The JAM global QCD analysis of spin-dependent PDFs

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Overview

- PDFs in terms of helicity distributions

$$q(x, Q^2) = q^{\uparrow}(x, Q^2) + q^{\downarrow}(x, Q^2)$$

- The spin dependent PDFs (SPDF)

$$\Delta q(x, Q^2) = q^{\uparrow}(x, Q^2) - q^{\downarrow}(x, Q^2)$$

- SPDF obey the DGLAP evolution (polarized splitting functions)

$$\frac{\partial \Delta q_i}{\partial \ln \mu_f} = \sum_j \Delta P_{ij} \otimes \Delta q_j$$

- SPDFs are key to understand the spin structure of nucleons

spin carried by a quark of type q $\quad \rightarrow \quad \frac{1}{2}\Delta q^{(1)} = \frac{1}{2}\int_0^1 dx\ \Delta q(x, Q^2)$

Overview

- The spin contribution from quarks are given by

$$\Delta\Sigma^{(1)} = \Delta u^{+(1)} + \Delta d^{+(1)} + \Delta s^{+(1)}, \qquad \Delta q^{+} \equiv \Delta q + \Delta\bar{q}$$

- From existing global analysis

$$\Delta\Sigma^{[10^{-3},1]} \sim 0.3, \qquad \Delta g^{[0.05,0.2]} \sim 0.1$$

$\rightarrow$ contribution from orbital angular momentum is large.

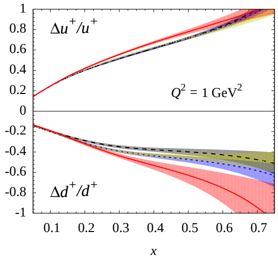
- Large x SPDF (valence) are relevant for quark models

- $SU(6)$ spin-flavor symmetry:

$$\Delta u/u \rightarrow 2/3 \qquad \Delta d/d \rightarrow -1/3$$

- pQCD:

$$\Delta q/q \rightarrow 1$$



JAM13 (Jimenez-Delgado, Accardi, Melnitchouk)

Overview

- Like the PDFs, SPDS are extracted from global analysis on spin dependent observables $\rightarrow$ Asymmetries

Polarized DIS:	$e^{\uparrow} + N^{\uparrow} \rightarrow e'^{\uparrow} + X$	$\Delta u, \Delta d$	(this talk)
Polarized SIDIS:	$e^{\uparrow} + N^{\uparrow} \rightarrow e'^{\uparrow} + h + X$	$\Delta \bar{d}, \Delta \bar{u}, \Delta s$	
Jets/π^0	$p^{\uparrow} + p^{\uparrow} \rightarrow j/\pi^0 + X$	Δg	
W production	$p^{\uparrow} + p/p^{\uparrow} \rightarrow W + X$	$\Delta \bar{d}, \Delta \bar{u}$	

- The axial charges (F, D) of weak baryon decays gives further constraints

$$\int_0^1 dx\ (\Delta u - \Delta d) = F + D = 1.269 \pm 0.003$$

$$\int_0^1 dx\ (\Delta u + \Delta d - 2\Delta s) = 3F - D = 0.586 \pm 0.031$$

Overview

- The hadronic tensor for photon exchange is given by

$$
\begin{aligned}
W_{\mu\nu} =& -\tilde{g}_{\mu\nu}F_1(x,Q^2) + \frac{\tilde{p}_\mu \tilde{p}_\nu}{p\cdot q}F_2(x,Q^2) \\
& + i\epsilon_{\mu\nu\alpha\beta}\frac{q^\alpha}{p\cdot q}\left[s^\beta g_1(x,Q^2) + \left(s^\beta - \frac{s\cdot q}{p\cdot q}p^\beta\right) g_2(x,Q^2)\right] \\
\tilde{g}_{\mu\nu} =& g_{\mu\nu} - q_\mu q_\nu / q^2 \\
\tilde{p}_\mu =& p_\mu - q_\mu p\cdot q/q^2
\end{aligned}
$$

- Polarized DIS asymmetries are described as

$$
A_{||} = \frac{\sigma^{\uparrow\Downarrow} - \sigma^{\uparrow\Uparrow}}{\sigma^{\uparrow\Downarrow} + \sigma^{\uparrow\Downarrow}} = D(A_1 + \eta A_2)
$$

$$
A_{\perp} = \frac{\sigma^{\uparrow\Rightarrow} - \sigma^{\uparrow\Leftarrow}}{\sigma^{\uparrow\Rightarrow} + \sigma^{\uparrow\Leftarrow}} = d(A_2 - \xi A_1)
$$

$$
A_1 = \frac{(g_1 - \gamma^2 g_2)}{F_1} \qquad A_2 = \gamma\frac{(g_1 + g_2)}{F_1} \qquad \gamma^2 = \frac{4M^2x^2}{Q^2}
$$

- Formal description of g_1 and g_2 can be derived using operator product expansion.

