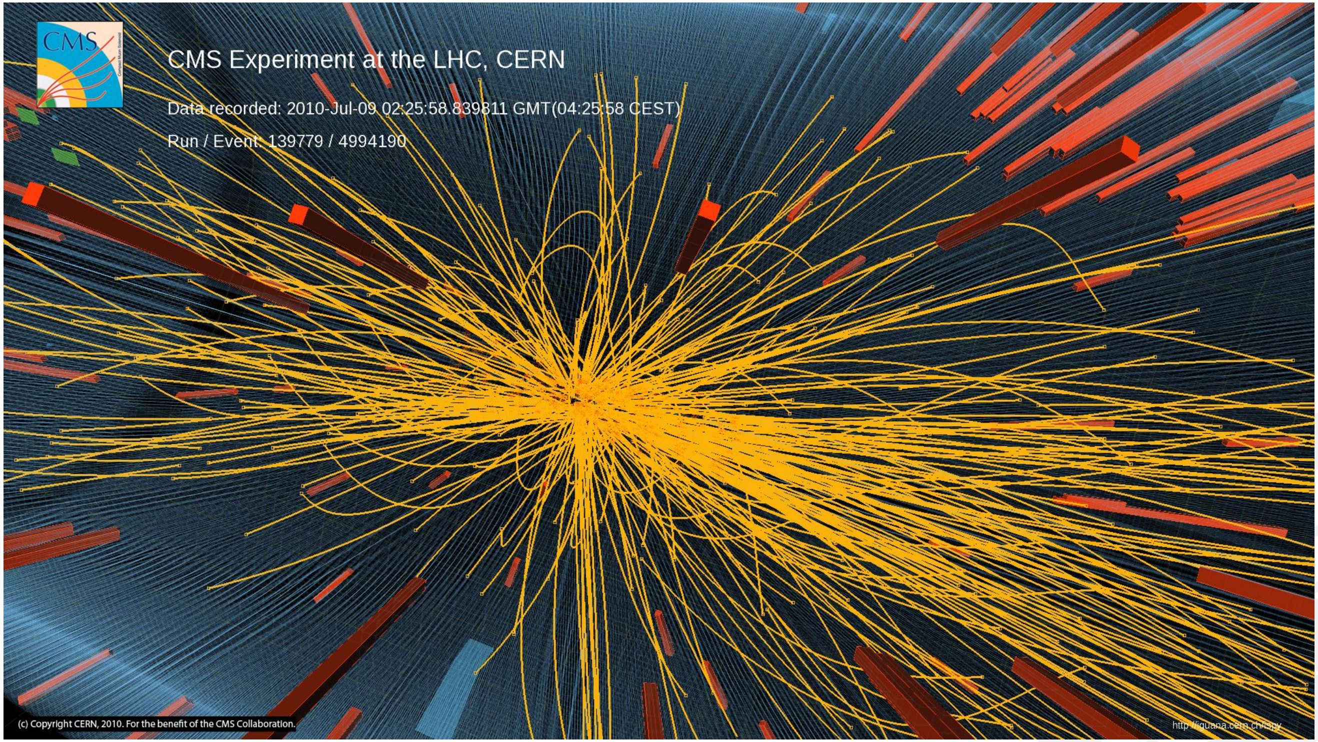
- Matching fixed Next-to-Leading Order (NLO) calculations with Parton Shower (PS): NLO+PS

- Explain all the ingredients of a calculation at NLO+PS accuracy:
  - fixed order (FO): LO, NLO (real/virtual corrections)
  - parton shower (PS)
  - PS applied to NLO: NLO+PS

- Recent developments in `POWHEG BOX`

Hadronic collisions

- Typical proton-proton collision



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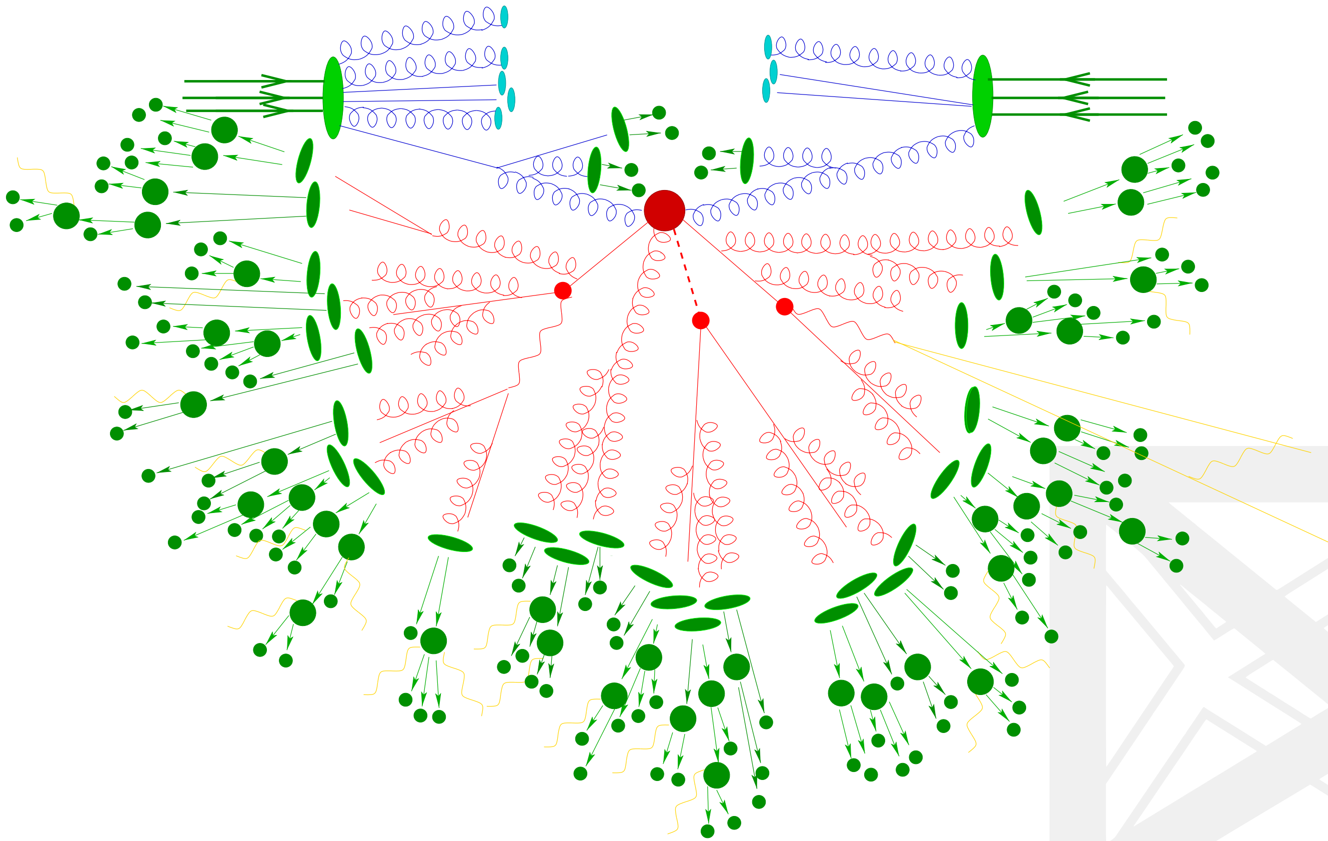

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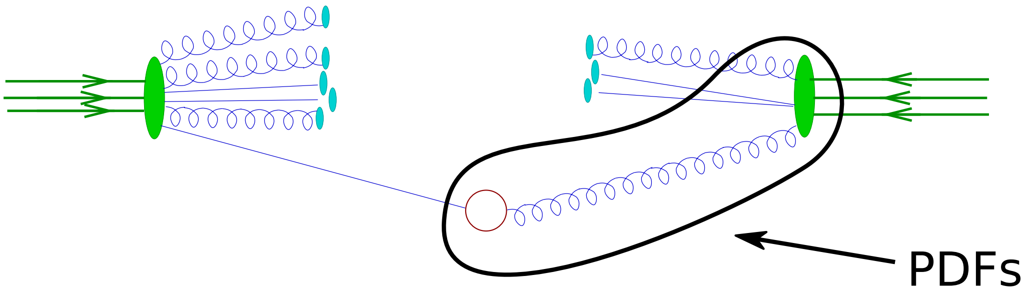

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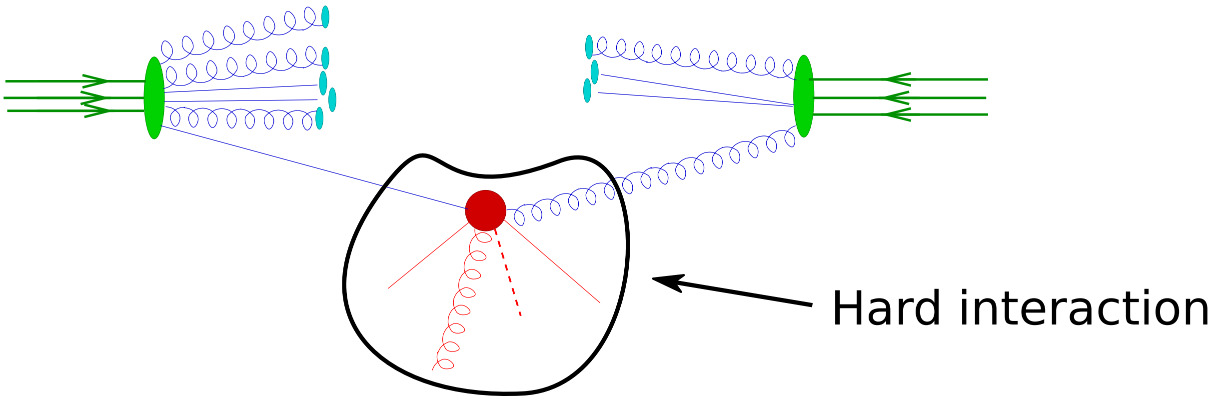

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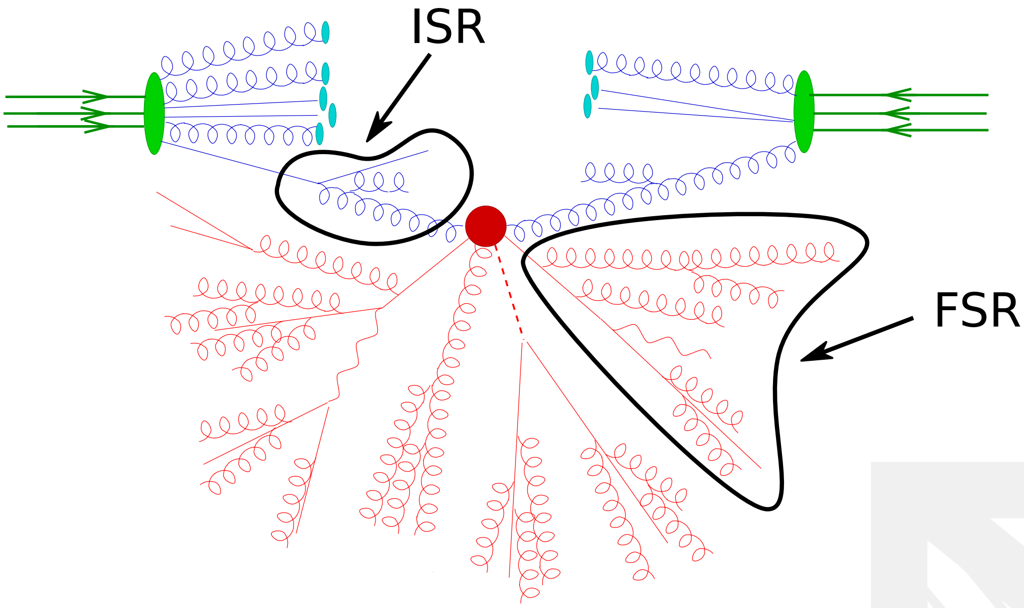

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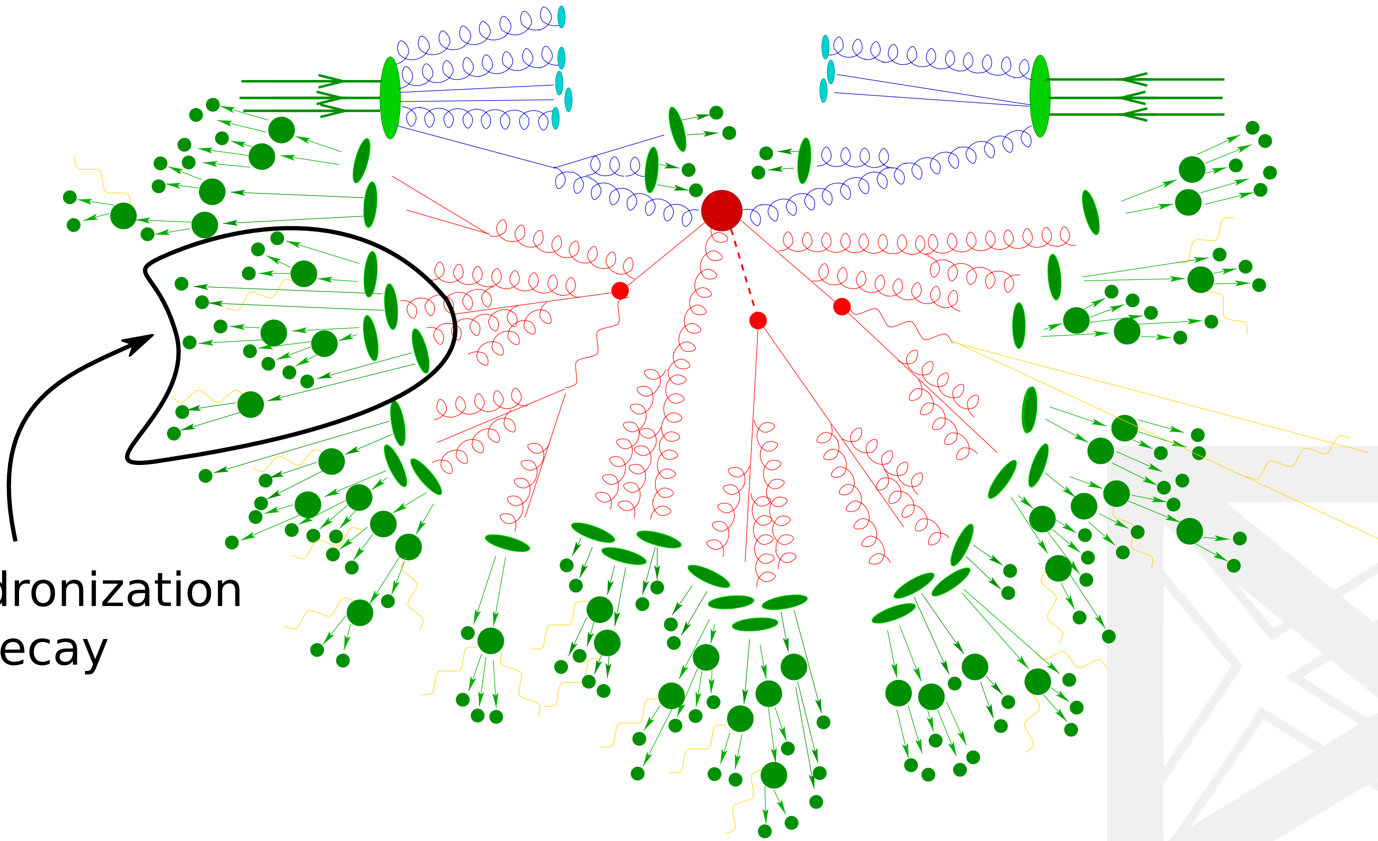

