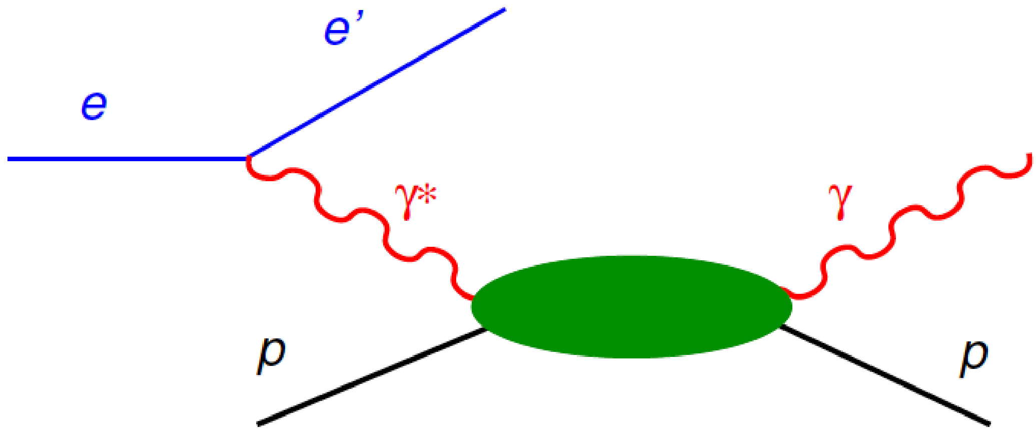
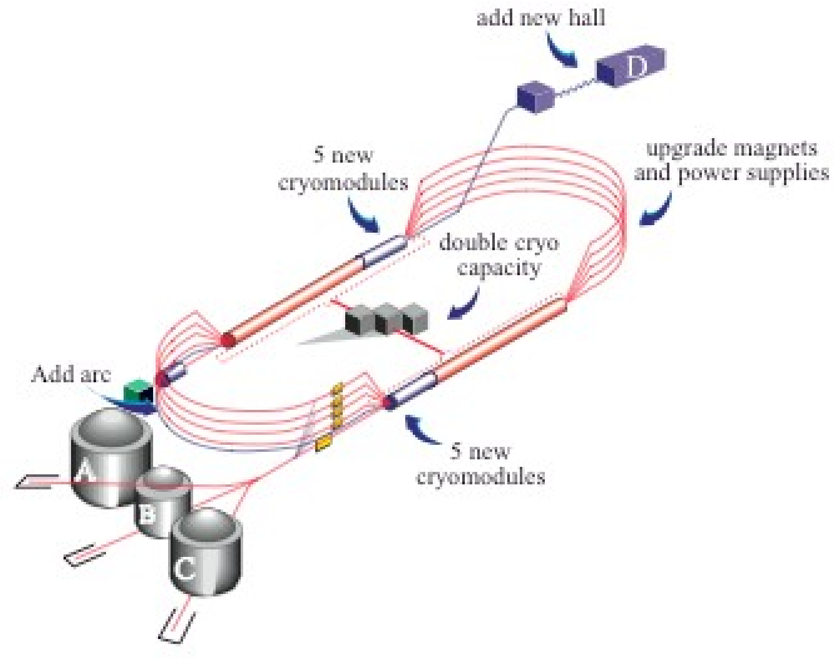
Kinematic Issue of GPDs in DVCS



B.L.G. Bakker and C.-R.Ji, arXiv:1002.0443[hep-ph];
PRD83,091502(R) (2011).