Overview

- For large Q^2 values the asymmetries can be described in terms of *leading-twist* g_1^{LT} (g_2 is suppressed by kinematics)

$$g_1^{LT}(x) = \frac{1}{2}\sum_q e_q^2 \left[\Delta C_{qq} \otimes \Delta q(x) + \Delta C_{qg} \otimes \Delta g(x)\right]$$

- In practice, only a small range in Q^2 of data is available
  most of the data exist in the range $1 < Q^2 < 2\ GeV^2$

- To maximally utilize the data one needs to:
  - Include g_2 structure function in the asymmetries
  - Include finite Q^2 corrections from higher twists (HT) and target mass corrections (TMC)
  - Include nuclear corrections for deuteron & He data (relevant for large x)

Theory overview

- For TMC the Nachtmann variable is used

$$\xi = \frac{2x}{1+(1+4x^2\mu^2)^{1/2}}, \qquad \mu^2 = M^2/Q^2$$

- For LT, TMC are implemented as follow (Bluemlein, Tkabladze)

$$\begin{aligned}
g_1^{LT+TMC}(x,Q^2) =& \frac{x}{\xi}\frac{g_1^{LT}(\xi)}{(1+4\mu^2x^2)^{3/2}} + 4\mu^2x^2\frac{x+\xi}{\xi(1+4\mu^2x^2)^2}\int_\xi^1 \frac{dz}{z} g_1^{LT}(z) \\
& - 4\mu^2x^2\frac{2-4\mu^2x^2}{2(1+4\mu^2x^2)^{5/2}}\int_\xi^1 \frac{dz}{z}\int_{z'}^1 \frac{dz'}{z'} g_1^{LT}(z') \\
g_2^{LT+TMC}(x,Q^2) =& -\frac{x}{\xi}\frac{g_1^{LT}(\xi)}{(1+4\mu^2x^2)^{3/2}} + \frac{x}{\xi}\frac{(1-4\mu^2x\xi)}{(1+4\mu^2x^2)^2}\int_\xi^1 \frac{dz}{z} g_1^{LT}(z) \\
& + \frac{3}{2}\frac{4\mu^2x^2}{(1+4\mu^2x^2)^{5/2}}\int_\xi^1 \frac{dz}{z}\int_{z'}^1 \frac{dz'}{z'} g_1^{LT}(z')
\end{aligned}$$

- In the Bjorken limit ($Q^2 \to \infty$):

$$g_1^{LT+TMC}(x,Q^2) \simeq g_1^{LT}(x) \qquad g_2^{LT+TMC}(x,Q^2) \simeq -g_1^{LT}(x) + \int_\xi^1 \frac{dz}{z} g_1^{LT}(z)$$

Theory overview

- For twist-3 (T3) we have

$$g_1^{T3+TMC}(x,Q^2) = 4\mu^2x^2 \frac{D(\xi)}{(1+4\mu^2x^2)^{3/2}} - 4\mu^2x^2 \frac{3}{(1+4\mu^2x^2)^2} \int_\xi^1 \frac{dz}{z} D(z) + 4\mu^2x^2 \frac{2-4\mu^2x^2}{(1+4\mu^2x^2)^{5/2}} \int_\xi^1 \frac{dz}{z} \int_{z'}^1 \frac{dz'}{z'} D(z')$$

$$g_2^{T3+TMC}(x,Q^2) = \frac{D(\xi)}{(1+4\mu^2x^2)^{3/2}} - \frac{1-8\mu^2x^2}{(1+4\mu^2x^2)^2} \int_\xi^1 \frac{dz}{z} D(z) - \frac{12\mu^2x^2}{(1+4\mu^2x^2)^{5/2}} \int_\xi^1 \frac{dz}{z} \int_{z'}^1 \frac{dz'}{z'} D(z')$$

- The T3 distributions are expressed as

$$D(z,Q^2) = \sum_q e_q^2 D_q(z)$$

- In the Bjorken limit ($Q^2 \to \infty$):

$$g_1^{T3+TMC}(x,Q^2) \simeq 0 \qquad g_2^{T3+TMC}(x,Q^2) \simeq D(x) - \int_\xi^1 \frac{dz}{z} D(z)$$

Theory overview

- A quantity of interest is known as d_2 matrix element

$$d_2 = 2g_1^{(3)}(Q^2) + 3g_2^{(3)}(Q^2)$$

- The SLACE155x measurements at $Q^2 = 5GeV^2$ are

  - $d_2^p \sim 0.003$
  - $d_2^n \sim 0.007$

- At LT, such combination vanishes and it reveals to leading order in $1/Q^2$ at T3.