Hadronization
& decay

Hadronic collisions

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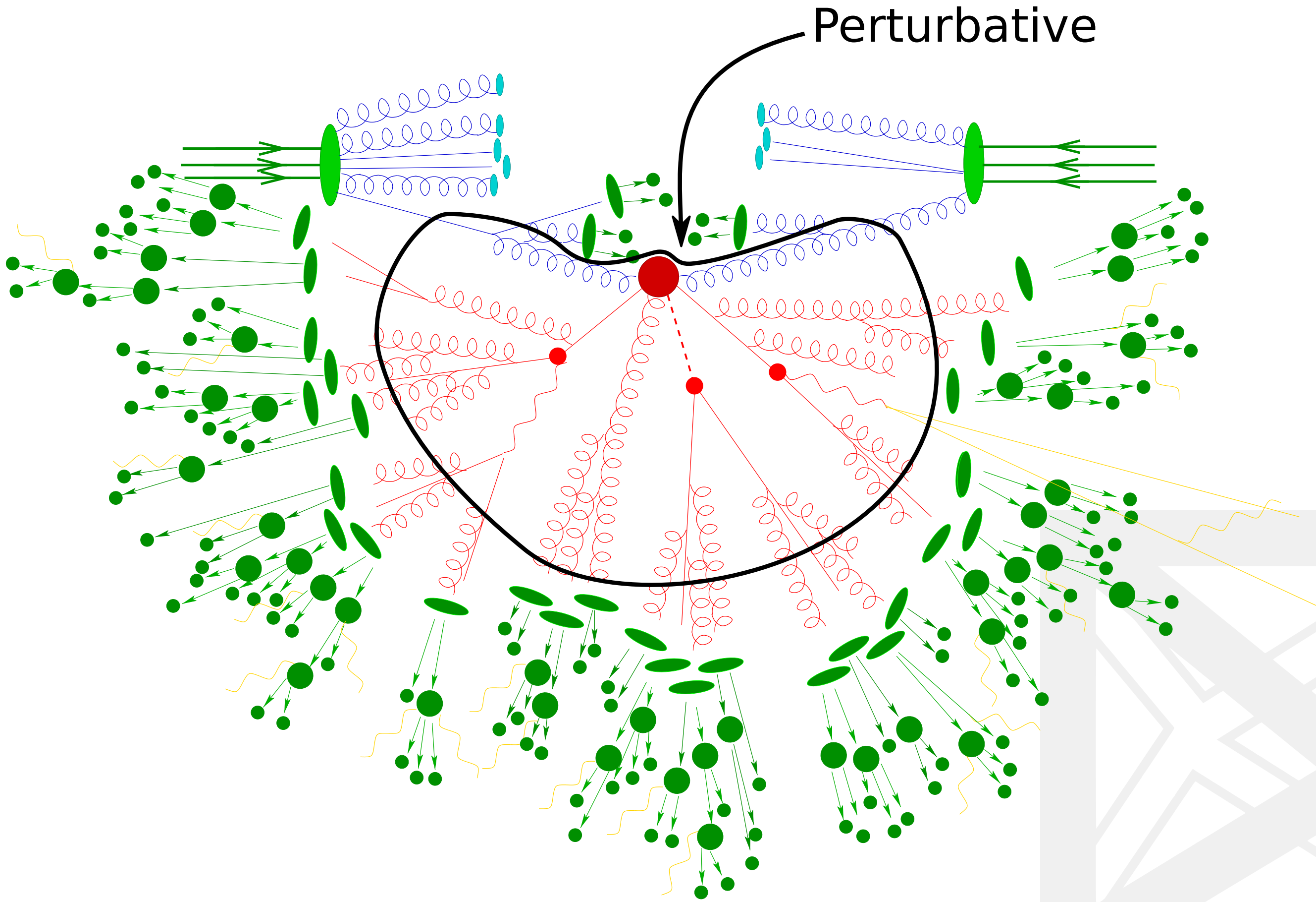

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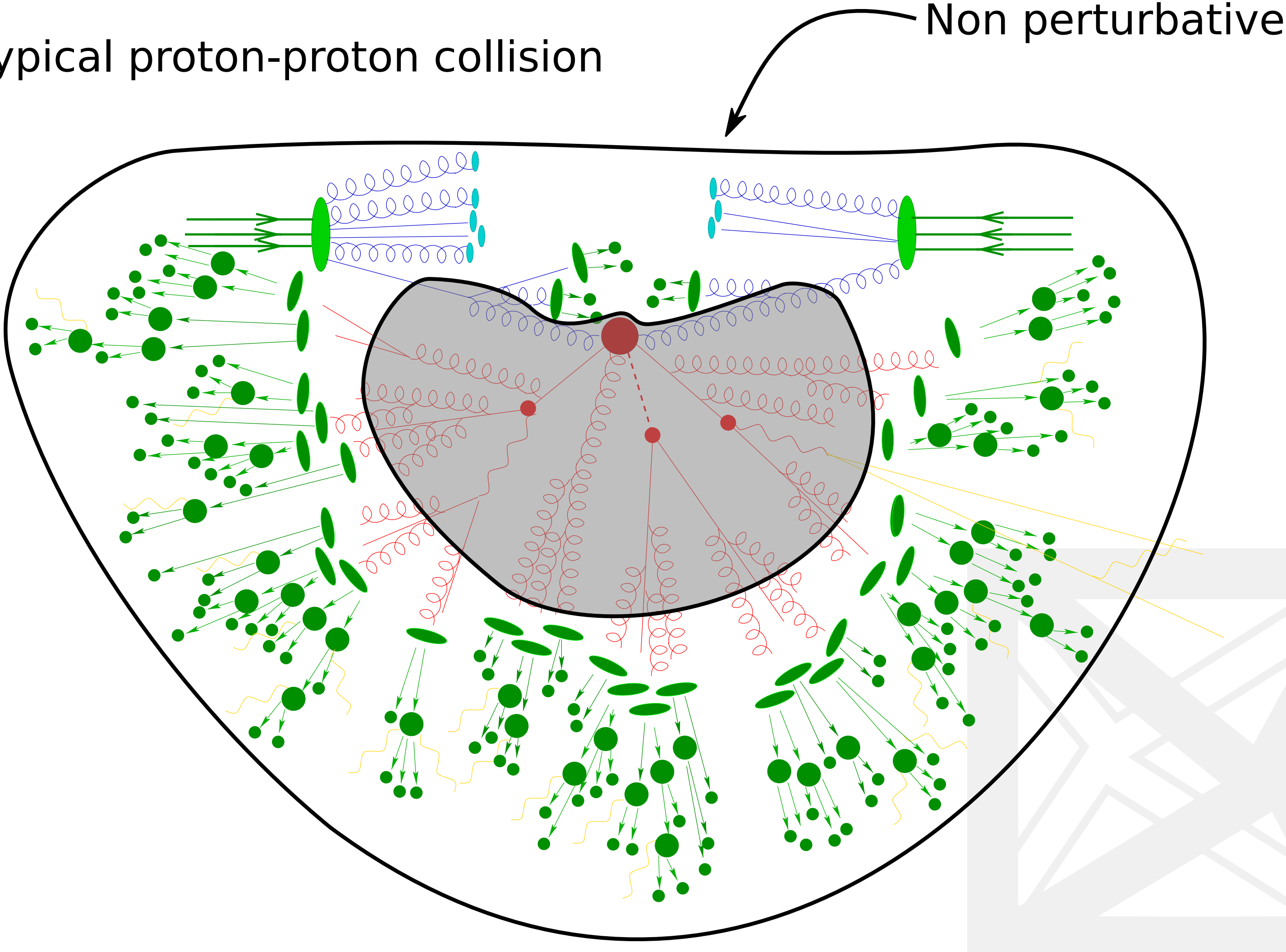
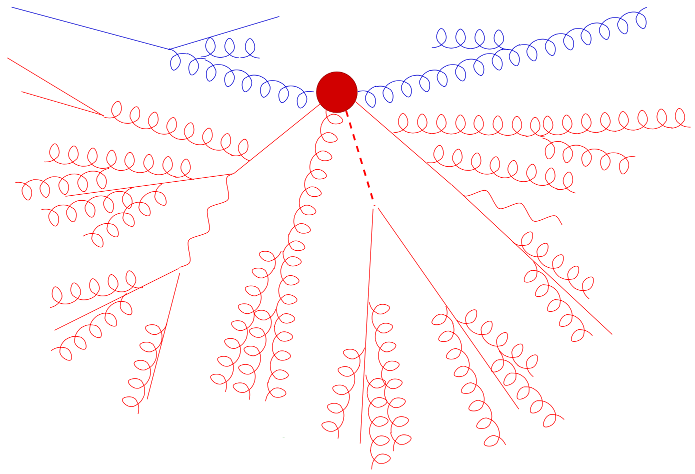


Hadronic collisions

- Typical proton-proton collision
    - I focus on the perturbative part
    - in particular: interplay of FO calculations and PS @ NLO



FO = **F**ixed **O**rder
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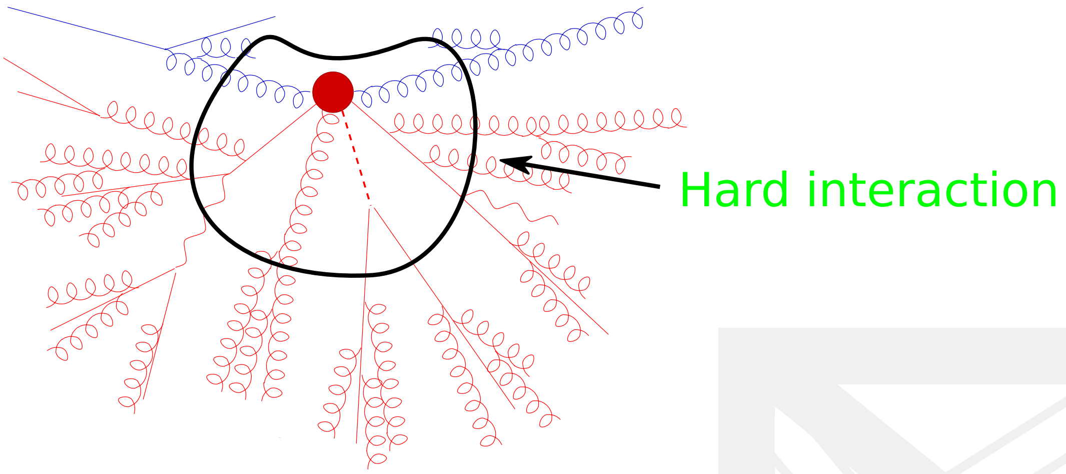

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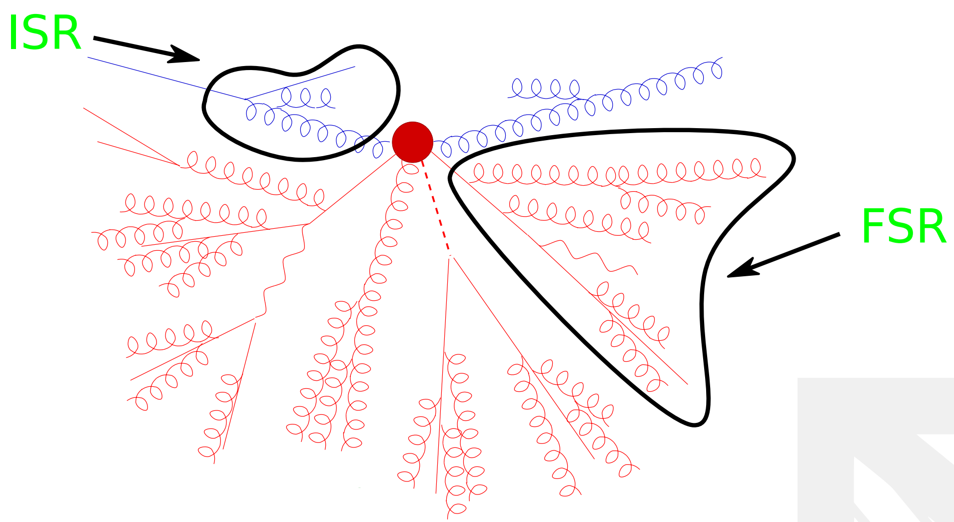

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ISR = **I**nitial **S**tate **R**adiation; FSR = **F**inal **S**tate **R**adiation

nCTEQ

nuclear parton distribution functions

(nuclear) CTEQ - The coordinated theoretical-experimental project on QCD

nCTEQ collaboration

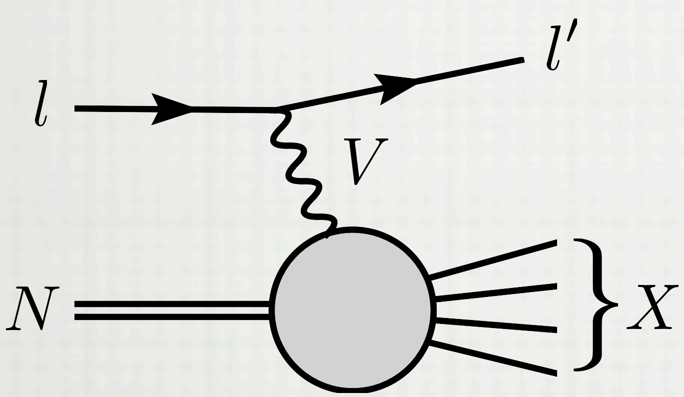
K. Kovarik, A. Kusina, T. Jezo, D. Clark, C. Keppel, F. Lyonnet, J. Morfin, F. Olness, J. Owens, I. Schienbein, J.Y. Yu

Westfälische
Wilhelms-Universität
Münster

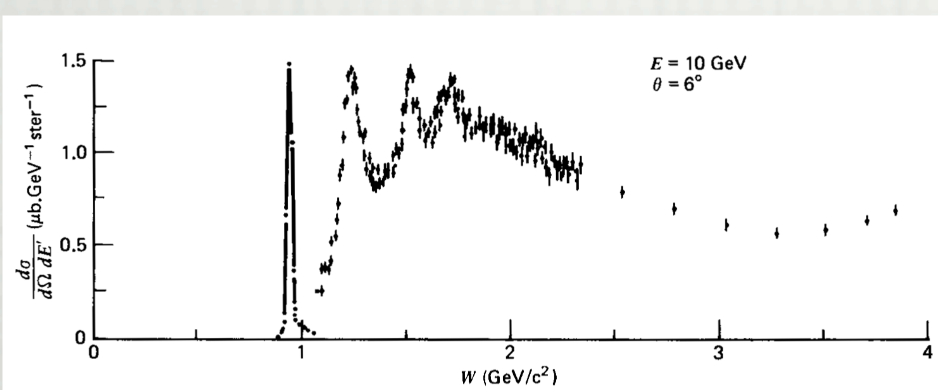
nCTEQ

nuclear parton distribution functions

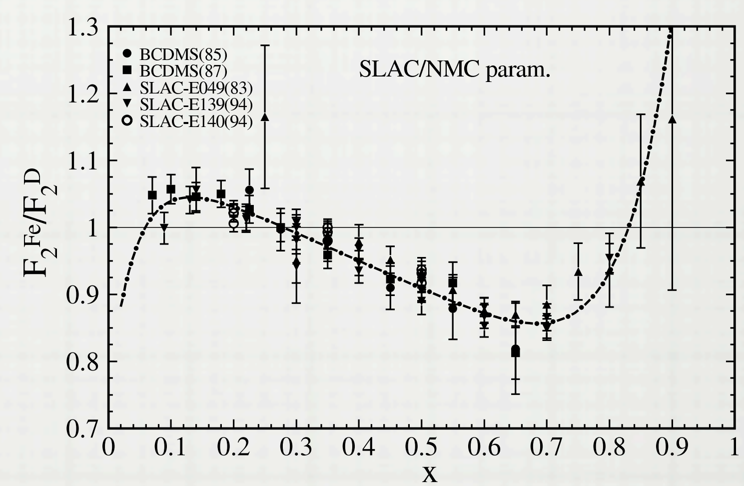
- ***PDFs, structure of the proton & DIS***
  - structure of proton & neutron bound in nuclei



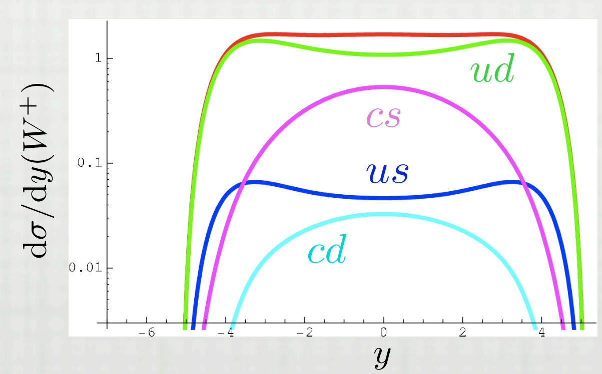
$$l(k) + N(p_N) \to l'(k') + X$$

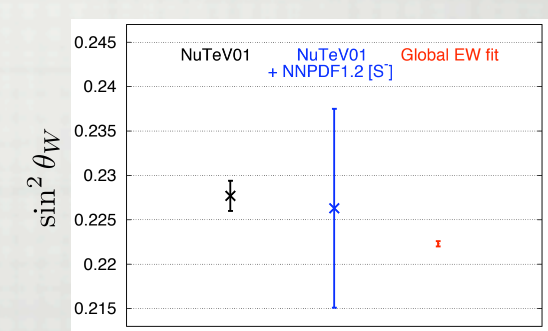


- ***nuclear effects in PDFs - nPDFs***
  - nuclear effects for partons in bound proton & neutron