SMU, Jan. 19, 2012

Hadron Physics at JLab

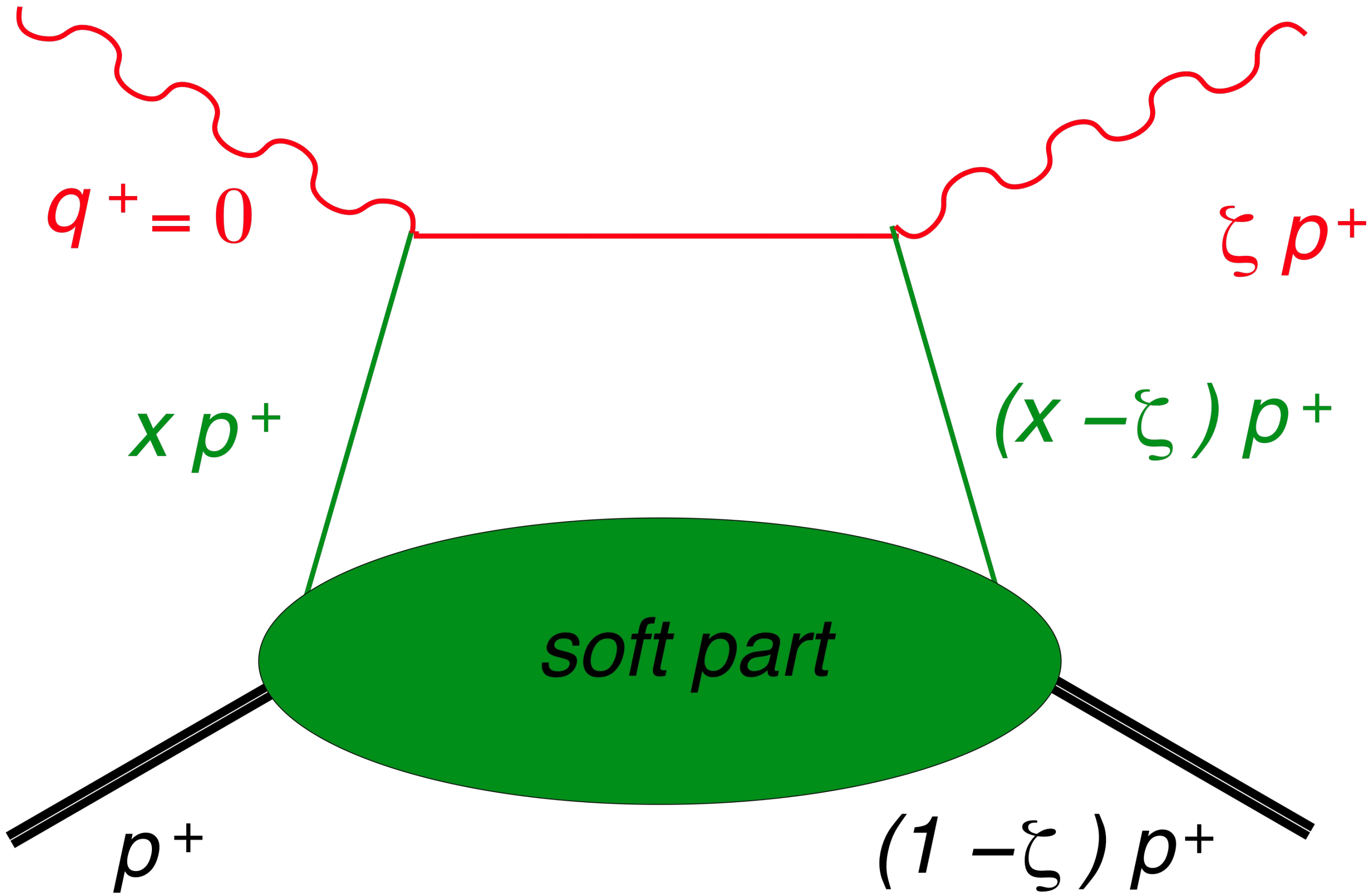


Outline

- Original Formulation of DVCS with GPDs
- JLab Kinematics
- Tree Level Calculation
- Conclusions

GPDs rely on the handbag dominance in DVCS; i.e.