- Such quantity is of interest in understanding "color polarizabilities"

Theory overview

- The twist-4 (T4) contributions are parametrized as

$$g_1^{T4(p,n)}(x, Q^2) = H^{(p,n)}(x)/Q^2$$

- In summary the nucleon structure functions are described as

$$g_1(x, Q^2) = g_1^{LT+TMC} + g_1^{T3+TMC} + g_1^{T4}$$
$$g_2(x, Q^2) = g_2^{LT+TMC} + g_2^{T3+TMC}$$

in terms of $\Delta q(x, Q^2)$, $D_q(x)$ and $H^{(p,n)}(x)$

- The nuclear corrections for Deuteron and Helium data can be implemented as convolutions between the nuclear smearing functions (f_{ij}^N) and nucleon structure functions g_i^N

$$g_i^A(x, Q^2) = \sum_N \int \frac{dy}{y} f_{ij}^N(y, \gamma) g_j^N(x/y, Q^2)$$

Mellin-space techniques

- g_1, g_2 involves convolutions (up to 3 integrations) which is computationally too expensive for a global analysis
- A Mellin-space technique (Stratmann, Vogelsang) solves the problem
- Consider the following problem in which $g(x)$ is a function to be fitted

$$I(x) = \int_x^1 \frac{dy}{y} f(y) \int_y^1 \frac{dz}{z} g\left(\frac{x}{yz}\right) \qquad \leftarrow \qquad g(\xi) = \frac{1}{2\pi i}\int dN \xi^{-N} g_N$$

$$= \frac{1}{2\pi i}\int dN g_N \left[\int_x^1 \frac{dy}{y} f(y) \int_y^1 \frac{dz}{z} \left(\frac{x}{yz}\right)^{-N}\right]$$

$$= \frac{1}{2\pi i}\int dN g_N \mathcal{M}_N$$

$$= \sum_{i,k} w_i^k j^k \mathrm{Im}\left(e^{i\phi} g_{N_j^k} \mathcal{M}_{N_j^k}\right) \qquad \leftarrow \qquad \text{Gausian Quadrature}$$

- The time consuming part can be precalculated prior to the fit.

The fitting

- For all of the unknown distributions $(\Delta q, D, H)$ we use the traditional parmetrization at the input scale $Q^2 = 1 GeV^2$.

$$xf(x) = Nx^a(1-x)^b(1+c\sqrt{x}+dx)$$

- Since we only consider inclusive DIS, only $\Delta u^+, \Delta d^+, \Delta s^+, \Delta g$ are fitted while the anti-quarks are parametrized in terms of Δs^+.
- The parameters are fitted to minimize the following χ^2

$$\chi^2 = \sum_i \left(\frac{D_i - T_i(1 - \sum_k r^k \beta_i^k / D_i)^{-1}}{\alpha_i} \right)^2 + \sum_k (r^k)^2$$

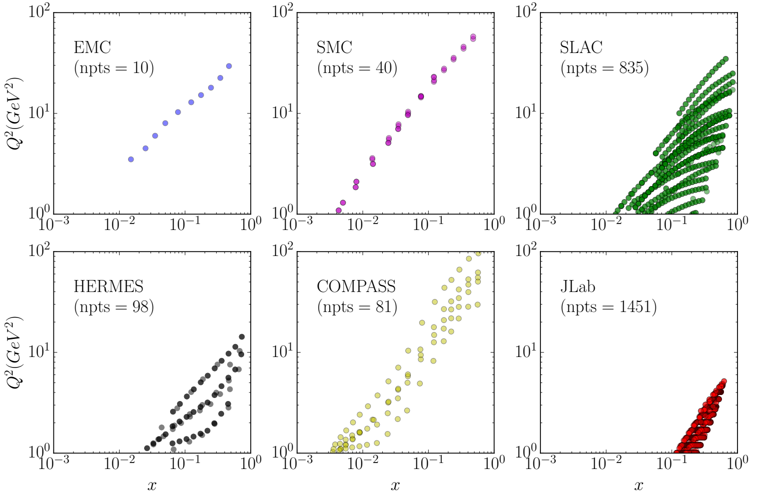
where free parameters r_k are introduced for the treatment of correlated uncertainties.