  - (anti-)strange PDF from (anti-)neutrino DIS with heavy nuclei





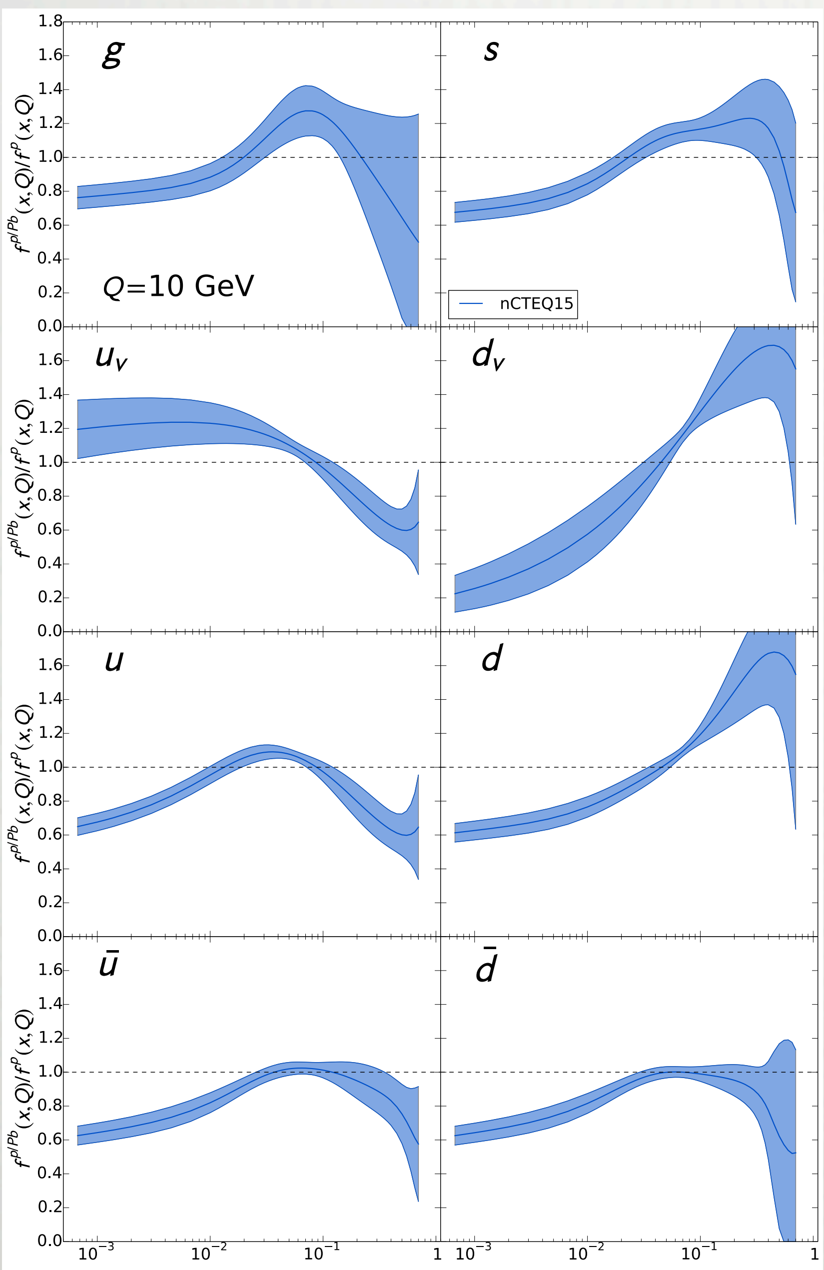
Westfälische Wilhelms-Universität Münster

nCTEQ

nuclear parton distribution functions

nCTEQ15 - Global analysis of nuclear parton distributions with uncertainties in the CTEQ framework

K. Kovarik, A. Kusina, T. Jezo, D. Clark, C. Keppel, F. Lyonnet, J. Morfin, F. Olness, J. Owens, I. Schienbein, J.Y. Yu

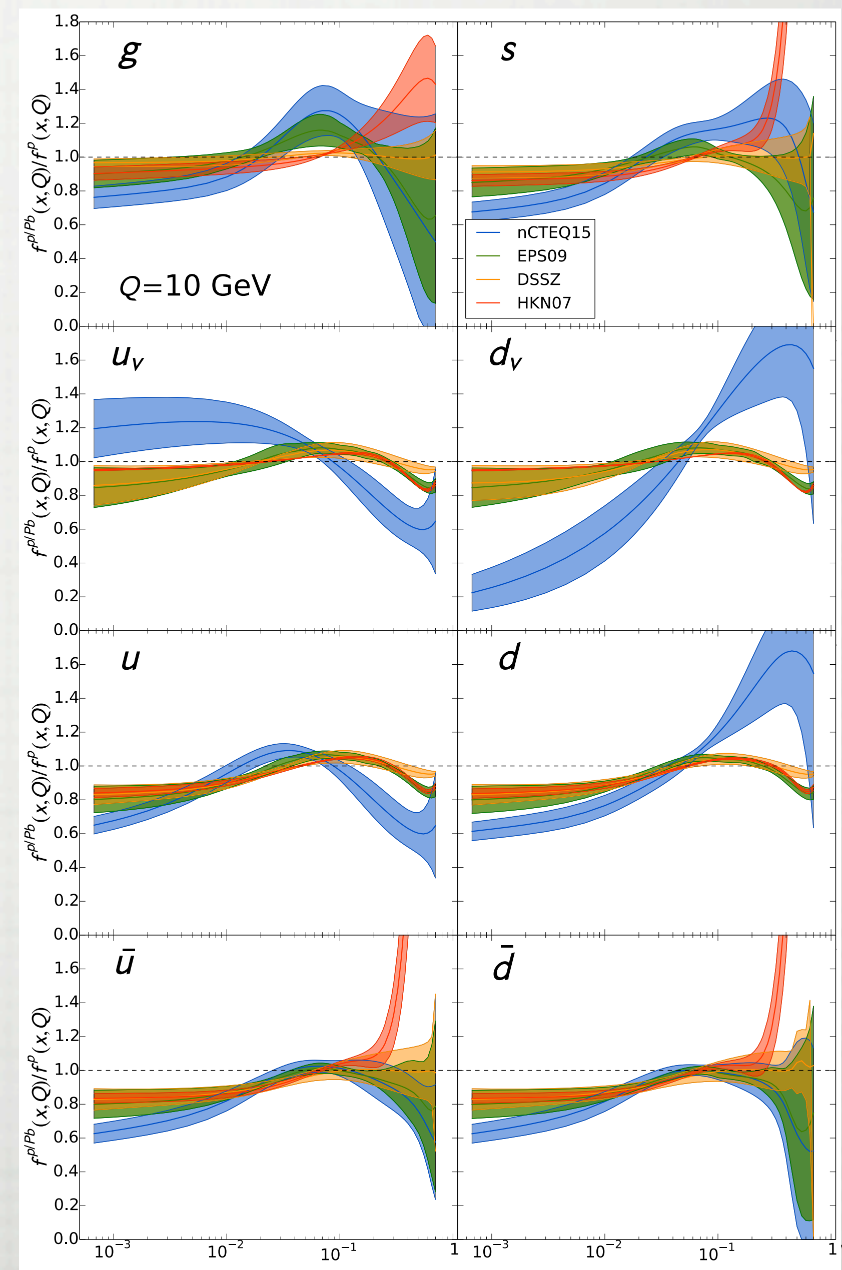


$$R_i(\mathrm{Pb}) = \frac{f_i^{Pb}(x,Q)}{f_i^{p}(x,Q)}$$

$$@ \;\; Q^2 = 100 \text{ GeV}^2$$

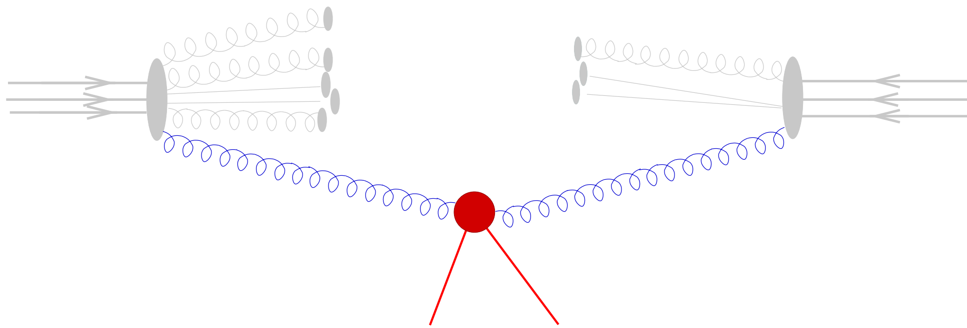
- larger uncertainty @ gluon nuclear correction factor & bigger low-x suppression

- different solution for d-valence & u-valence R factor & different underlying proton PDF

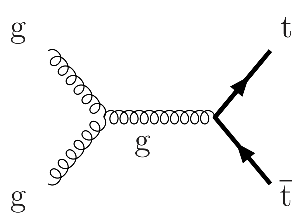


Fixed order calculations

- Take for example a production of a pair of top-quarks



- Diagramatically



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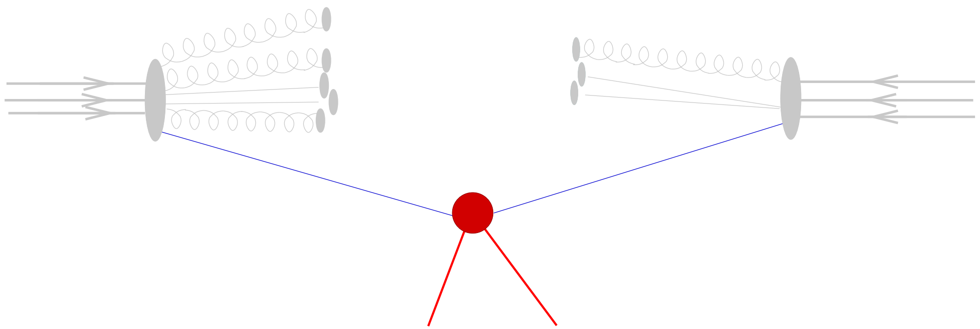

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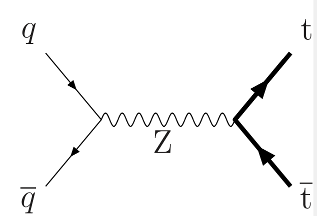

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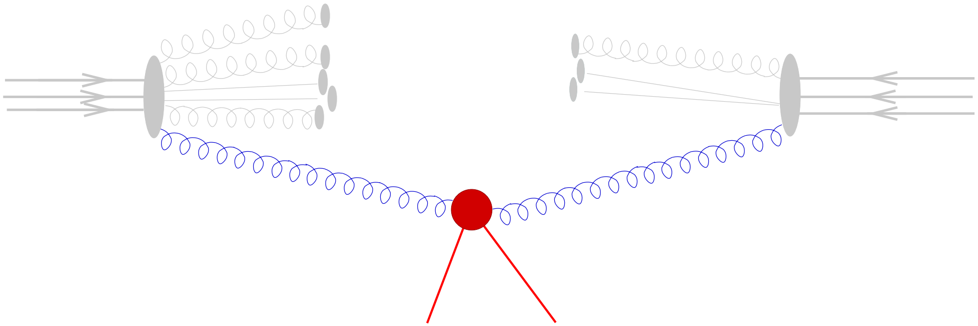

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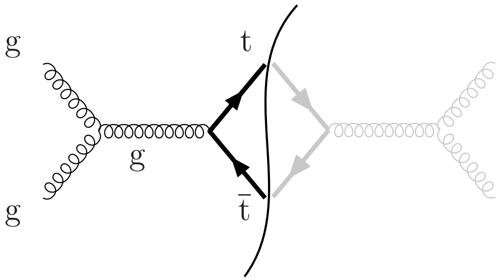

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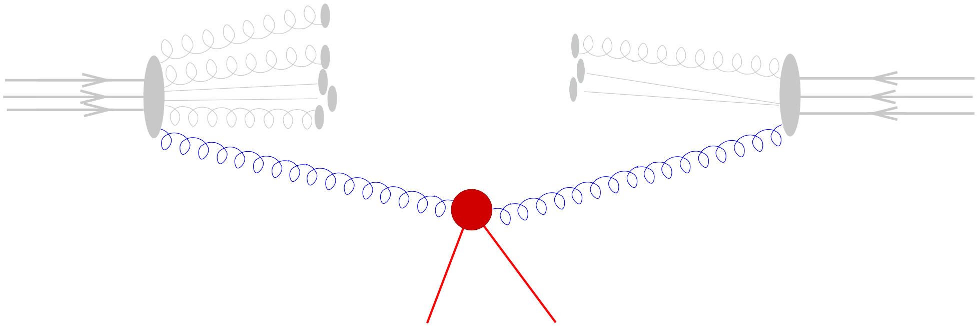

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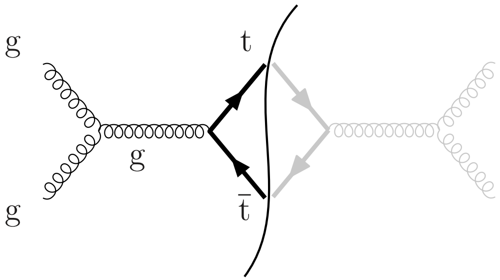

$$\int d\boldsymbol{\Phi}_2\, \mathcal{B}(\boldsymbol{\Phi}_2)$$

$$\boldsymbol{\Phi}_2 = \{k_1, k_2, p_1, p_2\}$$

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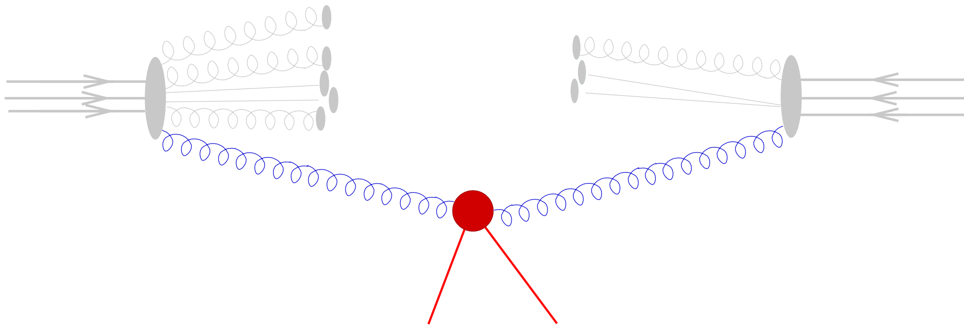

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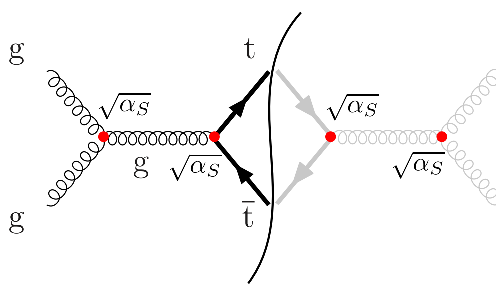

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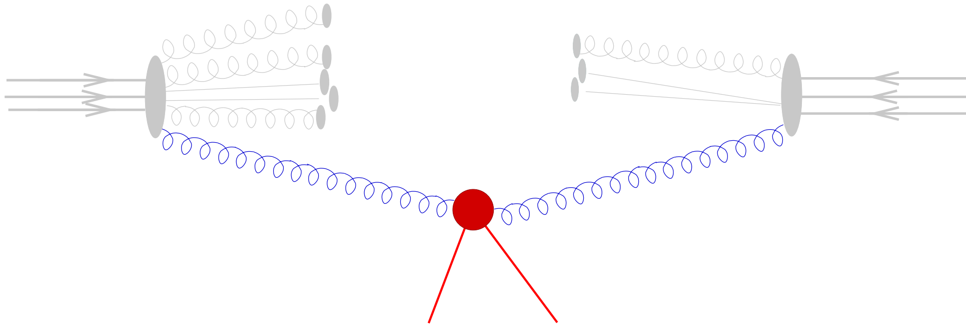

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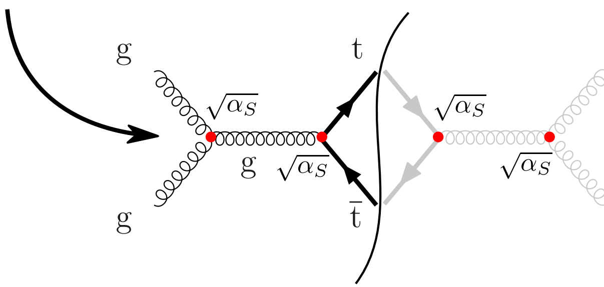
$$\hat{\sigma} = \hat{\sigma}(\alpha_S^2) + \hat{\sigma}(\alpha_W^2) + \hat{\sigma}(\alpha_S^3) + \hat{\sigma}(\alpha_S^2\alpha_W) + \hat{\sigma}(\alpha_S\alpha_W^2) + \hat{\sigma}(\alpha_W^3)$$



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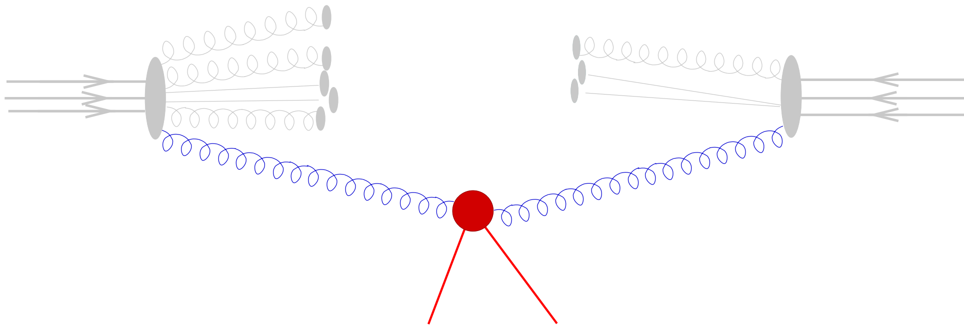