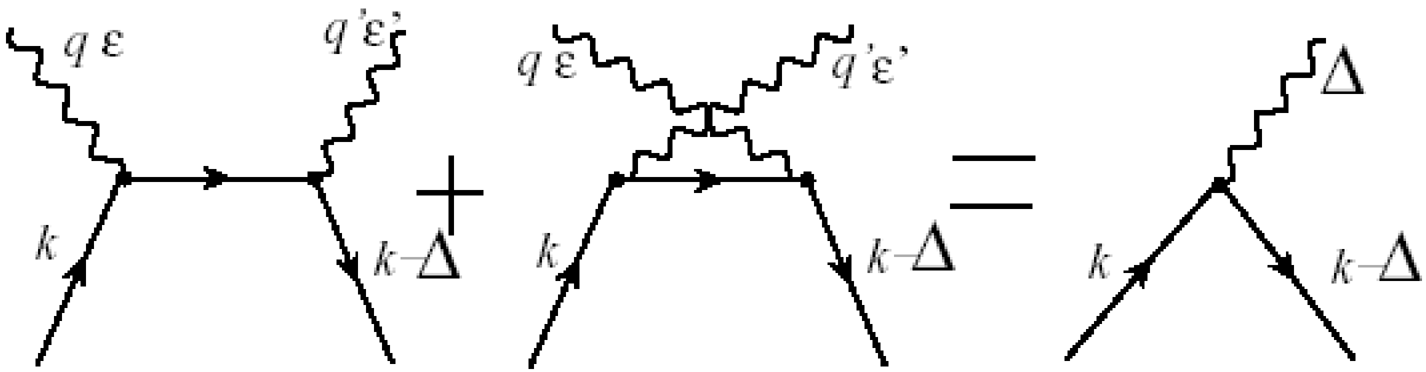
$Q^2 >>$ any soft mass scale



$q^2 = q^+q^- - q_\perp^2 = -q_\perp^2 = -Q^2 < 0$, e.g.

S.J.Brodsky,M.Diehl,D.S.Hwang, NPB596,99(01)

Full Amp vs. Reduced Amp



S-channel: $$\frac{\not\epsilon^*(l')(\not k+\not q+m)\not\epsilon(l)}{(k+q)^2-m^2} \approx \frac{\not\epsilon^*(l')q^-\gamma^+\not\epsilon(l)}{(x-z)P^+q^-}$$

U-channel: $$\frac{\not\epsilon(l)(\not k-\not q'+m)\not\epsilon^*(l')}{(k-q')^2-m^2} \approx -\frac{\not\epsilon(l)q^-\gamma^+\not\epsilon^*(l')}{x P^+q^-}$$

Nucleon GPDs in DVCS Amplitude

X.Ji,PRL78,610(1997): Eqs.(14) and (15)

$$p^\mu = \Lambda(\overset{ct}{1},\overset{x}{0},\overset{y}{0},\overset{z}{1}) \quad ,$$

$$n^\mu = (\overset{ct}{1},\overset{x}{0},\overset{y}{0},\overset{z}{-1})/(2\Lambda) \quad ,$$

$$\bar{P}^\mu = \frac{1}{2}(P+P')^\mu = p^\mu + \frac{M^2-\Delta^2/4}{2} n^\mu ,$$

$$q^\mu = -\xi p^\mu + \frac{Q^2}{2\xi} n^\mu \quad , \quad \xi = \frac{Q^2}{2\bar{P}\cdot q} \quad ,$$

$$\Delta^\mu = -\xi \left(p^\mu - \frac{M^2-\Delta^2/4}{2} n^\mu \right) + \Delta_\perp^\mu \ .$$

$$T^{\mu\nu}(p,q,\Delta) = -\frac{1}{2}(p^\mu n^\nu + p^\nu n^\mu - g^{\mu\nu}) \int_{-1}^{+1} dx \left(\frac{1}{x-\frac{\xi}{2}+i\epsilon} + \frac{1}{x+\frac{\xi}{2}-i\epsilon} \right)$$
$$\times \left[H(x,\Delta^2,\xi)\bar{U}(P')\not{n}U(P) + E(x,\Delta^2,\xi)\bar{U}(P')\frac{i\sigma^{\alpha\beta}n_\alpha\Delta_\beta}{2M}U(P) \right]$$
$$-\frac{i}{2}\epsilon^{\mu\nu\alpha\beta}p_\alpha n_\beta \int_{-1}^{+1} dx \left(\frac{1}{x-\frac{\xi}{2}+i\epsilon} - \frac{1}{x+\frac{\xi}{2}-i\epsilon} \right)$$
$$\times \left[\tilde{H}(x,\Delta^2,\xi)\bar{U}(P')\not{n}\gamma_5 U(P) + \tilde{E}(x,\Delta^2,\xi)\frac{\Delta\cdot n}{2M}\bar{U}(P')\gamma_5 U(P) \right]$$

Just above Eq.(14),

``To calculate the scattering amplitude, it is convenient to define <u>a special system of coordinates</u>."

Note here that $q'^2 = -\Delta_\perp^2$ = 0 , i.e. t = 0 .