The fitting

- Issues:
  - Stability in the moments (i.g. $\Delta\Sigma^{(1)}$)
  - Is the solution given by a single fit unique?
  - Is over-fitting present in our fits?
  - Which parameters should be fixed and at which value?
  - Determination of uncertainty bands.

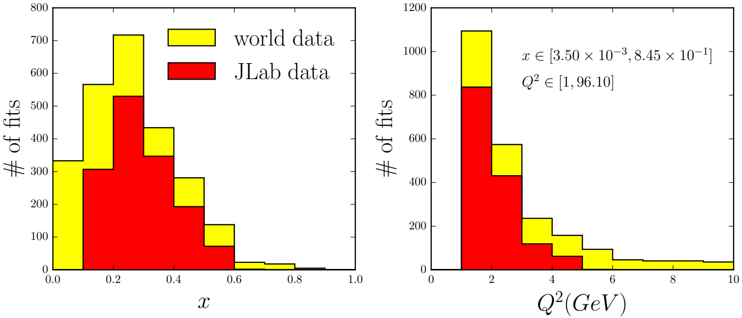
- Solution: MC approach (some similarities with NNPDF)
  - MC fits $\equiv$ many fits ($\sim 10,000$)
  - Flat randomization of initial guess parameters.
  - Cross-validation technique (training set and validation set)
  - Data resampling (bootstrap)

PDIS Kinematics



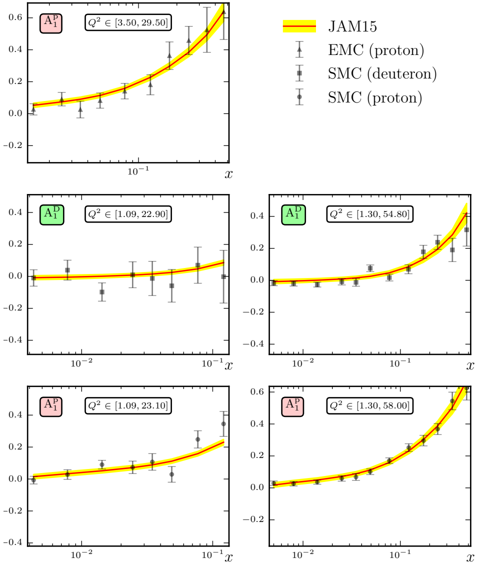
- \# points $= 2515$

PDIS Kinematics



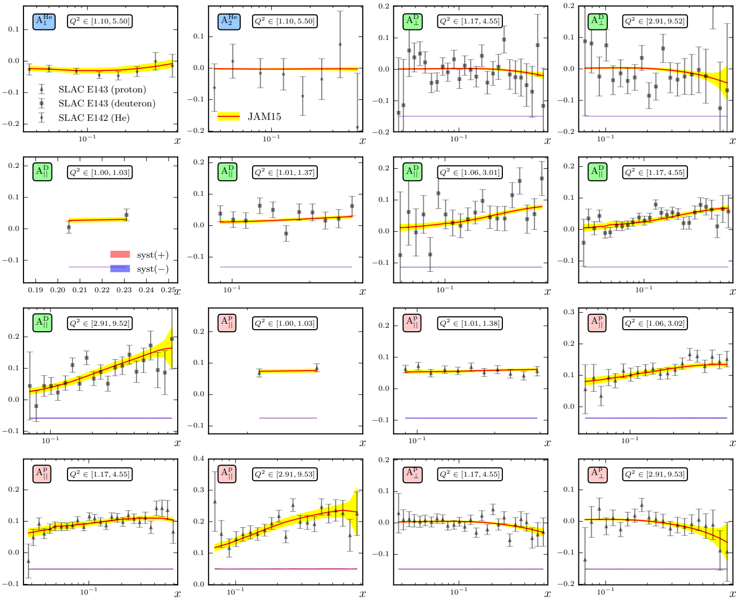
- proton data $= 1638$
- deuteron data $= 782$
- helium data $= 95$