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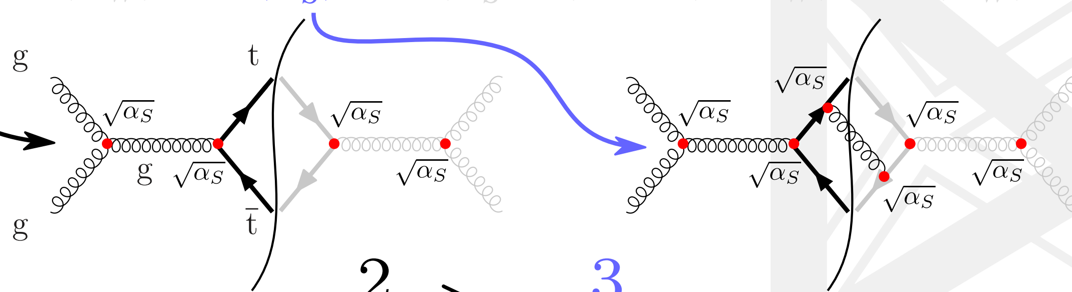

$$\alpha_S^2 > \alpha_S^3$$

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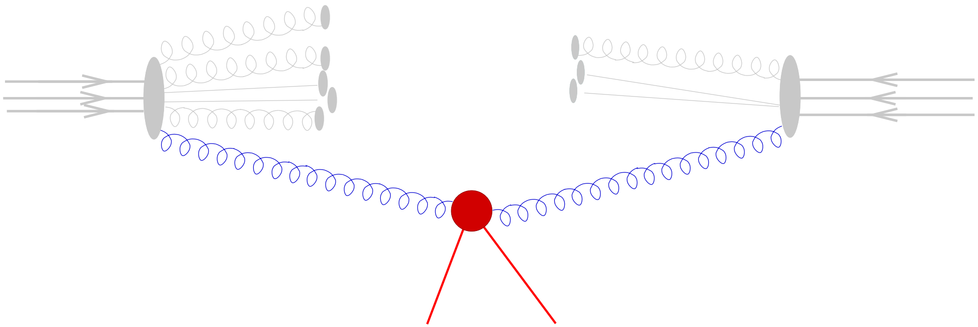

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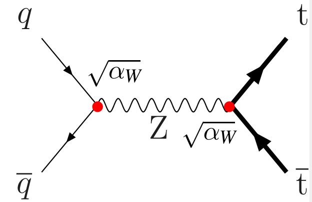

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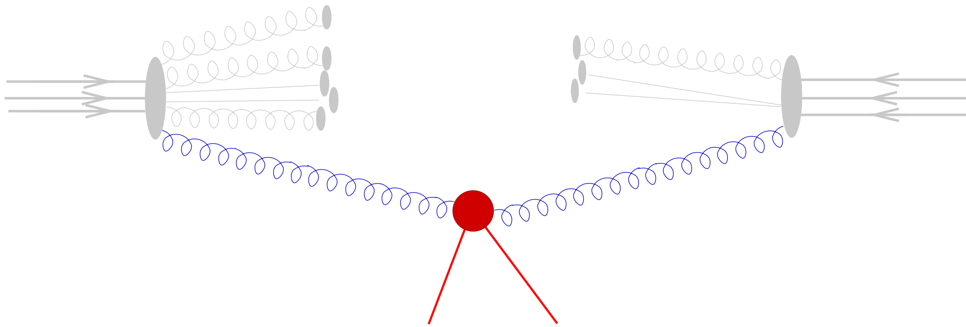

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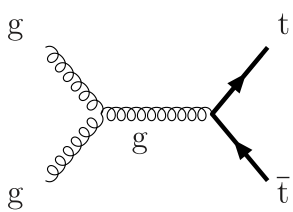
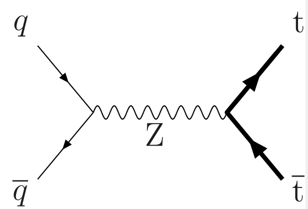


QCD



EW

Fixed order calculations

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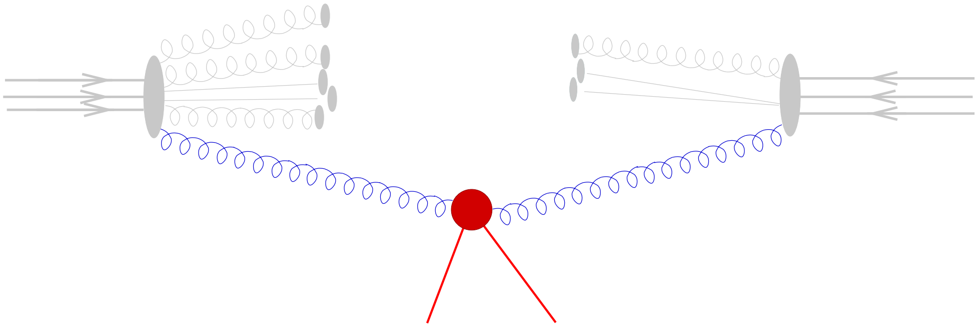

- Diagramatically

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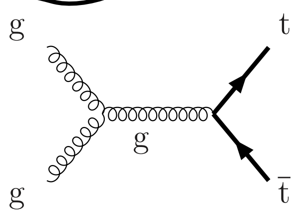

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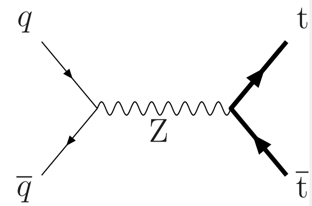

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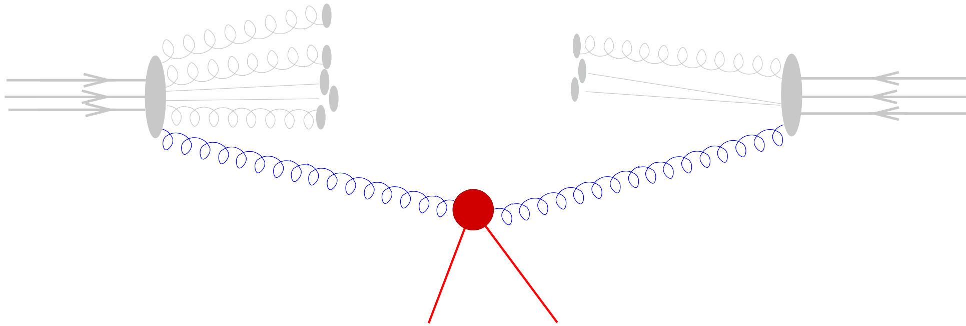

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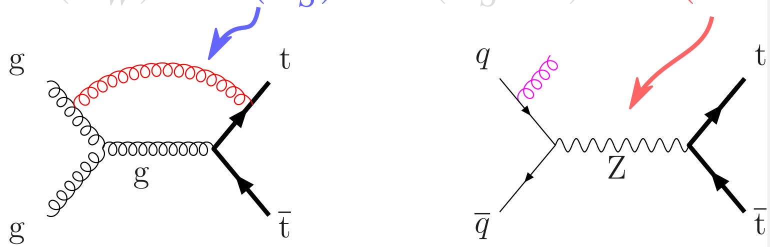

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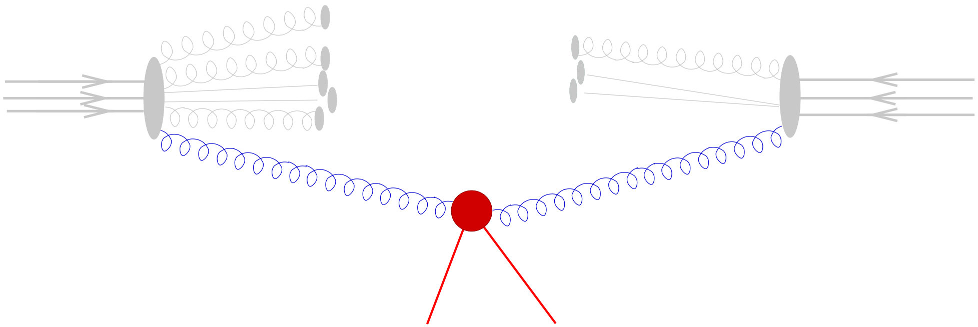

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Fixed order calculations

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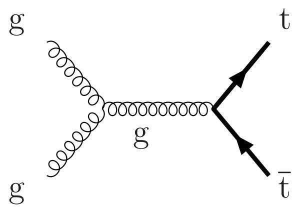
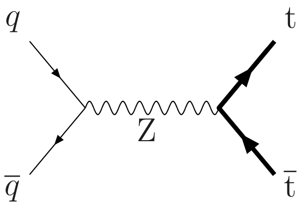


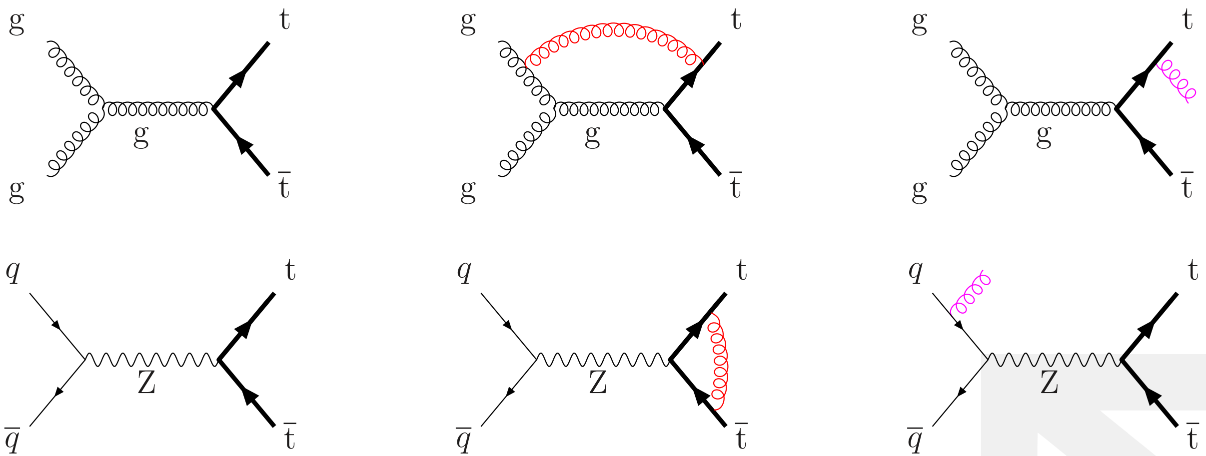


Fixed order calculations

- Take for example a production of a pair of top-quarks

Born

QCD corrections



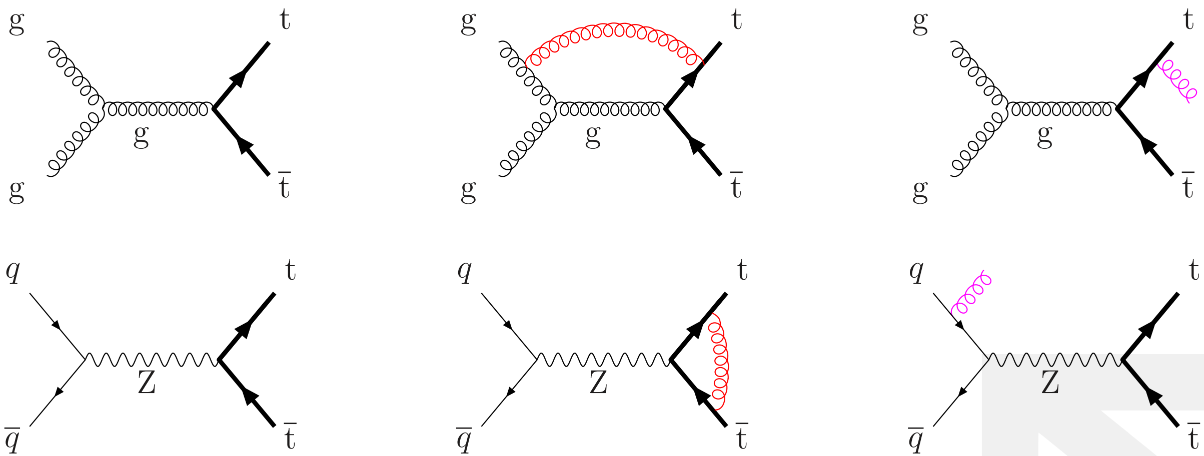
virtual

real

Fixed order calculations

- Take for example a production of a pair of top-quarks

Born virtual real



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Fixed order calculations

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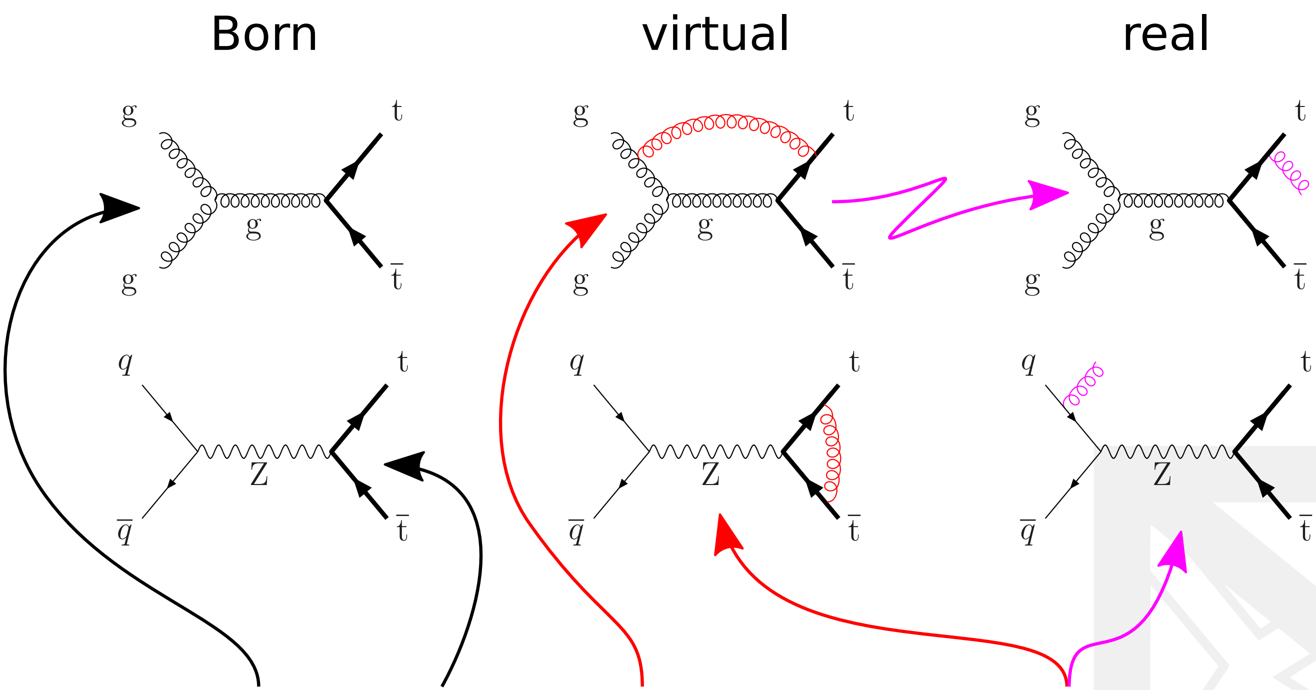

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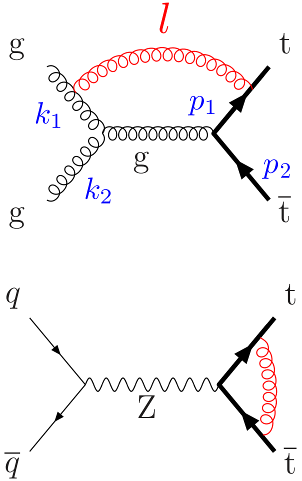
Fixed order calculations: virtual

- Take for example a production of a pair of top-quarks

virtual



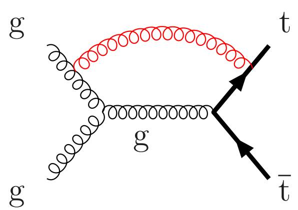
$$\int d\mathbf{\Phi}_2\, \mathcal{V}(\mathbf{\Phi}_2) \quad \mathbf{\Phi}_2 = \{k_1, k_2, p_1, p_2\}$$

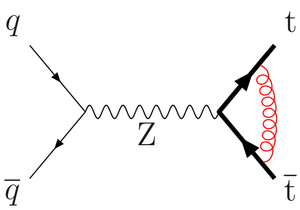
$$\mathcal{V}(\mathbf{\Phi}_2) = \int d^4 l \cdots$$