Nucleon GPDs in DVCS Amplitude

A.V.Radyushkin, PRD56, 5524 (1997): Eq.(7.1)

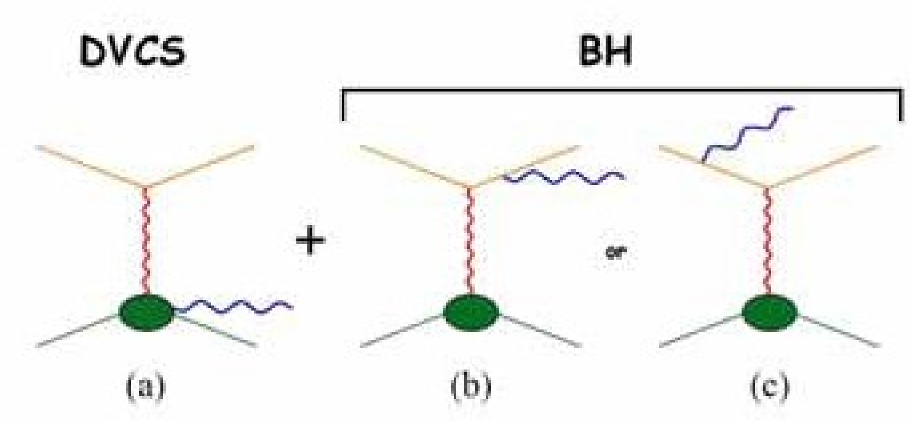
$$q = q' - \zeta p \;,\qquad \zeta = \frac{Q^2}{2p\cdot q'} \;,\qquad r = p - p'$$

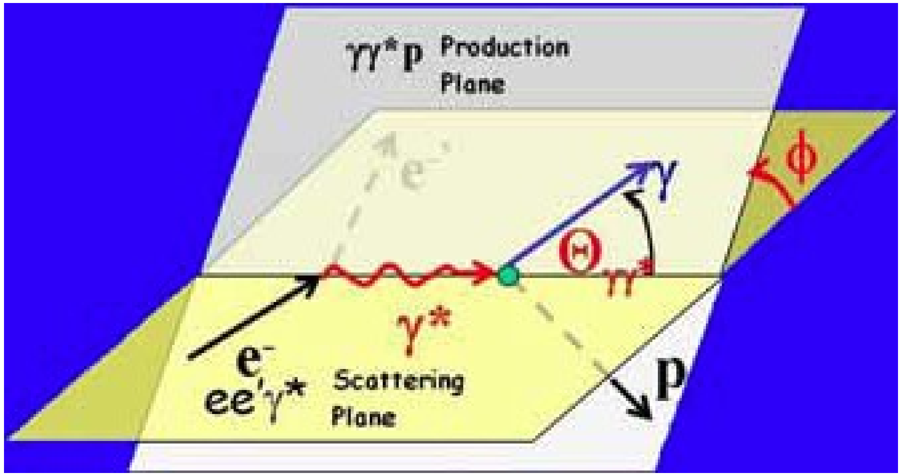
$$T^{\mu\nu}(p,q,q') = \frac{1}{2(p\cdot q')}\sum_a e_a^2 \Big[\Big(-g^{\mu\nu} + \frac{1}{p\cdot q'}(p^\mu q'^\nu + p^\nu q'^\mu)\Big)\Big(\bar{u}(p')\,\not{q}'\,u(p)\,T_F^a(\zeta) + \frac{1}{2M}\bar{u}(p')(\not{q}'\not{r} - \not{r}\not{q}')u(p)\,T_K^a(\zeta)\Big)$$
$$+\, i\epsilon^{\mu\nu\alpha\beta}\frac{p_\alpha q'_\beta}{p\cdot q'}\Big(\bar{u}(p')\,\not{q}'\gamma_5 u(p)\,T_G^a(\zeta) + \frac{q'\cdot r}{2M}\bar{u}(p')\gamma_5 u(p)\,T_P^a(\zeta)\Big)\Big]$$

At the beginning of Section 2E (Nonforward distributions),
``Writing the momentum of the virtual photon as q=q'-ζp is equivalent to using the Sudakov decomposition in the light-cone `plus'(p) and `minus'(q') components in a situation <u>when there is no transverse momentum</u> ."

Note here that $t = \Delta^2 = (\zeta P)^2 = \zeta^2 M^2 > 0$,i.e. only consistent at t=0, neglecting nucleon mass.