EMC/SMC

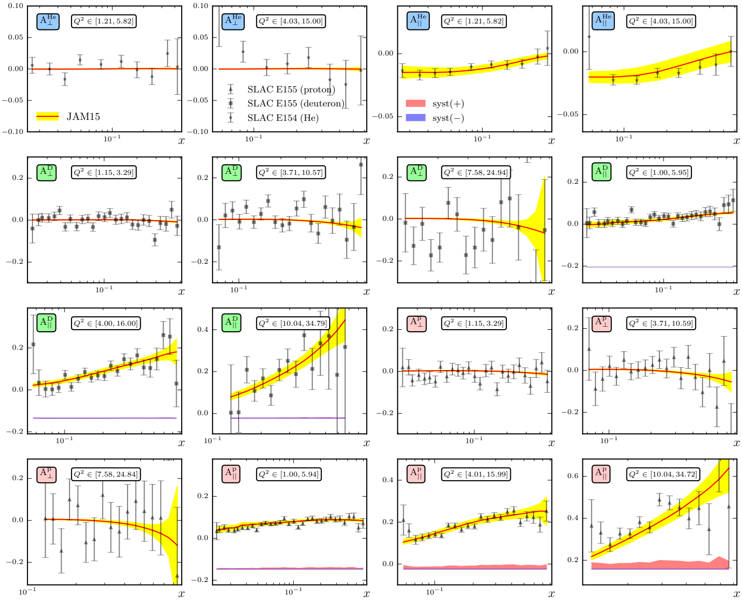


A_1^P
$Q^2 \in [3.50, 29.50]$
A_1^D
$Q^2 \in [1.09, 22.90]$
A_1^D
$Q^2 \in [1.30, 54.80]$
A_1^P
$Q^2 \in [1.09, 23.10]$
A_1^P
$Q^2 \in [1.30, 58.00]$
x
JAM15
EMC (proton)
SMC (deuteron)
SMC (proton)

