QUANTUM MODELS OF COGNITION AND DECISION

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Supported by AFOSR

WHAT IS THE GOAL OF QUANTUM COGNITION AND DECISION?

- Not a physical/neurobiological theory of the brain
- Not a theory of consciousness
- It is a mathematical theory about human behavior
    - Specifically judgments and decisions

ORGANIZATION OF THIS TALK

1. Why use quantum theory for cognition and decision?
2. Quantum vs classic probability theory.
3. Evidence for quantum probability theory.
4. Quantum versus Markov dynamics.
5. Evidence for quantum dynamics
6. Conclusions

1. WHY USE QUANTUM THEORY?

1. Quantum theory is a general Axiomatic theory of probability
   - Human judgments and decisions are probabilistic
   - These probabilities do not obey the Kolmogorov axioms
   - Quantum theory provides a viable alternative
2. Non Commutativity of measurements
   - Measurements change psychological states producing context effects
   - Principle of complementarity was borrowed by Niels Bohr from William James
3. Vector space representation of probabilities
   - Agrees with connectionist-neural network models of cognition

2. HOW DO WE USE QUANTUM THEORY?

COMPARISON OF CLASSIC AND QUANTUM PROBABILITY THEORIES

Kolmogorov

Von Neumann

Classical

- Each unique outcome is a member of a set of points called the Sample space

Quantum

- Each unique outcome is an orthonormal vector from a set that spans a Vector space

Classical

- Each unique outcome is a member of a set of points called the Sample space
- Each event is a subset of the sample space

Quantum

- Each unique outcome is an orthonormal vector from a set that spans a Vector space
- Each event is a subspace of the vector space.

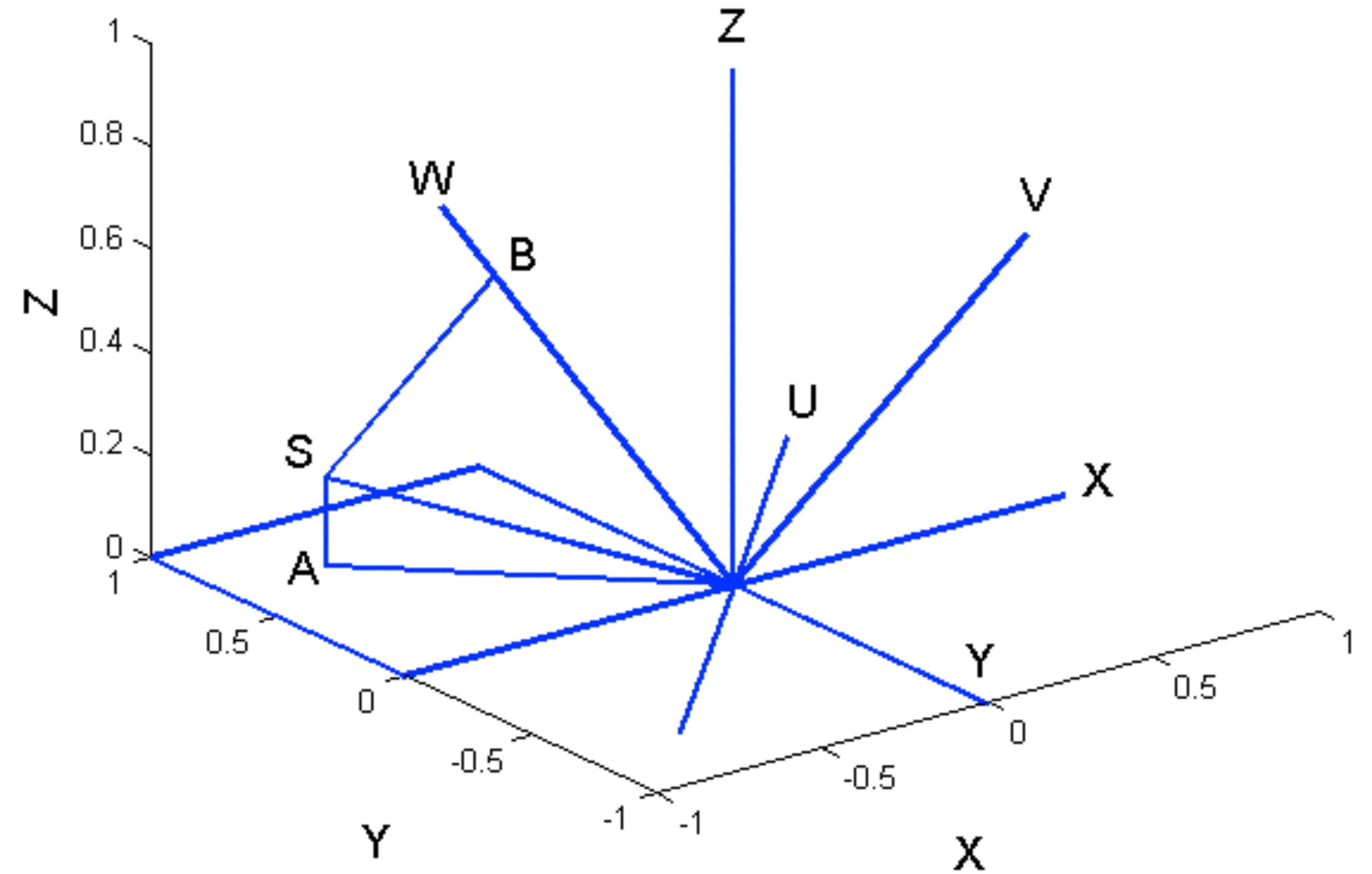
Classical

- Each unique outcome is a member of a set of points called the Sample space
- Each event is a subset of the sample space
- State is a probability function, p, defined on subsets of the sample space.