JLab Kinematics $t < -|t_{min}| \neq 0$

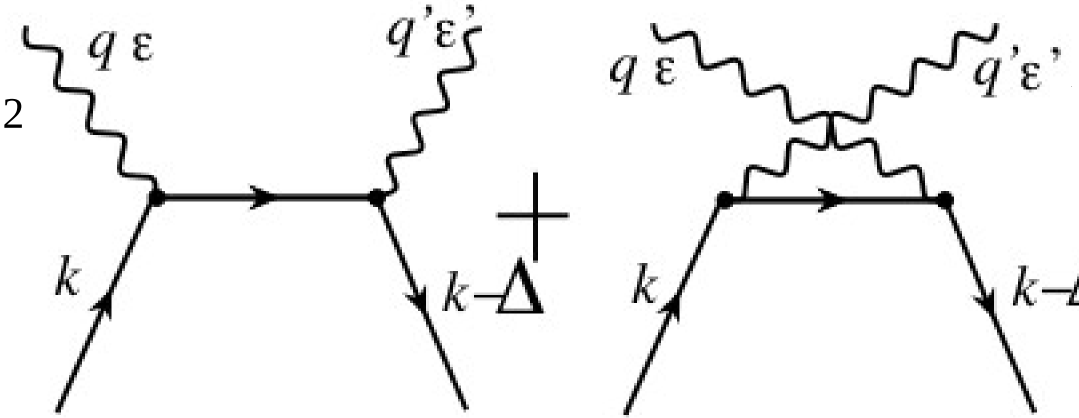




Original formulation of DVCS in terms of GPDs due to X.Ji and A.Radyushkin is applied only at $t=0$.

“Bare Bone” VCS Amplitude at Tree Level

$$S=(k+q)^2$$



$$U=(k-q')^2$$

$$= k'$$

Hadron Helicity Amplitude:

$$\mathrm{H}(h_q, h_{q'}, s_k, s_{k'}) = \epsilon^*_\mu(q', h_{q'})\,\epsilon_\nu(q, h_q)\,(T_S^{\mu\nu} + T_U^{\mu\nu})$$

Neglecting masses,

$$T_S^{\mu\nu} = \frac{k_\alpha + q_\alpha}{S}\,\bar{u}(k', s_{k'})\gamma^\mu\gamma^\alpha\gamma^\nu u(k, s_k)$$

$$T_U^{\mu\nu} = \frac{k_\alpha - q'_\alpha}{U}\,\bar{u}(k', s_{k'})\gamma^\nu\gamma^\alpha\gamma^\mu u(k, s_k)$$

Identity: $\gamma^\mu\gamma^\alpha\gamma^\nu = g^{\mu\alpha}\gamma^\nu + g^{\alpha\nu}\gamma^\mu - g^{\mu\nu}\gamma^\alpha + i\epsilon^{\mu\alpha\nu\beta}\gamma_\beta\gamma_5$

JLab Kinematics t < 0

- We want to see the effect of taking t<0.
- Therefore we mimic the kinematics at JLab.
- In JLab kinematics, the final hadron and final photon move off the z-axis.

$$k'^{\mu} = \left((x-\zeta_{\text{eff}})P^{+}, \mathbf{\Delta}_{\perp}, \frac{\mathbf{\Delta}_{\perp}^{2}}{2(x-\zeta_{\text{eff}})P^{+})} \right)$$

$$q'^{\mu} = \left(\alpha \frac{\mathbf{\Delta}_{\perp}^{2}}{Q^{2}} P^{+}, -\mathbf{\Delta}_{\perp}, \frac{Q^{2}}{2\alpha P^{+}} \right)$$

The quantity ζ_{eff} is given by

$$\zeta_{\text{eff}} = \zeta + \alpha \frac{\mathbf{\Delta}_{\perp}^{2}}{Q^{2}} \to \zeta \text{ for } Q \to \infty$$

$$\alpha = \frac{x-\zeta}{2} \left(1 - \sqrt{1 - \frac{4\zeta}{x-\zeta} \frac{\mathbf{\Delta}_{\perp}^{2}}{Q^{2}}} \right) \to 0 \text{ for } Q \to \infty$$