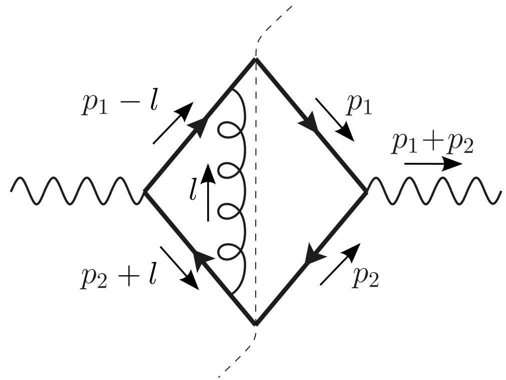
Fixed order calculations: virtual

- Take for example a production of a pair of top-quarks

virtual







$$I^{\mathrm{f}} = \int \frac{d^4 l}{(2\pi)^2} \frac{f^{\mathrm{f}}(l, p_1, p_2)}{l^2[(p_1 - l)^2 + m_{\mathrm{t}}^2][(p_2 + l)^2 + m_{\mathrm{t}}^2]}$$

$$= \int \frac{d^4 l}{(2\pi)^2} \frac{f^f(l, p_1, p_2)}{l^2(l^2 - 2p_1 \cdot l)(l^2 + 2p_2 \cdot l)}$$

$$= \int \frac{d^4 l}{(2\pi)^2} \frac{1}{l^2 l^2}$$

divergent

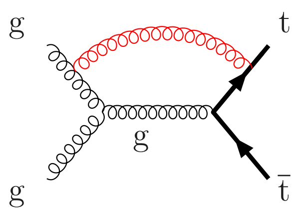
$$l \longrightarrow \infty$$

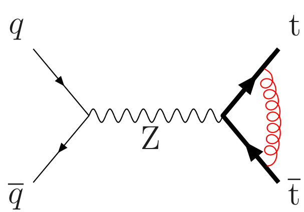
$$f^{\mathrm{f}} = l^2 - 2p_1 \cdot l$$

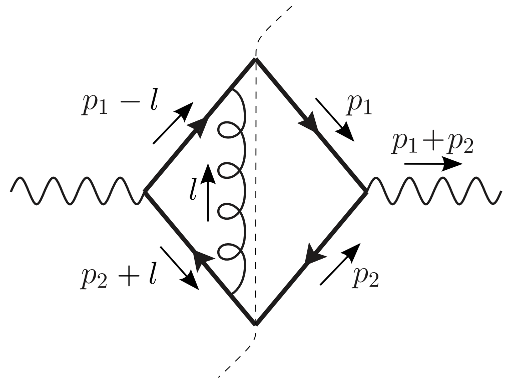
Fixed order calculations: virtual

- Take for example a production of a pair of top-quarks

virtual







$$I^{\mathrm{f}} = \int \frac{d^4 l}{(2\pi)^2} \frac{f^{\mathrm{f}}(l, p_1, p_2)}{l^2[(p_1 - l)^2 + m_{\mathrm{t}}^2][(p_2 + l)^2 + m_{\mathrm{t}}^2]}$$

$$= \int \frac{d^4 l}{(2\pi)^2} \frac{f^{f}(l, p_1, p_2)}{l^2(l^2 - 2p_1 \cdot l)(l^2 + 2p_2 \cdot l)}$$

$$= \int \frac{d^4 l}{(2\pi)^2} \frac{1}{l^2 l^2}$$

UV divergent

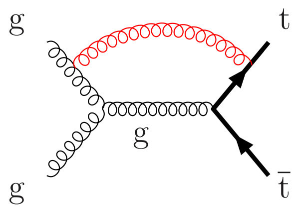
$l \to \infty$

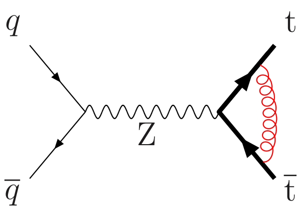
$$f^{\mathrm{f}} = l^2 - 2p_1 \cdot l$$

Fixed order calculations: virtual

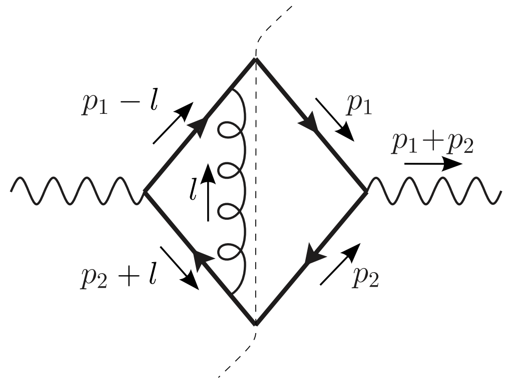
- Take for example a production of a pair of top-quarks

virtual





$l \to 0$



$$I^{\mathrm{f}} = \int \frac{d^4 l}{(2\pi)^2} \frac{f^{\mathrm{f}}(l, p_1, p_2)}{l^2[(p_1 - l)^2 + m_{\mathrm{t}}^2][(p_2 + l)^2 + m_{\mathrm{t}}^2]}$$

$$= \int \frac{d^4 l}{(2\pi)^2} \frac{f^{f}(l, p_1, p_2)}{l^2(l^2 - 2p_1 \cdot l)(l^2 + 2p_2 \cdot l)}$$

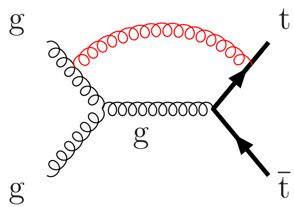
$$= \int -\frac{d^4 l}{(2\pi)^2} \frac{1}{l^2 2p_1 \cdot l 2p_2 \cdot l}$$

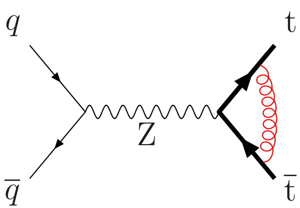
divergent

Fixed order calculations: virtual

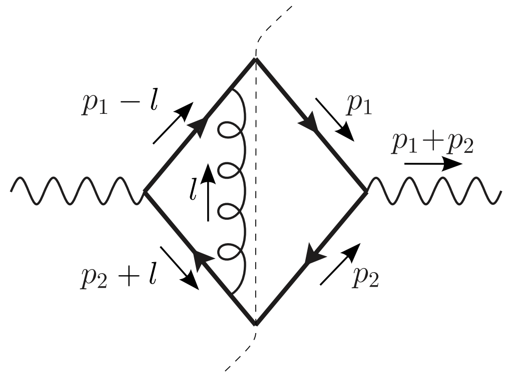
- Take for example a production of a pair of top-quarks

virtual





$l \to 0$



$$I^{\mathrm{f}} = \int \frac{d^4 l}{(2\pi)^2} \frac{f^{\mathrm{f}}(l, p_1, p_2)}{l^2[(p_1 - l)^2 + m_{\mathrm{t}}^2][(p_2 + l)^2 + m_{\mathrm{t}}^2]}$$

$$= \int \frac{d^4 l}{(2\pi)^2} \frac{f^{f}(l, p_1, p_2)}{l^2(l^2 - 2p_1 \cdot l)(l^2 + 2p_2 \cdot l)}$$

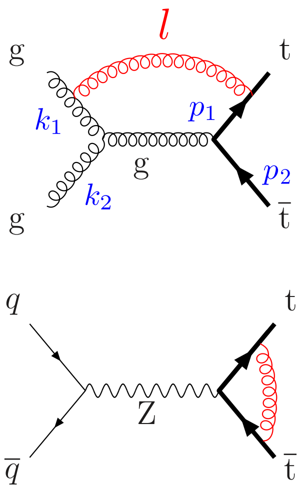
$$= \int -\frac{d^4 l}{(2\pi)^2} \frac{1}{l^2 2p_1 \cdot l 2p_2 \cdot l}$$

IR divergent

Fixed order calculations: virtual

- Take for example a production of a pair of top-quarks

virtual



$$\int d\boldsymbol{\Phi}_2 \mathcal{V}(\boldsymbol{\Phi}_2) \quad \boldsymbol{\Phi}_2 = \{k_1, k_2, p_1, p_2\}$$

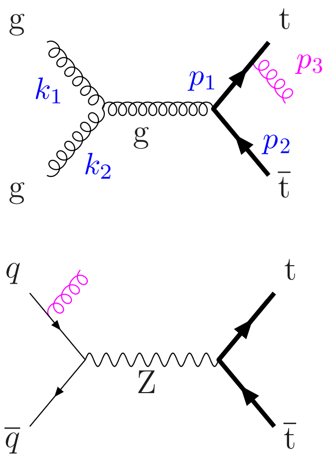
$$\mathcal{V}(\boldsymbol{\Phi}_2) = \int d^4 l \cdots$$

UV
IR
divergent

Fixed order calculations: real

- Take for example a production of a pair of top-quarks

real

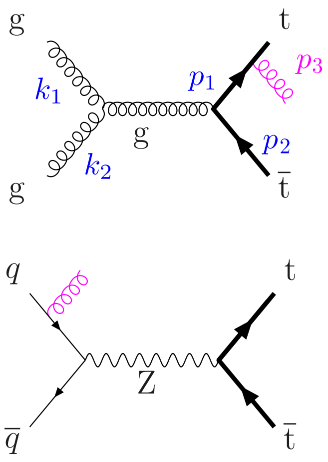


$$\int d\boldsymbol{\Phi}_3 \mathcal{R}(\boldsymbol{\Phi}_3) \quad \boldsymbol{\Phi}_3 = \{k_1, k_2, p_1, p_2, p_3\}$$

Fixed order calculations: real

- Take for example a production of a pair of top-quarks

real



$$\int d\mathbf{\Phi}_3 \mathcal{R}(\mathbf{\Phi}_3) \quad \mathbf{\Phi}_3 = \{k_1, k_2, p_1, p_2, p_3\}$$

IR divergent

Fixed order calculations: NLO

- Total cross section for a $2 \to n$ scattering at NLO

$$\sigma_{\mathbf{NLO}} = \int d\mathbf{\Phi}_n \Big[\mathcal{B}(\mathbf{\Phi}_n) + \mathcal{V}(\mathbf{\Phi}_n)\Big] + \int d\mathbf{\Phi}_{n+1}\,\mathcal{R}(\mathbf{\Phi}_{n+1})$$

Separately infinite

Fixed order calculations: NLO

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$$\sigma_{\mathbf{NLO}} = \int d\boldsymbol{\Phi}_n \Big[\mathcal{B}(\boldsymbol{\Phi}_n) + \mathcal{V}(\boldsymbol{\Phi}_n) \Big] + \int d\boldsymbol{\Phi}_{n+1} \mathcal{R}(\boldsymbol{\Phi}_{n+1})$$

- UV divergences renormalized away $\mathcal{V} \to \mathcal{V}_{\mathbf{b}}$; IR divergences cancel in the sum of real and virtual contributions

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- UV divergences renormalized away $\mathcal{V} \to \mathcal{V}_{\mathrm{b}}$; IR divergences cancel in the sum of real and virtual contributions

- In a general subtraction framework

$$\sigma_{\mathrm{NLO}} = \int d\Phi_n \Big\{\mathcal{B}(\Phi_n) + \mathcal{V}_{\mathrm{b}}(\Phi_n) + \sum_{\alpha} \Big[\bar{\mathcal{C}}\,(\Phi_n)\Big]_{\alpha}\Big\}$$
$$+ \int d\Phi_{n+1} \Big\{\mathcal{R}(\Phi_{n+1}) - \sum_{\alpha} \Big[\mathcal{C}(\Phi_{n+1})\Big]_{\alpha}\Big\}$$

- $\Big[\mathcal{C}(\Phi_{n+1})\Big]_{\alpha}$: real CTs; $\Big[\bar{\mathcal{C}}\,(\Phi_n)\Big]_{\alpha}$: integrated CTs
- α labels singular regions

Fixed order calculations: NLO

- Total cross section for a $2 \to n$ scattering at NLO

$$\sigma_{\mathbf{NLO}} = \int d\boldsymbol{\Phi}_n \Big[\mathcal{B}(\boldsymbol{\Phi}_n) + \mathcal{V}(\boldsymbol{\Phi}_n)\Big] + \int d\boldsymbol{\Phi}_{n+1} \mathcal{R}(\boldsymbol{\Phi}_{n+1})$$

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- In a general subtraction framework

$$\sigma_{\mathbf{NLO}} = \int d\boldsymbol{\Phi}_n \Big\{\mathcal{B}(\boldsymbol{\Phi}_n) + \mathcal{V}_{\mathbf{b}}(\boldsymbol{\Phi}_n) + \sum_{\alpha} \Big[\bar{\mathcal{C}}\left(\boldsymbol{\Phi}_n\right)\Big]_{\alpha} \Big\} + \int d\boldsymbol{\Phi}_{n+1} \Big\{\mathcal{R}(\boldsymbol{\Phi}_{n+1}) - \sum_{\alpha} \Big[\mathcal{C}(\boldsymbol{\Phi}_{n+1})\Big]_{\alpha} \Big\}$$

- $\Big[\mathcal{C}(\boldsymbol{\Phi}_{n+1})\Big]_{\alpha}$: real CTs; $\Big[\bar{\mathcal{C}}\left(\boldsymbol{\Phi}_n\right)\Big]_{\alpha}$: integrated CTs
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Fixed order calculations: NLO

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- UV divergences renormalized away $\mathcal{V} \to \mathcal{V}_{\mathrm{b}}$; IR divergences cancel in the sum of real and virtual contributions

- In the FKS subtraction method

$$\sigma_{\mathrm{NLO}} = \int d\mathbf{\Phi}_n \Big\{\mathcal{B}(\mathbf{\Phi}_n) + \hat{\mathcal{V}}(\mathbf{\Phi}_n)\Big\} + \int d\mathbf{\Phi}_{n+1} \hat{\mathcal{R}}(\mathbf{\Phi}_{n+1})$$

$$\hat{\mathcal{R}} \equiv \frac{1}{\xi} \left\{ \left(\frac{1}{\xi}\right)_+ \left(\frac{1}{1-y}\right)_+ \left[\xi^2 (1-y)\, \mathcal{R}\right] \right\}$$

$$\hat{\mathcal{V}} = \frac{\alpha_{\mathrm{S}}}{2\pi} \left(\mathcal{Q}\,\mathcal{B} + \sum_{\substack{i,j \in \mathcal{I} \\ i \neq j}} \mathcal{I}_{ij}\, \mathcal{B}_{ij} + \mathcal{V}_{\mathrm{fin}} \right)$$

Fixed order calculations: NLO

- In the FKS subtraction method

$$\sigma_{\mathbf{NLO}} = \int d\mathbf{\Phi}_n \Big\{ \mathcal{B}(\mathbf{\Phi}_n) + \hat{\mathcal{V}}(\mathbf{\Phi}_n) \Big\} + \int d\mathbf{\Phi}_{n+1} \hat{\mathcal{R}}(\mathbf{\Phi}_{n+1})$$

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Fixed order calculations: NLO

- In the FKS subtraction method

$$\sigma_{\mathbf{NLO}} = \int d\mathbf{\Phi}_n \Big\{ \mathcal{B}(\mathbf{\Phi}_n) + \hat{\mathcal{V}}(\mathbf{\Phi}_n) \Big\} + \int d\mathbf{\Phi}_{n+1} \hat{\mathcal{R}}(\mathbf{\Phi}_{n+1})$$

variables proper to one singular region

$$\hat{\mathcal{R}} \equiv \frac{1}{\xi} \left\{ \left(\frac{1}{\xi}\right)_+ \left(\frac{1}{1-y}\right)_+ \left[\xi^2 (1-y)\, \mathcal{R}\right] \right\}$$

sum of real contribution and real counterterms
in a given singular region

$$\hat{\mathcal{V}} = \frac{\alpha_{\mathrm{S}}}{2\pi} \left(\mathcal{Q}\,\mathcal{B} + \sum_{\substack{i,j \in \mathcal{I} \\ i \neq j}} \mathcal{I}_{ij}\, \mathcal{B}_{ij} + \mathcal{V}_{\mathbf{fin}} \right)$$

Fixed order calculations: NLO

- In the FKS subtraction method

$$\sigma_{\mathbf{NLO}} = \int d\mathbf{\Phi}_n \Big\{ \mathcal{B}(\mathbf{\Phi}_n) + \hat{\mathcal{V}}(\mathbf{\Phi}_n) \Big\} + \int d\mathbf{\Phi}_{n+1} \hat{\mathcal{R}}(\mathbf{\Phi}_{n+1})$$

- sum of real contribution and real counterterms in a given singular region

$$\hat{\mathcal{R}} \equiv \frac{1}{\xi} \left\{ \left(\frac{1}{\xi} \right)_+ \left(\frac{1}{1-y} \right)_+ \left[\xi^2 (1-y) \mathcal{R} \right] \right\}$$

$$\hat{\mathcal{V}} = \frac{\alpha_{\mathrm{S}}}{2\pi} \left(\mathcal{Q}\,\mathcal{B} + \sum_{\substack{i,j \in \mathcal{I} \\ i \neq j}} \mathcal{I}_{ij}\,\mathcal{B}_{ij} + \mathcal{V}_{\mathbf{fin}} \right)$$

Fixed order calculations: NLO

- In the FKS subtraction method

$$\sigma_{\mathbf{NLO}} = \int d\mathbf{\Phi}_n \Big\{ \mathcal{B}(\mathbf{\Phi}_n) + \hat{\mathcal{V}}(\mathbf{\Phi}_n) \Big\} + \int d\mathbf{\Phi}_{n+1} \hat{\mathcal{R}}(\mathbf{\Phi}_{n+1})$$

- sum of real contribution and real counterterms in a given singular region

$$\hat{\mathcal{R}} \equiv \frac{1}{\xi} \left\{ \left(\frac{1}{\xi} \right)_+ \left(\frac{1}{1-y} \right)_+ \left[\xi^2 (1-y) \mathcal{R} \right] \right\}$$

sum of virtual contribution and integrated counterterms

$$\hat{\mathcal{V}} = \frac{\alpha_{\mathrm{S}}}{2\pi} \left(\mathcal{Q}\mathcal{B} + \sum_{\substack{i,j \in \mathcal{I} \\ i \neq j}} \mathcal{I}_{ij} \mathcal{B}_{ij} + \mathcal{V}_{\mathbf{fin}} \right)$$

Fixed order calculations: NLO

- In the FKS subtraction method

$$\sigma_{\mathrm{NLO}} = \int d\mathbf{\Phi}_n \Big\{ \mathcal{B}(\mathbf{\Phi}_n) + \hat{\mathcal{V}}(\mathbf{\Phi}_n) \Big\} + \int d\mathbf{\Phi}_{n+1} \hat{\mathcal{R}}(\mathbf{\Phi}_{n+1})$$

- sum of real contribution and real counterterms in a given singular region

$$\hat{\mathcal{R}} \equiv \frac{1}{\xi} \left\{ \left(\frac{1}{\xi} \right)_+ \left(\frac{1}{1-y} \right)_+ \left[\xi^2 (1-y) \, \mathcal{R} \right] \right\}$$