Quantum

- Each unique outcome is an orthonormal vector from a set that spans a Vector space
- Each event is a subspace of the vector space.
- State is a unit length vector, S,

$$p(A) = \|P_A S\|^2$$


Z
W
B
V
U
S
X
A
Y
Z
Y
X
1
0.8
0.6
0.4
0.2
0
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1

Classical

- Suppose event A is observed (state reduction):

$$p(B \mid A) = \frac{p(B \cap A)}{p(A)}$$

Quantum

- Suppose event A is observed (state reduction):

$$p(B \mid A) = \frac{\left\| P_B P_A S \right\|^2}{\left\| P_A S \right\|^2}$$

Classical

- Suppose event A is observed (state reduction):

$$p(B \mid A) = \frac{p(B \cap A)}{p(A)}$$

- Commutative Property

$$p(B \cap A) = p(A \cap B)$$

Quantum

- Suppose event A is observed (state reduction):

$$p(B \mid A) = \frac{\left\|P_B P_A S\right\|^2}{\left\|P_A S\right\|^2}$$

- Non-Commutative

$$\left\|P_B P_A S\right\|^2 \neq \left\|P_A P_B S\right\|^2$$

3. WHAT IS THE EMPIRICAL EVIDENCE?

CONJUNCTION -DISJUNCTION PROBABILITY JUDGMENT ERRORS

Tversky & Kahneman
(1983, *Psychological Review*)

Busemeyer, Pothos, Franco, Trueblood
(2011, *Psychological Review*)

Read the following information:

Linda was a philosophy major as a student at UC Berkeley and she was an activist in social welfare movements.

Rate the probability of the following events

Linda is a feminist (.83)
Linda is a bank teller (.26)
Linda is a feminist and a bank teller (.36)
Linda is a feminist or a bank teller (.60)

Conjunction Fallacy

Disjunction Fallacy

LAW OF TOTAL PROBABILITY

$$p(B) = p(F)p(B \mid F) + p(\sim F)p(B \mid \sim F)$$

$$\geq p(F)p(B \mid F)$$

CONJUNCTION - FALLACY

VIOLATES THIS LAW

Quantum Model Predictions

$$\left\|P_B S\right\|^2 = \left\|P_B I S\right\|^2 = \left\|P_B (P_F + P_{\bar{F}}) S\right\|^2$$

$$= \left\|P_B P_F S + P_B P_{\bar{F}} S\right\|^2$$

$$= \left\|P_B P_F S\right\|^2 + \left\|P_B P_{\bar{F}} S\right\|^2 + Int$$

$$Int = \left\langle S' P'_F \, P'_B \, P_{\bar{F}} S \right\rangle + \left\langle S' P_{\bar{F}} P'_B \, P_F S \right\rangle$$

$$Int < -\left\|P_B P_{\bar{F}} S\right\|^2$$

DISJUNCTION FALLACY

$$Finding:\ p(F) \geq p(F \text{ or B})$$

$$p(F) = 1 - \left\| P_{\bar{F}} S \right\|^2$$

$$p(F \text{ or B}) = 1 - \left\| P_{\bar{F}} P_{\bar{B}} S \right\|^2$$

$$Finding \rightarrow \left\| P_{\bar{F}} P_{\bar{B}} S \right\|^2 \geq \left\| P_{\bar{F}} S \right\|^2$$

INTERFERENCE OF CATEGORIZATION ON DECISION

Psychological version of a double slit experiment

Busemeyer, Wang, Mogiliansky-Lambert
(2009, *J. of Mathematical Psychology*)