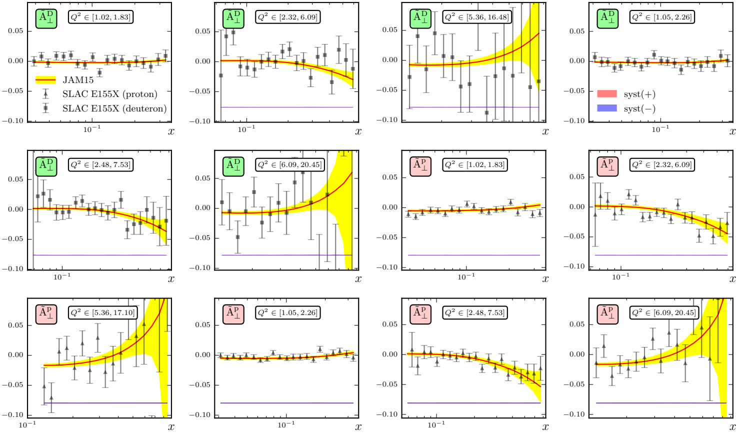
SLAC E142/E143



SLAC E154/E155

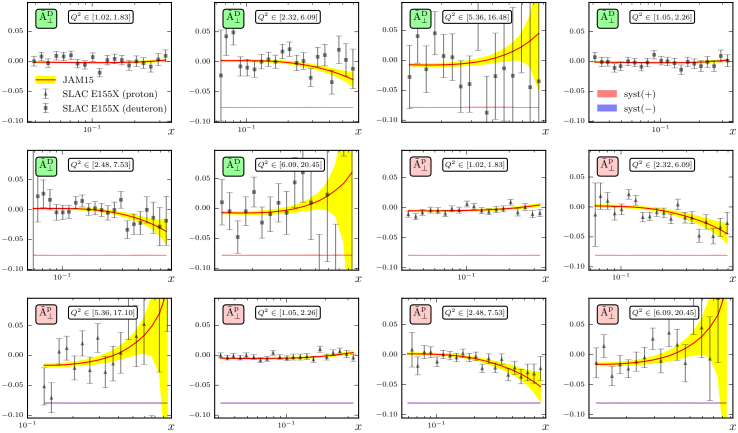


SLAC E155x



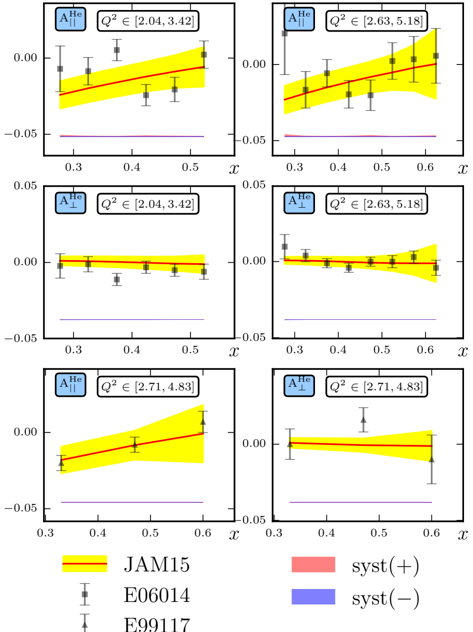
$\tilde{A}_\perp^D$
$Q^2 \in [1.02, 1.83]$
JAM15
SLAC E155X (proton)
SLAC E155X (deuteron)
$Q^2 \in [2.32, 6.09]$
$Q^2 \in [5.36, 16.48]$
$Q^2 \in [1.05, 2.26]$
syst(+)
syst(−)
$Q^2 \in [2.48, 7.53]$
$Q^2 \in [6.09, 20.45]$
$\tilde{A}_\perp^P$
$Q^2 \in [1.02, 1.83]$
$Q^2 \in [2.32, 6.09]$
$Q^2 \in [5.36, 17.10]$
$Q^2 \in [1.05, 2.26]$
$Q^2 \in [2.48, 7.53]$
$Q^2 \in [6.09, 20.45]$
0.05
0.00
−0.05
−0.10
10^{-1}
x

SLAC E155x



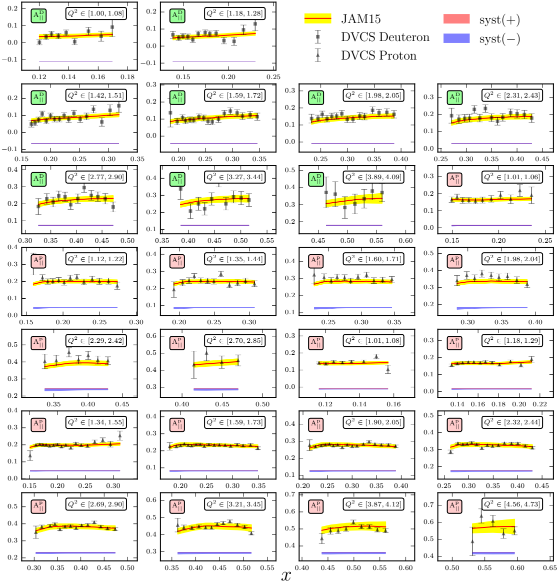
$\tilde{A}^{D}_{\perp}$ $Q^2 \in [1.02, 1.83]$
JAM15
SLAC E155X (proton)
SLAC E155X (deuteron)
$\tilde{A}^{D}_{\perp}$ $Q^2 \in [2.32, 6.09]$
$\tilde{A}^{D}_{\perp}$ $Q^2 \in [5.36, 16.48]$
$\tilde{A}^{D}_{\perp}$ $Q^2 \in [1.05, 2.26]$
syst(+)
syst(−)
$\tilde{A}^{D}_{\perp}$ $Q^2 \in [2.48, 7.53]$
$\tilde{A}^{D}_{\perp}$ $Q^2 \in [6.09, 20.45]$
$\tilde{A}^{P}_{\perp}$ $Q^2 \in [1.02, 1.83]$
$\tilde{A}^{P}_{\perp}$ $Q^2 \in [2.32, 6.09]$
$\tilde{A}^{P}_{\perp}$ $Q^2 \in [5.36, 17.10]$
$\tilde{A}^{P}_{\perp}$ $Q^2 \in [1.05, 2.26]$
$\tilde{A}^{P}_{\perp}$ $Q^2 \in [2.48, 7.53]$
$\tilde{A}^{P}_{\perp}$ $Q^2 \in [6.09, 20.45]$
x
10^{-1}