- sum of virtual contribution and integrated counterterms

process independent terms

$$\hat{\mathcal{V}} = \frac{\alpha_{\mathrm{S}}}{2\pi} \left(\mathcal{Q} \, \mathcal{B} + \sum_{\substack{i,j \in \mathcal{I} \\ i \neq j}} \mathcal{I}_{ij} \, \mathcal{B}_{ij} + \mathcal{V}_{\mathrm{fin}} \right)$$

Singular regions

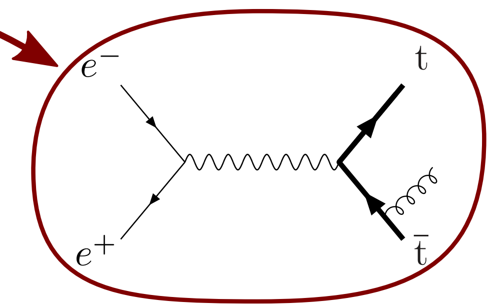
- In the formula

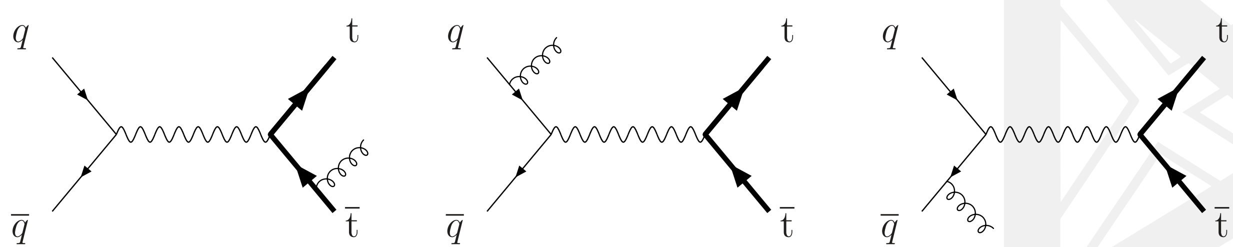
$$\hat{\mathcal{R}} \equiv \frac{1}{\xi}\left\{\left(\frac{1}{\xi}\right)_+ \left(\frac{1}{1-y}\right)_+ \left[\xi^2(1-y)\,\mathcal{R}\right]\right\}$$

ξ and y are proper to a given singular kinematic configuration

- Example: real contribution to top pair production

1 singular region





Singular regions

- In the formula

$$\hat{\mathcal{R}} \equiv \frac{1}{\xi}\left\{\left(\frac{1}{\xi}\right)_+ \left(\frac{1}{1-y}\right)_+ \left[\xi^2 (1-y)\,\mathcal{R}\right]\right\}$$

ξ and y are proper to a given singular kinematic configuration

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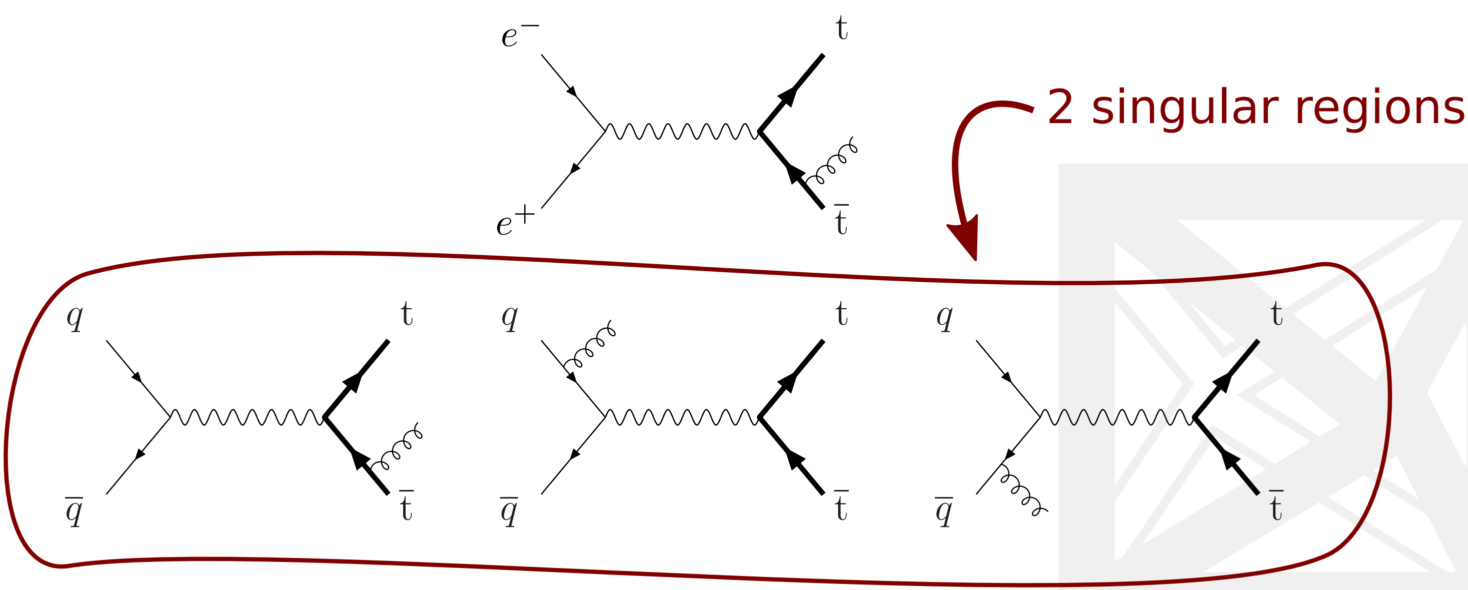
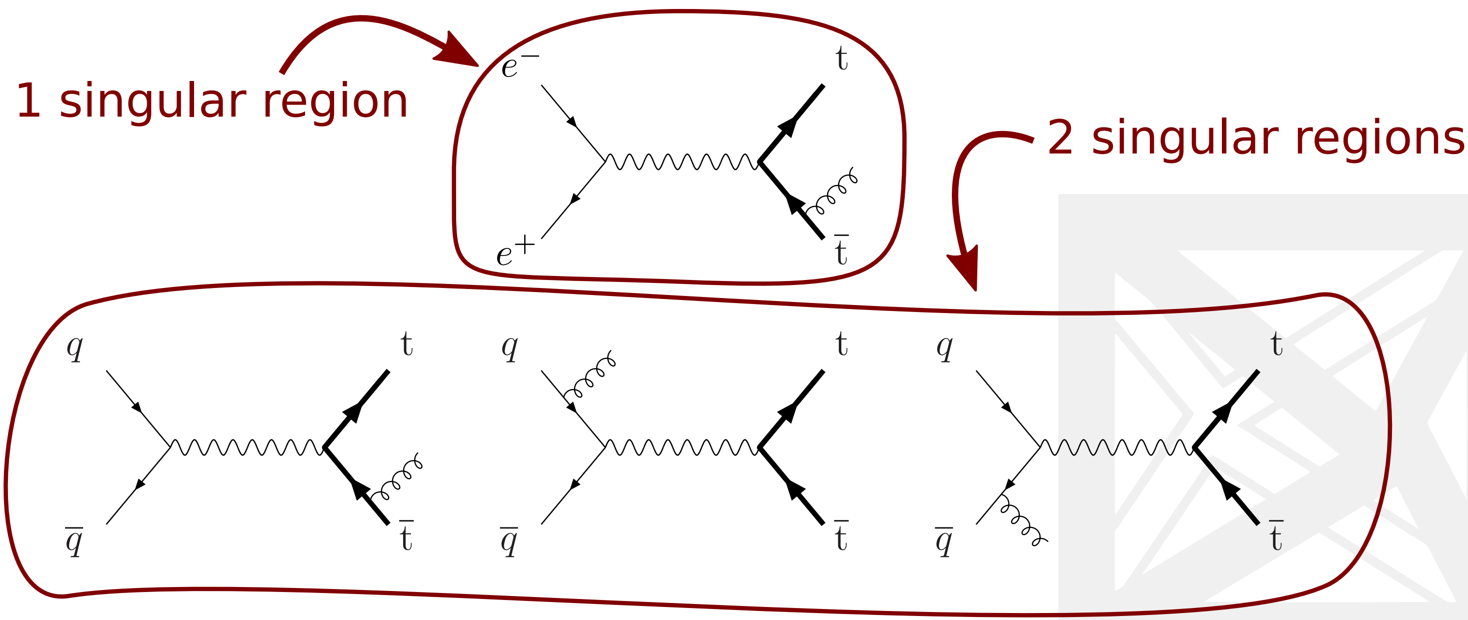


Singular regions

- Singular region: region of phase space with only one soft and/or collinear singularity
- Example: real contribution to top pair production



Singular regions

- In the formula

$$\hat{\mathcal{R}} \equiv \frac{1}{\xi}\left\{\left(\frac{1}{\xi}\right)_+ \left(\frac{1}{1-y}\right)_+ \left[\xi^2(1-y)\,\mathcal{R}\right]\right\}$$

ξ and y are proper to a given singular kinematic configuration

- Multiple singular configurations: real corrections split up into singular regions and integrated in each region separately

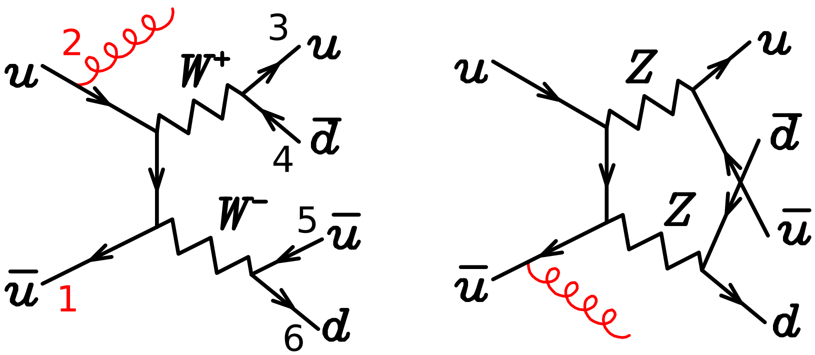
- This can be achieved by multiplying real corrections by

$$\lim_{k_m^0 \to 0}\left(\mathcal{S}_i + \sum_j \mathcal{S}_{ij}\right) = \delta_{im}\,, \quad \lim_{\vec{k}_m \| \vec{k}_l}\left(\mathcal{S}_{ij} + \mathcal{S}_{ji}\right) = \delta_{im}\delta_{jl} + \delta_{il}\delta_{jm}\,, \quad \cdots$$

$$\mathcal{R} = \sum_i \mathcal{S}_i \mathcal{R} + \sum_{ij} \mathcal{S}_{ij} \mathcal{R}$$

Singular regions

- Example: electroweak $u\bar{u} \to u\bar{d}\bar{u}d$

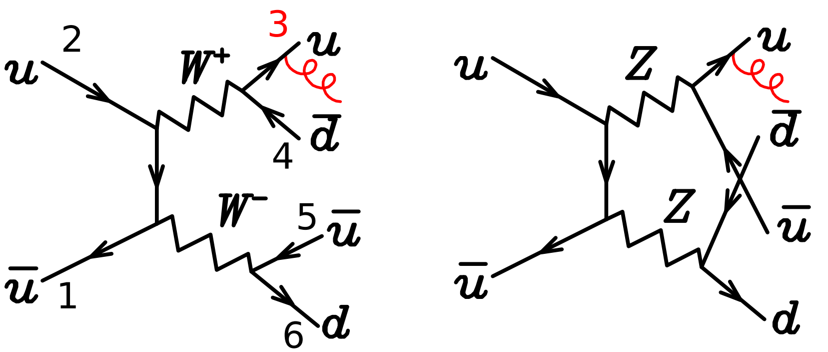


  - singular regions

α_r	emitter	$d^{-1}(\alpha_r)$
1	0	d_7^{-1}

Singular regions

- Example: electroweak $u\bar{u} \to u\bar{d}\bar{u}d$

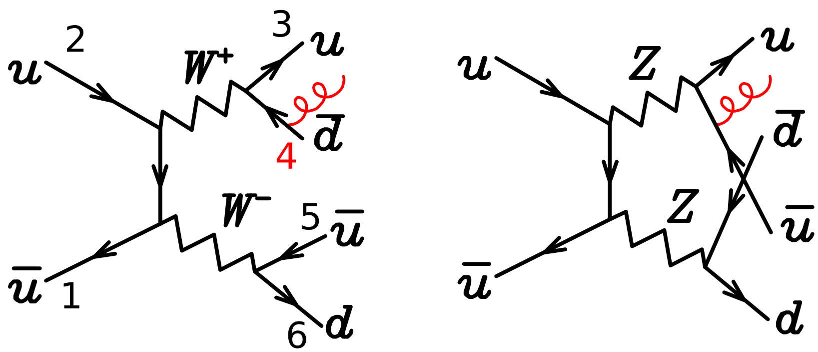


  - singular regions

α_r	emitter	$d^{-1}(\alpha_r)$
1	0	d_7^{-1}
2	3	d_{37}^{-1}

Singular regions

- Example: electroweak $u\bar{u} \to u\bar{d}\bar{u}d$

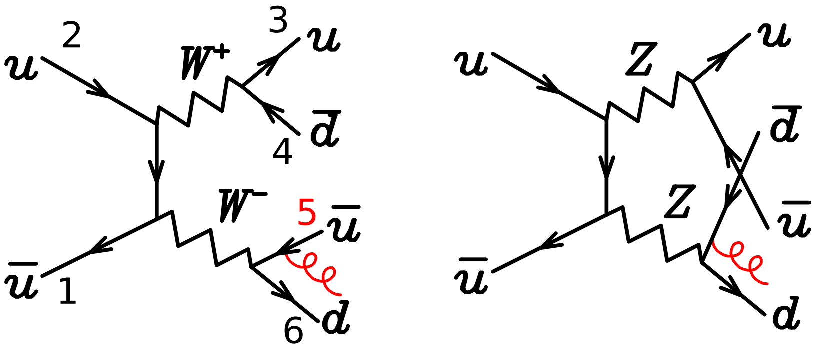


  - singular regions

α_r	emitter	$d^{-1}(\alpha_r)$
1	0	d_7^{-1}
2	3	d_{37}^{-1}
3	4	d_{47}^{-1}

Singular regions

- Example: electroweak $u\bar{u} \to u\bar{d}\bar{u}d$

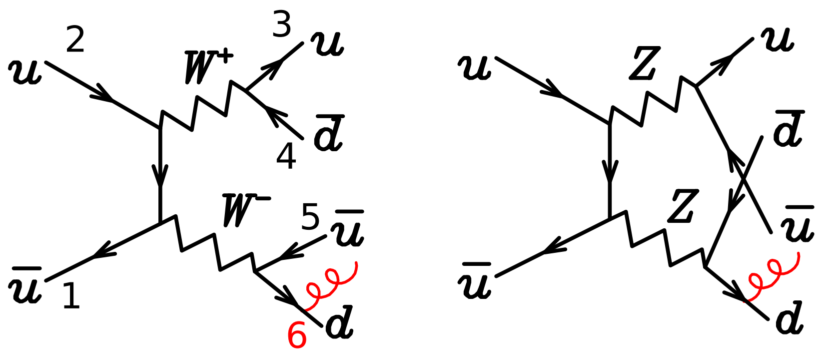


  - singular regions

α_r	emitter	$d^{-1}(\alpha_r)$
1	0	d_7^{-1}
2	3	d_{37}^{-1}
3	4	d_{47}^{-1}
4	5	d_{57}^{-1}

Singular regions

- Example: electroweak $u\bar{u} \to u\bar{d}\bar{u}d$

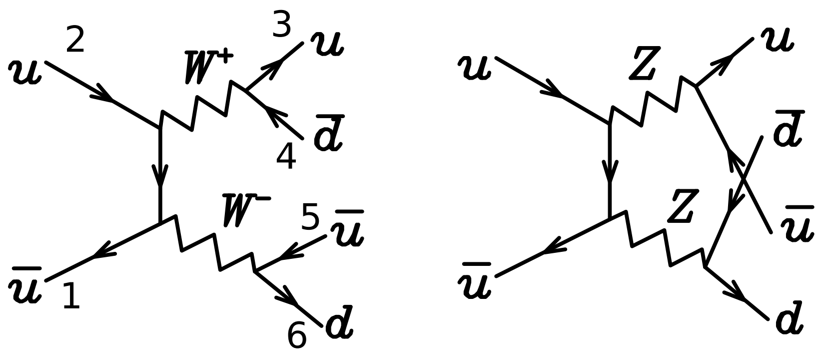


  - singular regions

α_r	emitter	$d^{-1}(\alpha_r)$
1	0	d_7^{-1}
2	3	d_{37}^{-1}
3	4	d_{47}^{-1}
4	5	d_{57}^{-1}
5	6	d_{67}^{-1}