Using Sudakov vectors

$$n(+)^\mu = (1,0,0,0), \;\; n(-)^\mu = (0,0,0,1)$$

we find

$$\begin{aligned} T_s{}^{\mu\nu} \;\; = \;\; & \frac{1}{s}\Big[\big(\{(k^+ + q^+)n^\mu(+) + q^- n^\mu(-) + q_\perp{}^\mu\}n^\nu(+) \\ & +\{(k^+ + q^+)n^\nu(+) + q^- n^\nu(-) + q_\perp{}^\nu\}n^\mu(+) - g^{\mu\nu} q^-\big) \\ & \times \bar{u}(k';s')\not{n}(-)u(k;s) \\ & -i\epsilon^{\mu\nu\alpha\beta}\{(k^+ + q^+)n_\alpha(+) + q^- n_\alpha(-) + q_{\perp\alpha}\}n_\beta(+) \\ & \times \bar{u}(k';s')\not{n}(-)\gamma_5 u(k;s)\Big]\,. \end{aligned}$$

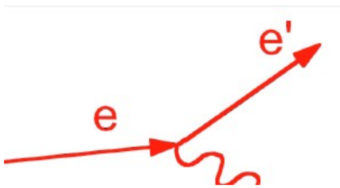
Keeping no transverse momentum in DVCS, we agree on

$$\begin{aligned} T_s{}^{\mu\nu} \;\; = \;\; & \frac{q^-}{s}\Big[\{n^\mu(-)n^\nu(+) + n^\nu(-)n^\mu(+) - g^{\mu\nu}\} \\ & \times \bar{u}(k';s')\not{n}(-)u(k;s) \\ & -i\epsilon^{\mu\nu\alpha\beta} n_\alpha(-)n_\beta(+) \times \bar{u}(k';s')\not{n}(-)\gamma_5 u(k;s)\Big] \end{aligned}$$

equivalent to the expression given by X. Ji and A.V. Radyushkin.

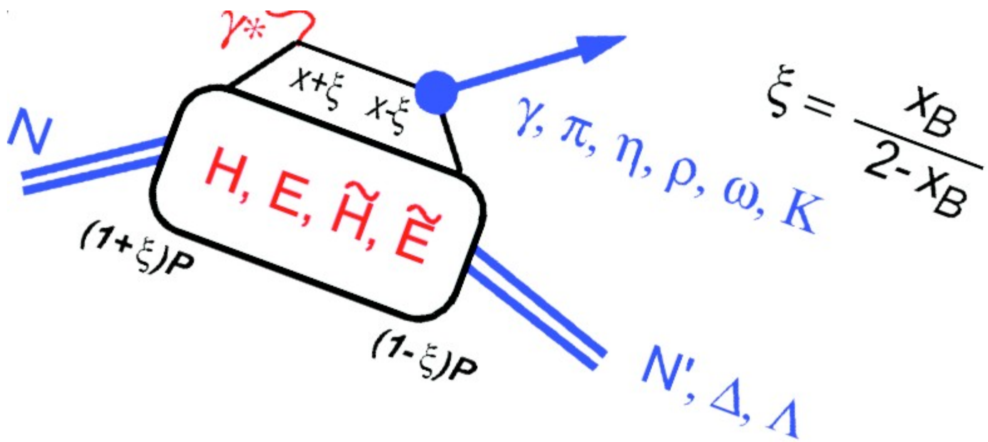
Investigation of Complete Ampltude

Attach the lepton current and check the spin filter for the DVCS amplitude.



$$\epsilon_{\mathrm{LF}}(q;\pm 1) = \frac{1}{\sqrt{2}}\left(0, \mp 1, -i, \mp \frac{q_x \pm i q_y}{q^+}\right)$$

$$\epsilon_{\mathrm{LF}}(q;0) = \frac{1}{\sqrt{q^2}}\left(q^+, q_x, q_y, \frac{q_\perp^2 - q^2}{2q^+}\right)$$



Singularities develop in the polarization vector as $q^+ \to 0$.

The amplitudes being obtained by contraction with the polarization vectors may be sensitive to the neglected parts.