JLab Hall A



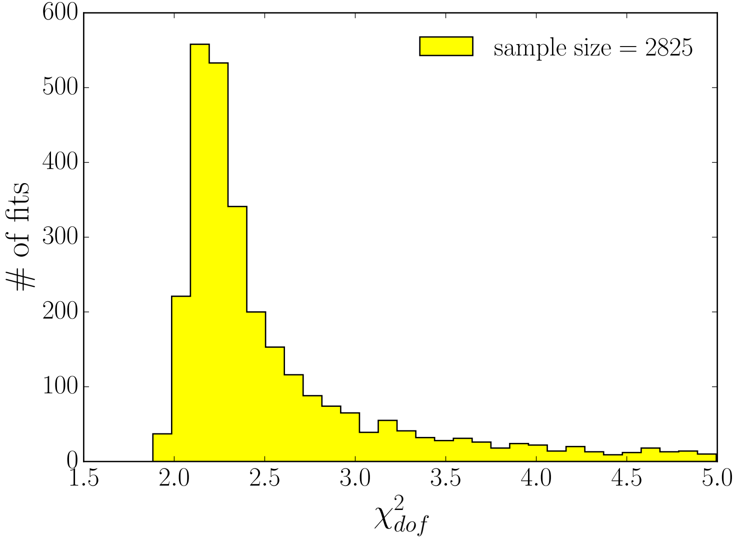
$A_{||}^{He}$
$Q^2 \in [2.04, 3.42]$
$Q^2 \in [2.63, 5.18]$
$A_{\perp}^{He}$
$Q^2 \in [2.71, 4.83]$
0.05
0.00
−0.05
0.3
0.4
0.5
0.6
x
JAM15
E06014
E99117
syst(+)
syst(−)

JLab Hall B (EG1-DVCS)



JLab Hall B (EG1b)

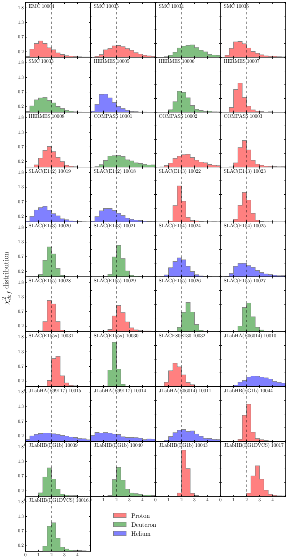
χ^2



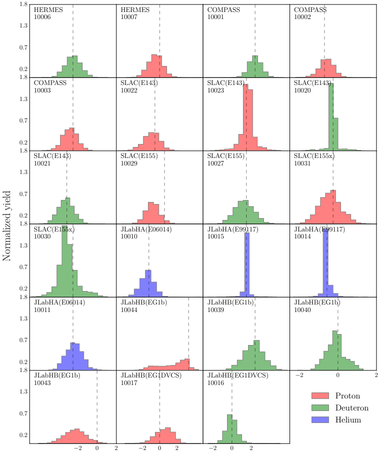
- In the ideal gaussian statistics, the peak should be located at 2.