Singular regions

- Example: electroweak $u\bar{u} \to u\bar{d}\bar{u}d$



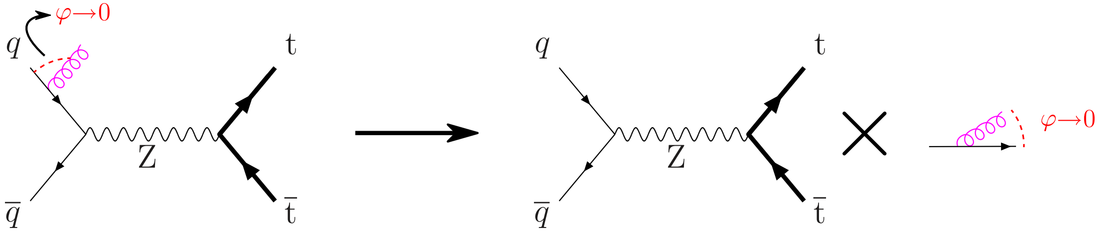
  - singular regions

α_r	emitter	$d^{-1}(\alpha_r)$
1	0	d_7^{-1}
2	3	d_{37}^{-1}
3	4	d_{47}^{-1}
4	5	d_{57}^{-1}
5	6	d_{67}^{-1}

$$\mathcal{R}_{\alpha_r=2} = \frac{d_{37}^{-1}}{(d_7^{-1} + \cdots + d_{67}^{-1})}\,\mathcal{R}$$

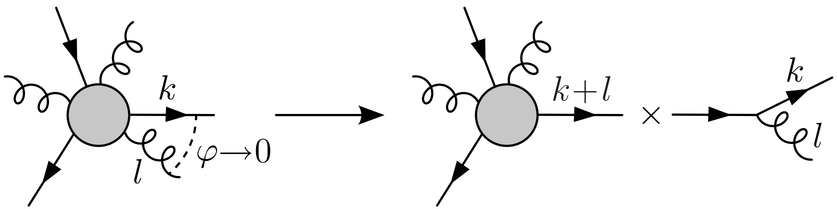
Collinear approximation

- In the collinear limit:



Collinear approximation

- In the collinear limit:



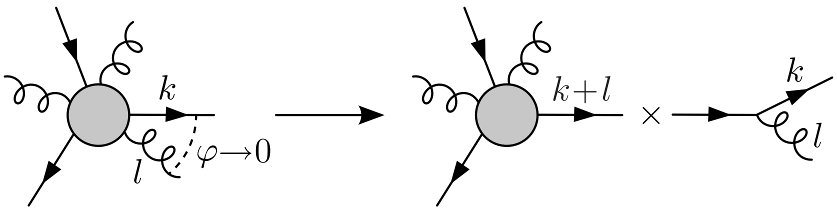
$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \to |\mathcal{M}_n|^2 d\Phi_n \times \frac{\alpha_S}{2\pi}\frac{dt}{t}P_{q,qg}(z)\,dz\,\frac{d\phi}{2\pi}$$

  - t vanishes in the collinear limit, z momentum fraction, ϕ azimuthal angle
  - $P_{q,qg}(z)$ Altarelli-Parisi splitting for $q \to qg$

- Can be applied recursively: n splittings naively correspond to real corrections at N^{n}LO

Collinear approximation

- In the collinear limit:



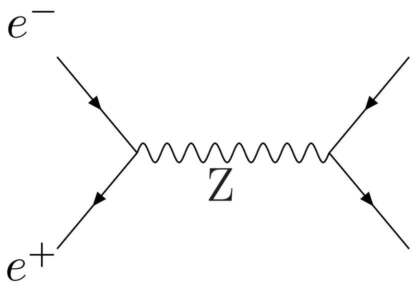
$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \to |\mathcal{M}_n|^2 d\Phi_n \times \frac{\alpha_S}{2\pi}\frac{dt}{t} P_{q,qg}(z) dz \frac{d\phi}{2\pi}$$

- Virtual contributions taken into account via Sudakov form factor

$$dP(t, t+dt) = \frac{\alpha_S}{2\pi}\frac{dt}{t}\int \frac{d\phi}{2\pi}\int P_{i,jl}(z) dz$$

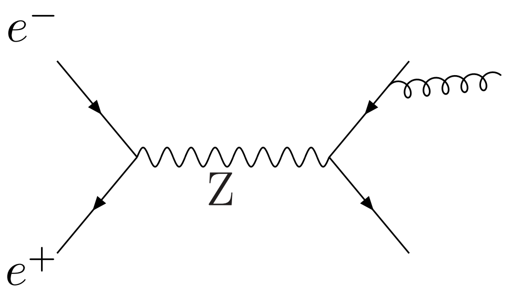
  - dP probability of $i \to jl$ splitting in $[t, t+dt]$
  - $1 - dP$ probability of no radiation equivalent to virtual contribution

Parton shower



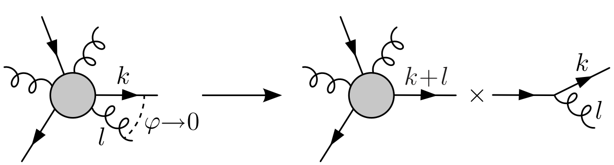
$$W = W_B$$

Parton shower



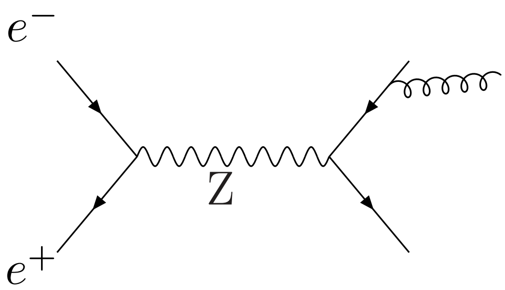
$$W = W_B \times V$$

- Real corrections in collinear approximation:



$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \to |\mathcal{M}_n|^2 d\Phi_n \times \overbrace{\frac{\alpha_S}{2\pi}\frac{dt}{t}P_{q,qg}(z)\,dz\,\frac{d\phi}{2\pi}}^{V}$$

Parton shower



$$W = W_B \times V \times \Delta$$

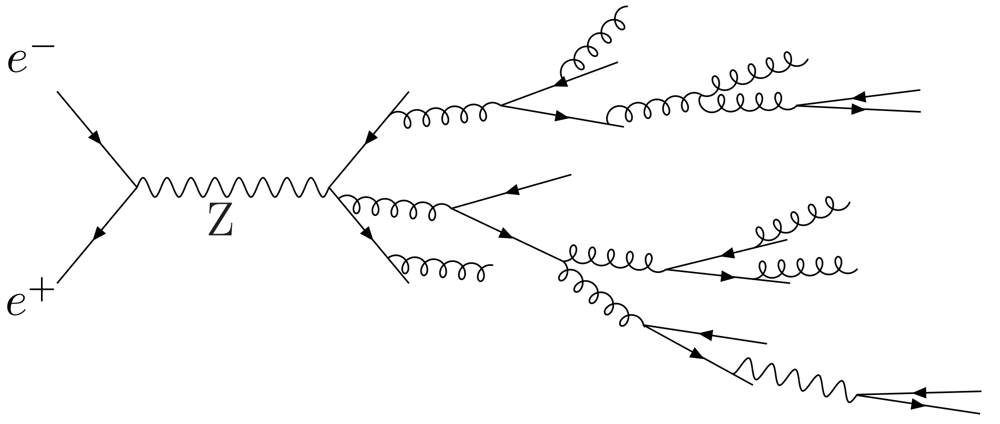
- Virtual corrections in collinear approximation:

$$\Delta \qquad dP(t, t+dt) = \frac{\alpha_S}{2\pi}\frac{dt}{t}\int\frac{d\phi}{2\pi}\int P_{i,jl}(z)dz$$

  - dP probability of $i \rightarrow jl$ splitting in $[t, t+dt]$
  - $1 - dP$ probability of no radiation equivalent to virtual contribution

Parton shower

- Parton showers can be automated



Parton shower

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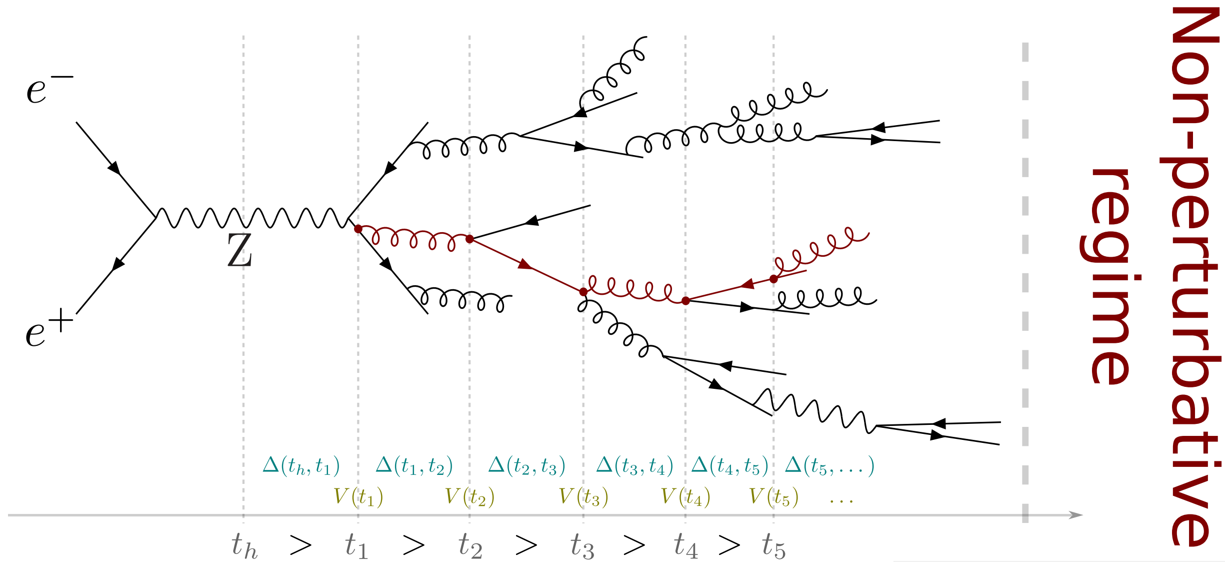


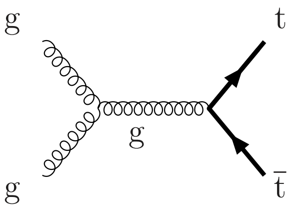
- Variable t measures hardness
  - vanishes in the collinear limit

- Weight of the event is the Born weight times the splitting and Sudakov factors

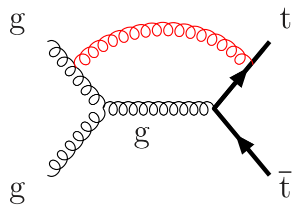
NLO & PS

- Fixed order calculation @ Next-to-Leading Order (NLO)

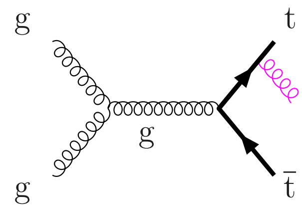
Born



virtual



real



NLO & PS

- Fixed order calculation @ Next-to-Leading Order (NLO)

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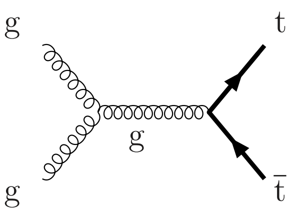
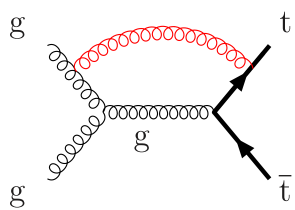
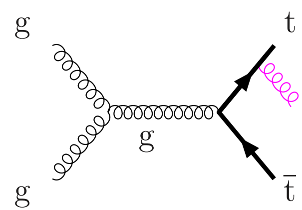


virtual

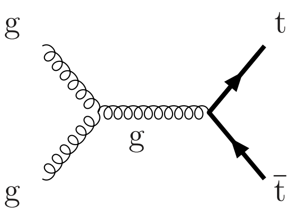


real

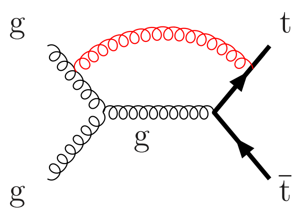


- Parton shower (PS)

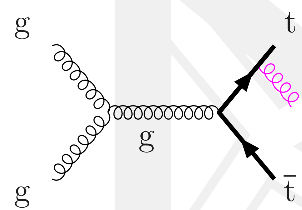
Born



Δ



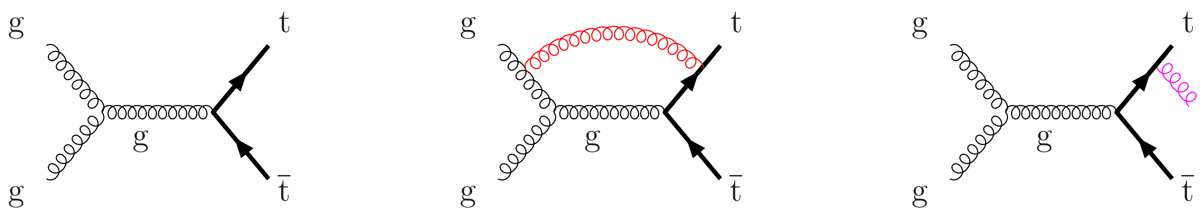
V



NLO & PS

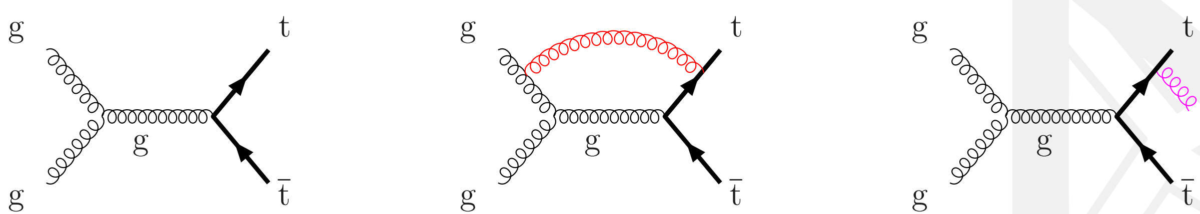
- Fixed order calculation @ Next-to-Leading Order (NLO)

Born virtual real



- Parton shower (PS)

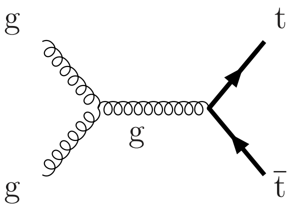
Born Δ V



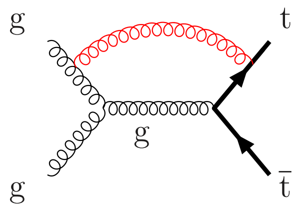
NLO+PS

- NLO merged with PS

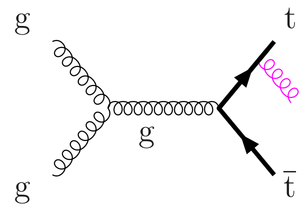
Born



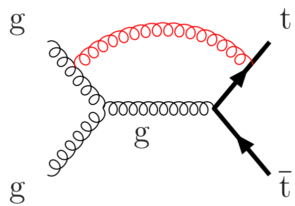
virtual



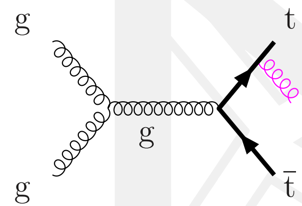
real



Δ



V



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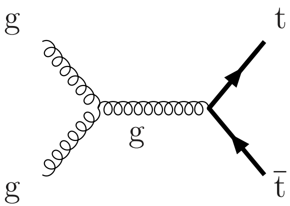
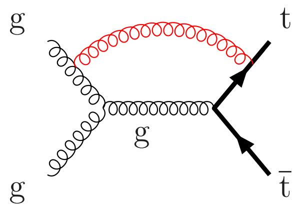
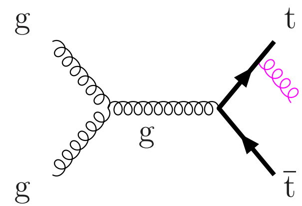