Calculation for massless spinors

Complete amplitude

$$\mathcal{M} = \sum_h \mathcal{L}(\{\lambda', \lambda\}h)\frac{1}{q^2}\mathcal{H}(\{s', s\}\{h', h\}),$$

Leptonic and hadronic parts

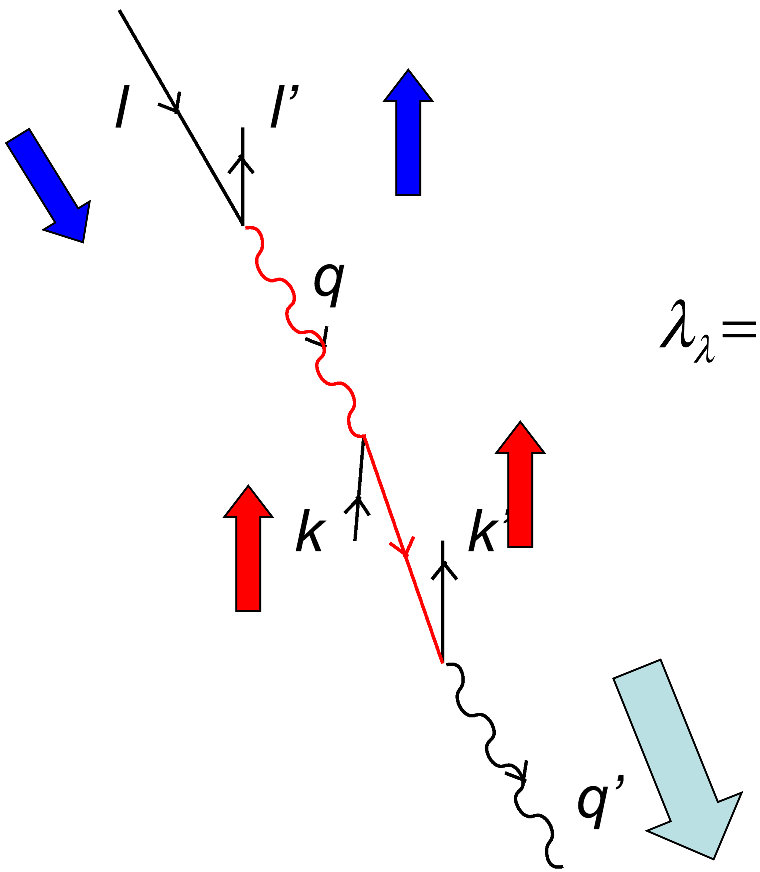
$$\begin{aligned}
\mathcal{L}(\{\lambda', \lambda\}h) &= \bar{u}_{\mathrm{LF}}(\ell'; \lambda')\not{\epsilon}^*(q; h)u_{\mathrm{LF}}(\ell; \lambda), \\
\mathcal{H}(\{s', s\}\{h', h\}) &= \bar{u}_{\mathrm{LF}}(k'; s')(\mathcal{O}_s + \mathcal{O}_u)u_{\mathrm{LF}}(k; s),
\end{aligned}$$

Operators

$$\begin{aligned}
\mathcal{O}_s &= \frac{\not{\epsilon}^*_{\mathrm{LF}}(q'; h')(\not{k} + \not{q})\not{\epsilon}_{\mathrm{LF}}(q; h)}{(k+q)^2}, \\
\mathcal{O}_u &= \frac{\not{\epsilon}_{\mathrm{LF}}(q; h)(\not{k} - \not{q}')\not{\epsilon}^*_{\mathrm{LF}}(q'; h')}{(k-q')^2}
\end{aligned}$$

Checking Amplitudes

- Gauge invariance of each and every polarized amplitude including the longitudinal polarization for the virtual photon.
- Klein-Nishina Formula in RCS.
- Angular Momentum Conservation.



$$\lambda_{\lambda} = \lambda_{\lambda'} = +\frac{1}{2}\,,\ \sigma_{\kappa} = \sigma_{\kappa'} = +\frac{1}{2}\,,\ \eta_{\theta'} = +1\,;$$

Allowed !