χ^2

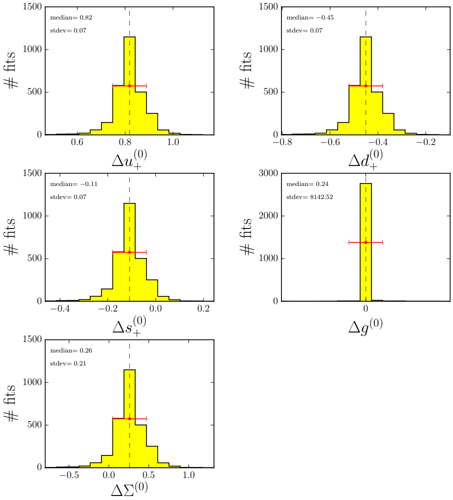
χ^2_{dof} per experiment



fitted r values



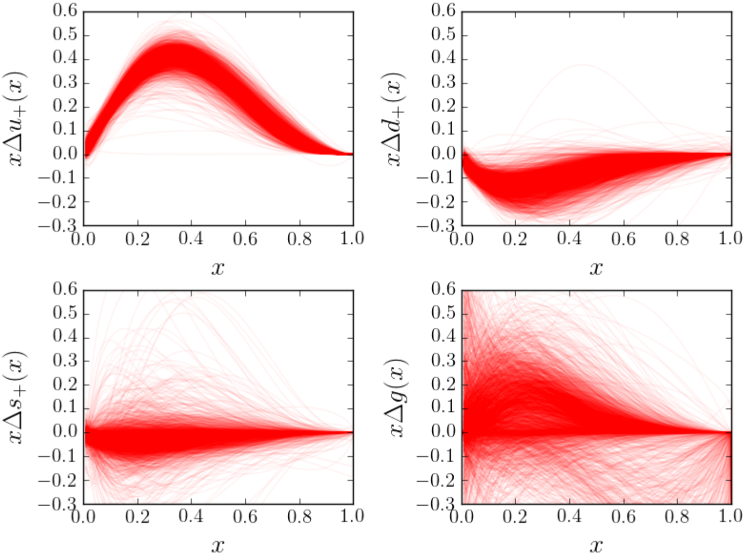
LT moments



- The extracted $\Delta\Sigma \sim 0.26$

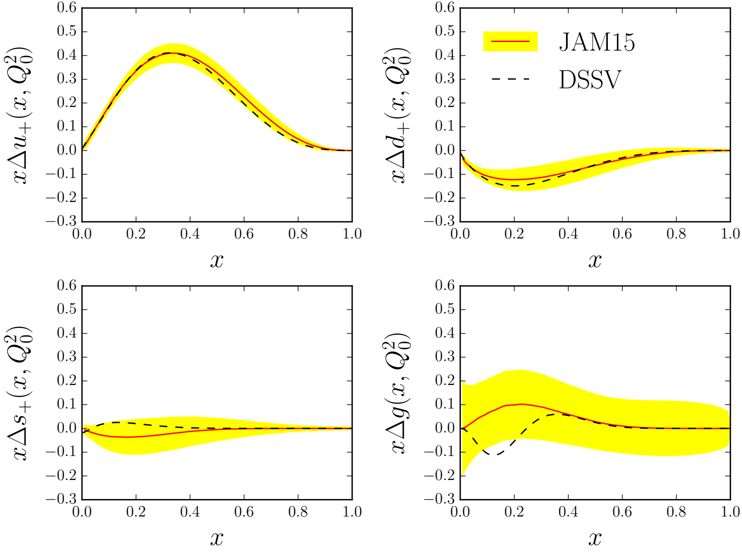
SPDFs

- Plot of raw MC fits

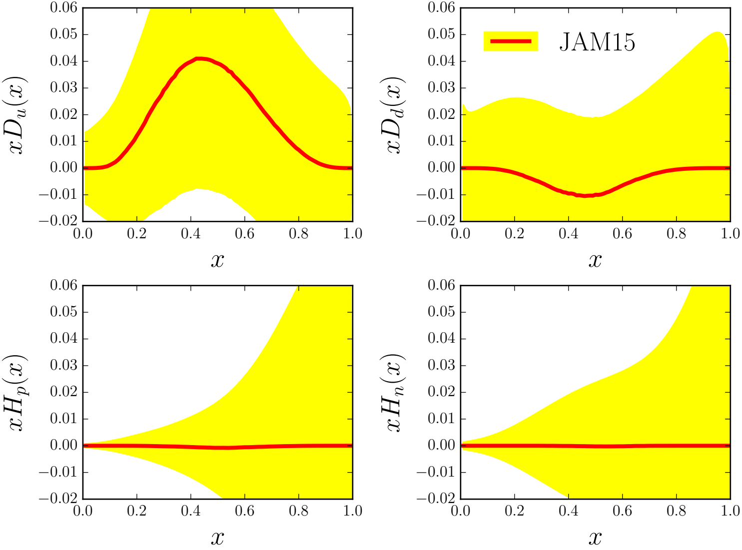


SPDFs

- Expectation values and variances

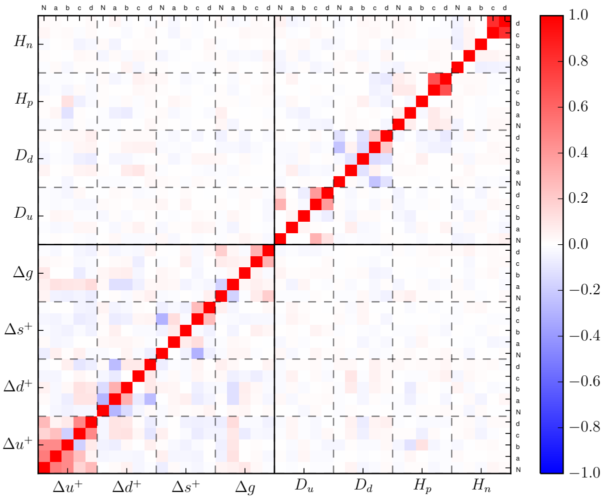


HT distributions: T3 $D_{u,d}$ T4 $H_{p,n}$



- From D_u and D_d the extracted value for $d_2^p \sim 0.02$ (preliminary)

Correlation



- LT & HT parameters decorrelates

Outlook

- Ongoing JAM15 analysis studying impact of JLab 6 GeV inclusive DIS data at low W and high x
- New tools has been developed (slides 10-12)
- Future analysis to study polarization of sea quarks & gluons.
  - SIDIS for flavor separation.
  - polarized pp cross sections (inclusive jet & π production) for Δg
  - Threshold resummation and effects at large x