virtual

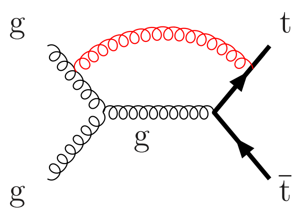


real

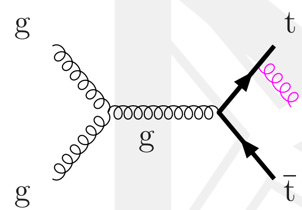


overcounted

Δ

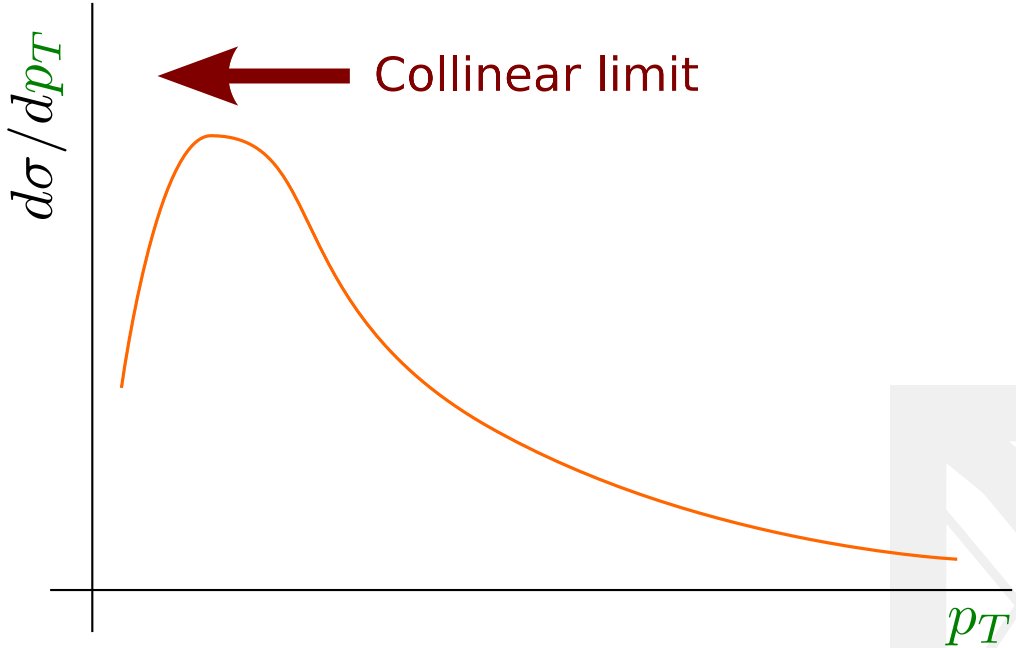


V



Parton shower

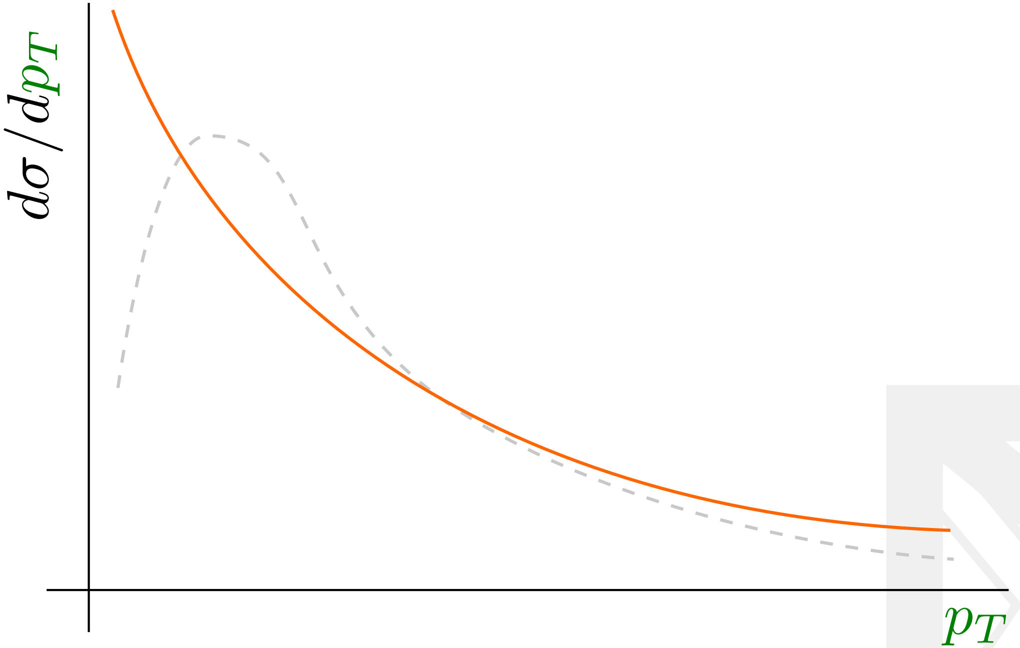
- At LO the two jets will be produced back to back, and the pair will have zero p_T
- Radiation simulated by PS will generate p_T of the pair



- As we approach $p_T \to 0$, the accuracy will reach leading log
- The accuracy in high p_T tail is limited

NLO+PS

- At NLO the high tail of the p_T of the hardest jet is exact (at a given order in FO sense)
- The low p_T tail blows up



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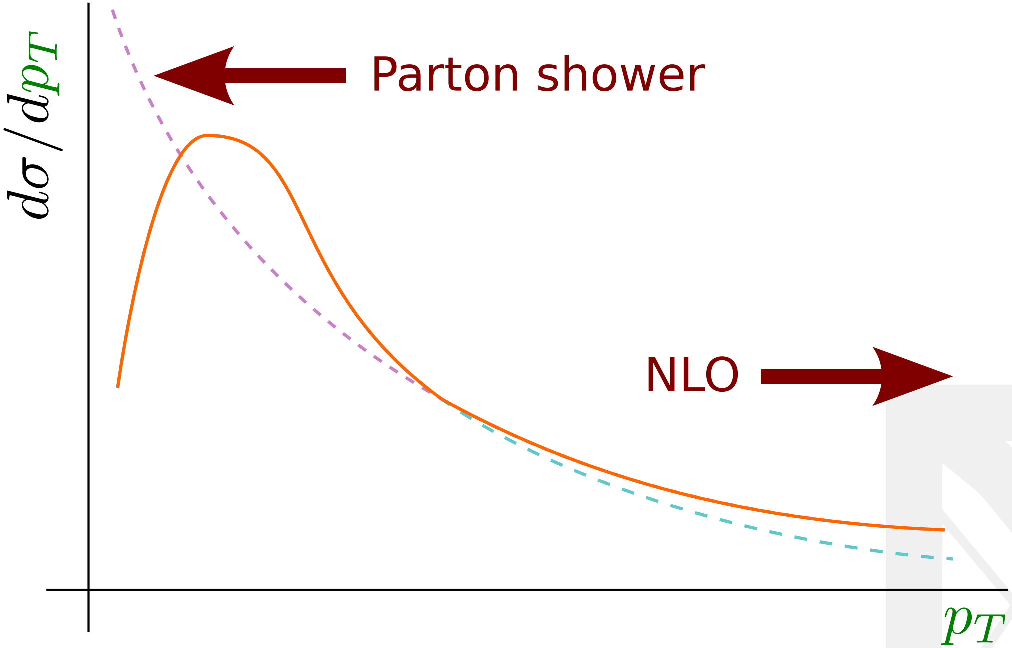

- Compromise?: NLO+PS

NLO+PS

- Naive matching of NLO and PS doesn't work: overcounting
  - Both PS and NLO contain the real and virtual contributions in the collinear limit
- Overcounting can be solved for example by modifying the Sudakov form factor for the first radiation
- Multiple solutions exist
  - Matrix Element (ME) corrections, `Pythia`
    - available only for a few processes
  - MC@NLO, `mg5_aMC`
    - procedure depends on the implementation of the PS algorithm, can lead to events with negative weights
  - POWHEG, `POWHEG-BOX`
    - independent of the PS algorithm, positive weighted events
- However some problems remain, most notably for processes mediated by narrow colored resonances

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$$\Delta(p_T^2) = 1 - dP \qquad dP(t, t + dt) = \frac{\alpha_S}{2\pi}\frac{dt}{t}\int\frac{d\phi}{2\pi}\int P_{i,jl}(z)dz$$

$$\Delta(p_T^2) = \exp\left[-\int\frac{R(\Phi_B, \Phi_{\mathrm{rad}})}{B(\Phi_B)}\theta(k_T(\Phi_{\mathrm{rad}}) - p_T)\mathrm{d}\Phi_{\mathrm{rad}}\right]$$

- Consecutive emissions can be attached using one of the common shower implementations with veto

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    - independent of the PS algorithm, positive weighted events
- However some technical problems remain

NLO+PS in POWHEG BOX

- POWHEG BOX automatically calculates everything down to the generation of the hardest emission provided the user specifies
  - Born matrix elements $\mathcal{B}(\mathbf{\Phi}_n)$
  - Renormalized virtual matrix elements $\mathcal{V}_{\mathrm{b}}(\mathbf{\Phi}_n)$
  - Real matrix elements $\mathcal{R}(\mathbf{\Phi}_{n+1})$
- Consecutive emissions can be generated by usual tools implementing PS (Pythia, Herwig, ...)

- It uses the FKS subtraction method

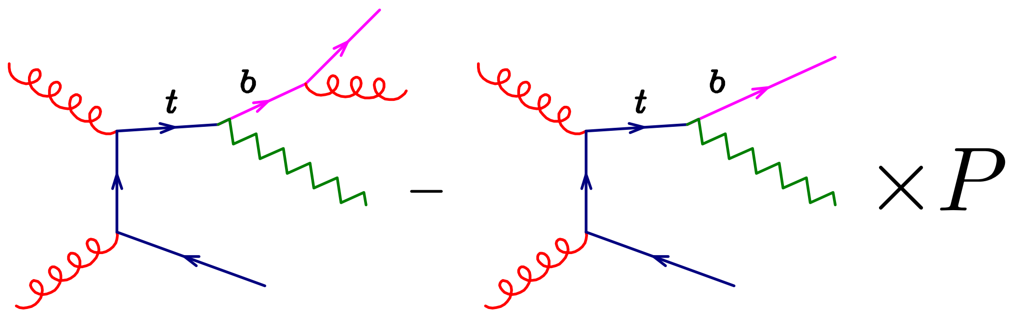
$$\sigma_{\mathrm{NLO}} = \int d\mathbf{\Phi}_n \left\{ \mathcal{B}(\mathbf{\Phi}_n) + \hat{\mathcal{V}}(\mathbf{\Phi}_n) \right\} + \int d\mathbf{\Phi}_{n+1} \hat{\mathcal{R}}(\mathbf{\Phi}_{n+1})$$

$$\hat{\mathcal{R}} \equiv \frac{1}{\xi} \left\{ \left(\frac{1}{\xi}\right)_+ \left(\frac{1}{1-y}\right)_+ \left[\xi^2 (1-y)\, \mathcal{R}\right] \right\} \qquad \hat{\mathcal{V}} = \frac{\alpha_{\mathrm{s}}}{2\pi} \left(\mathcal{Q}\,\mathcal{B} + \sum_{\substack{i,j \in \mathcal{I} \\ i \neq j}} \mathcal{I}_{ij}\, \mathcal{B}_{ij} + \mathcal{V}_{\mathrm{fin}} \right)$$

Treatment of resonances

in collaboration with P. Nason; arXiv:1509.09071

- Counterterm kinematics does not preserve the mass of the resonance - spoiling IR cancelation



- Mapping from real kinematics into underlying born kinematics does not preserve the mass of the resonance - leading to distortion of radiation observables

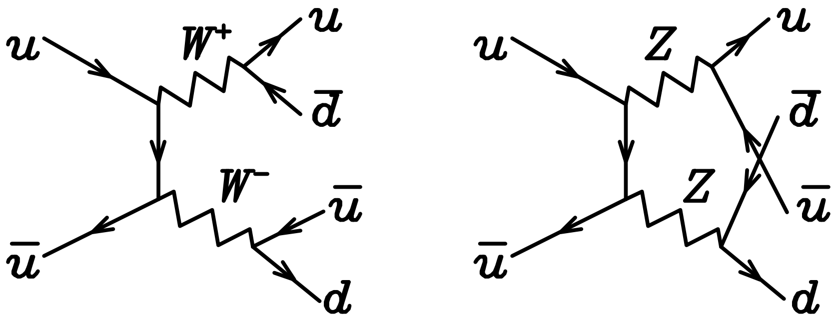
$$\Delta(p_T^2) = \exp\left[-\int \frac{R(\Phi_B,\Phi_{\rm rad})}{B(\Phi_B)}\theta(k_T(\Phi_{\rm rad}) - p_T)\mathrm{d}\Phi_{\rm rad}\right]$$

  - R/B large violating the collinear approximanion in region $m^2 \ll \Gamma E$

Treatment of resonances

in collaboration with P. Nason; arXiv:1509.09071

- Mapping from the real to born kinematics has to preserve the masses of the resonances



- All contributions needs to be split up into regions with only one dominant resonance structure

$$\mathcal{B}_1 = \frac{P^1 \mathcal{B}}{P^1 + P^2}$$

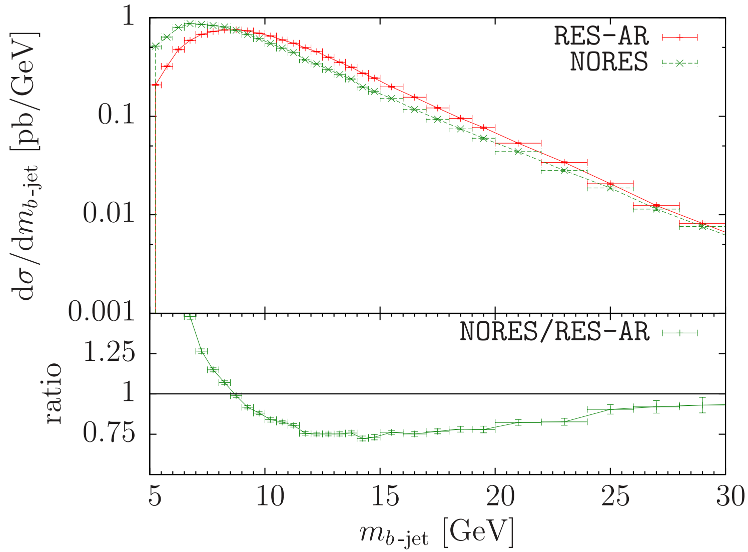
$$P^1 = \frac{M_W^4}{(s_{34} - M_W^2)^2 + \Gamma_W^2 M_W^2} \times \frac{M_W^4}{(s_{56} - M_W^2)^2 + \Gamma_W^2 M_W^2}$$

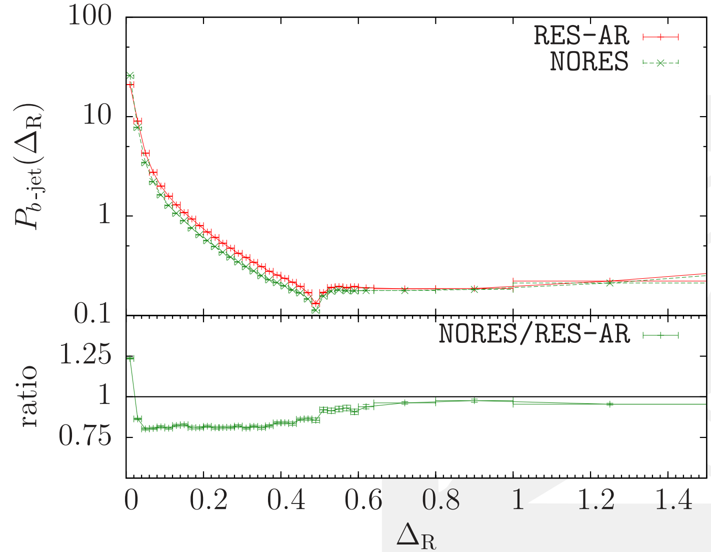
$$P^2 = \frac{M_Z^4}{(s_{35} - M_Z^2)^2 + \Gamma_Z^2 M_Z^2} \times \frac{M_Z^4}{(s_{46} - M_Z^2)^2 + \Gamma_Z^2 M_Z^2}$$

Treatment of resonances

in collaboration with P. Nason; arXiv:1509.09071

- Implement:
  - $pp \to \mu^+ \nu_\mu j_b j$ dominated by t-channel single top production at NLO QCD
  - born and real: `MadGraph4`, virtual: `MG5_aMC@NLO`
- Study impact of proper resonance treatment:
  - NORES: resonant treatment off
  - RES-AR: resonant treatment on, 1 hardest emission from resonance + 1 hardest emission from the production





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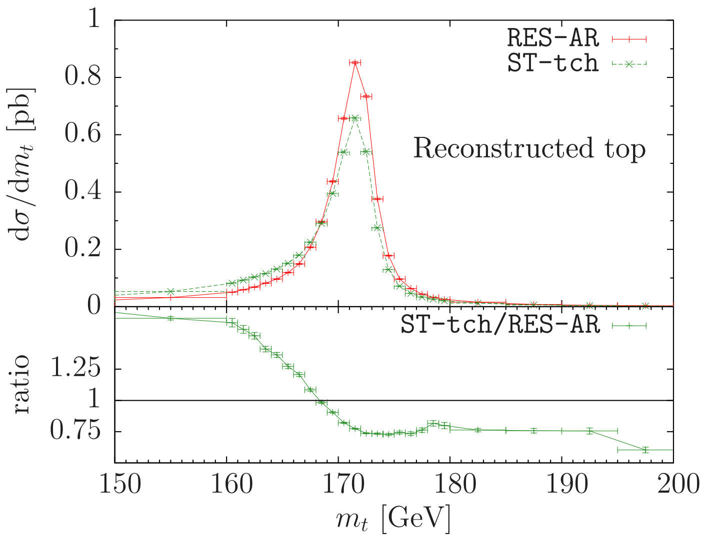
  - ▷ born and real: `MadGraph4`, virtual: `MG5_aMC@NLO`
- Study impact of proper resonance treatment:
  - ▷ ST-tch: top-pair@NLO, top decay@LO
  - ▷ RES-AR: resonant treatment on, 1 hardest emission from resonance + 1 hardest emission from the production



- ▷ average top mass in

$$m_t = 172.5 \pm 15 \text{ GeV}$$

- ▷ ST-tch: $M_{\text{trec}} = 169.59(1)$ GeV
- ▷ RES-AR: $M_{\text{trec}} = 170.55(2)$ GeV

Summary

- NLO calculations:
  - virtual and real corrections
  - appearance of UV and IR divergences
  - treatment of real divergences using subtraction
- PS:
  - collinear approximation
  - outline of the algorithm
  - Sudakov form factors
- NLO+PS:
  - what it is good for
  - overcounting and its solution
  - proper treatment of resonances important
- POWHEG BOX
  - use and cite