Checking Amplitudes

- Gauge invariance of each and every polarized amplitude including the longitudinal polarization for the virtual photon.
- Klein-Nishina Formula in RCS.
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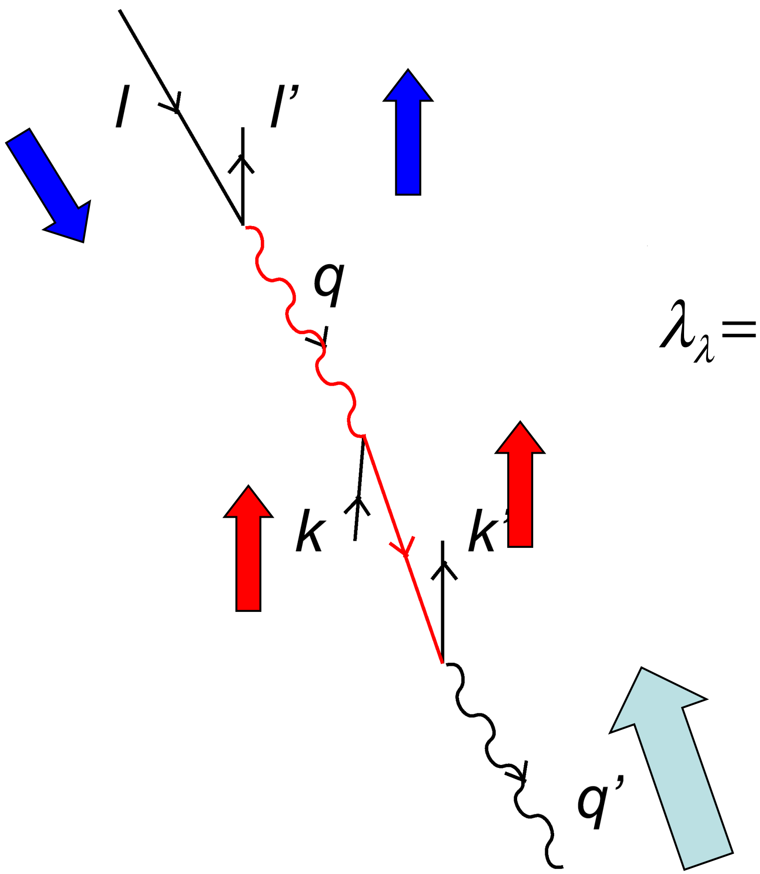

$$\lambda_{\lambda} = \lambda_{\lambda\square} = +\frac{1}{2}\,,\ \sigma_{\kappa} = \sigma_{\kappa\square} = +\frac{1}{2}\,,\ \eta_{\theta\square} = -1\,;$$

Prohibited !

Comparison

Complete DVCS amplitudes, $\sum_h \mathcal{L}(\{\lambda', \lambda\}, h) \frac{1}{q^2} \mathcal{H}(\{h', h\}\{s', s\})$ in three approaches, ours, A.V. Radyushkin, and X. Ji. Because the hadrons and leptons are massless, $\lambda' = \lambda$ and $s' = s$.

λ	h'	s	this work	AVR	XJ
$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{4}{Q}\sqrt{\frac{x}{x-\zeta}}\left(1+\frac{\zeta}{2(x-\zeta)}\frac{\Delta_\perp^2}{Q^2}\right)$	$\frac{4}{Q}\sqrt{\frac{x}{x-\zeta}}\left(1-\frac{\zeta}{2(x-\zeta)}\frac{\Delta_\perp^2}{Q^2}\right)$	0
$\frac{1}{2}$	1	$-\frac{1}{2}$	$\frac{4}{Q}\sqrt{\frac{x-\zeta}{x}}\left(1-\frac{\zeta}{2(x-\zeta)}\frac{\Delta_\perp^2}{Q^2}\right)$	$\frac{4}{Q}\sqrt{\frac{x-\zeta}{x}}\left(1+\frac{\zeta}{2(x-\zeta)}\frac{\Delta_\perp^2}{Q^2}\right)$	0
$\frac{1}{2}$	-1	$\frac{1}{2}$	$-\frac{4}{Q^3}\frac{\zeta^2}{\sqrt{x(x-\zeta)}(x-\zeta)}\frac{\Delta_\perp^2}{Q^2}$	0	$\frac{4}{Q}\sqrt{\frac{x}{x-\zeta}}\left(1-\frac{\zeta}{2(x-\zeta)}\frac{\Delta_\perp^2}{Q^2}\right)$
$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	$\frac{4}{Q}\sqrt{\frac{x-\zeta}{x}}\left(1+\frac{\zeta}{2(x-\zeta)}\frac{\Delta_\perp^2}{Q^2}\right)$

Conclusions

- For the good hadron phenomenology, treacherous points such as zero-modes and singularities should be taken into account correctly.
- As a consequence, we find that the XJ and AVR amplitudes for DVCS in terms of GPDs for $t < 0$ are not satisfactory.
- More careful investigation on the GPD formulation and the corresponding sum rules is necessary.