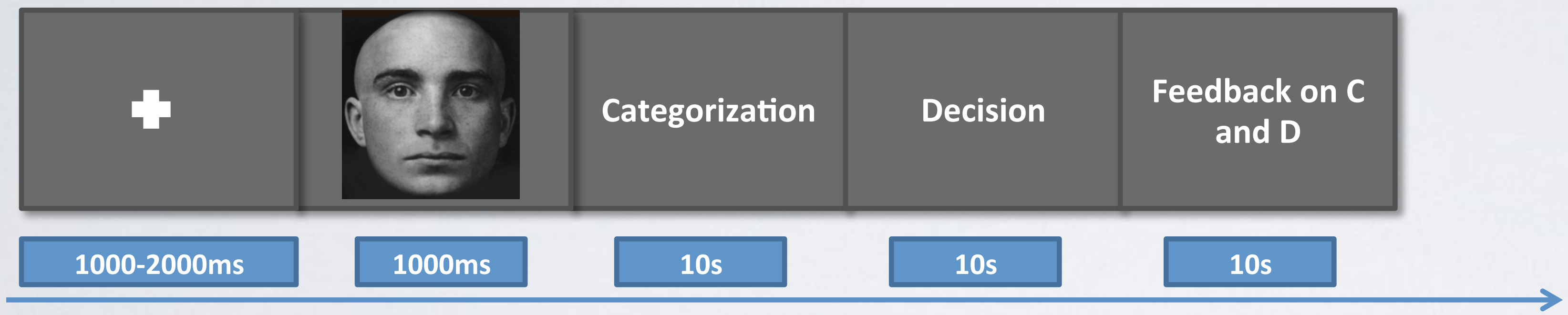
Participants shown pictures of faces

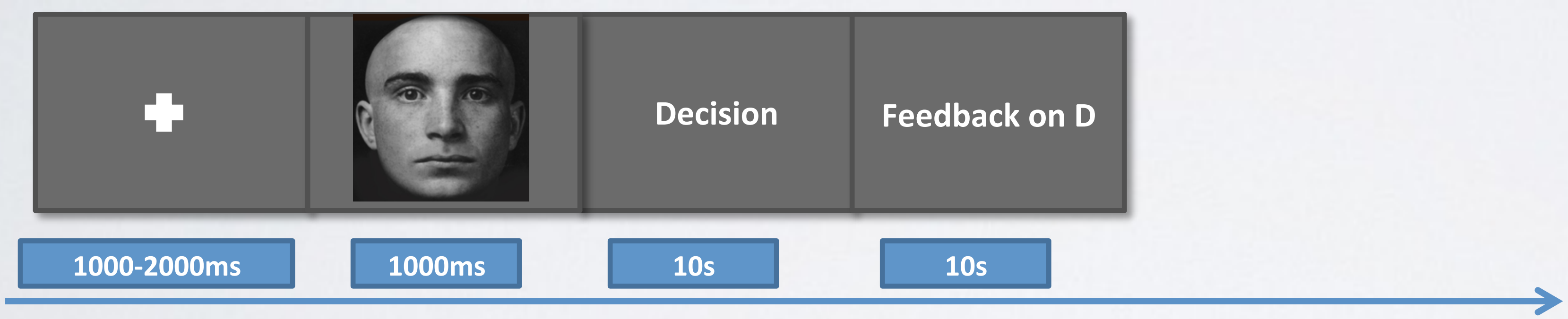
Categorize as "good" guy or "bad" guy
Decide to act "friendly" or "aggressive"

Bad Guys   Good Guys

Two Conditions:

C-then-D: Categorize face first and then decide action





D-alone: Decide without categorization

LAW OF TOTAL PROBABILITY

G =good guy, B=Bad guy, A=Attack

$$p(A) = p(G)p(A|G) + p(B)p(A|B)$$

D alone Condition

C-then-D Condition

RESULTS

Face	p(G)	p(A|G)	p(B)	p(A|B)	TP	P(A)
Good	0.84	0.35	0.16	0.52	0.37	0.39
Bad	0.17	0.41	0.82	0.63	0.59	**0.69**

QUANTUM INTERFERENCE

$$p(A \mid D\ alone) = \left\| P_A S \right\|^2 = \left\| P_A \cdot I \cdot S \right\|^2$$

$$= \left\| P_A \cdot \left(P_G + P_B \right) \cdot S \right\|^2$$

$$= \left\| P_A \cdot P_G \cdot S + P_A \cdot P_B \cdot S \right\|^2$$

$$= \left\| P_A \cdot P_G \cdot S \right\|^2 + \left\| P_A \cdot P_B \cdot S \right\|^2 + Int$$

Interference term violates of Law of Total Probability

$$Int = \langle S \mid P_G P_A P_A P_B \mid S \rangle + \langle S \mid P_B P_A P_A P_G \mid S \rangle$$

$$Finding \rightarrow Int > 0$$

VIOLATIONS OF RATIONAL DECISION THEORY

Shafir & Tversky
(1992, *Psychological Science*)

Pothos & Busemeyer
(2009, *Proceedings of Royal Society*)

PRISONER DILEMMA GAME

SHAFIR & TVERSKY (1992, COGNITIVE PSYCH)

	OD	OC
PD	O: 10 P: 10	O:5 P: 25
PC	O: 25 P: 5	O:20 P: 20

Examined three conditions in a prisoner dilemma task

Known Coop: Player is told other opponent will cooperate
Known Defect: Player is told other opponent will defect
UnKnown: Player is told nothing about the opponent

LAW OF TOTAL PROBABILITY

$p(PD) =$ probability player defects when opponent's move is unknown

$$p(PD) = p(OD)p(PD \mid OD) + p(OC)p(PD \mid OC)$$

Empirically we find : $p(PD \mid OD) \geq p(PD \mid OC)$

$$\rightarrow p(PD \mid OD) \geq p(PD) \geq p(PD \mid OC)$$

DEFECT RATE FOR TWO EXPERIMENTS

Study	Known Defect	Known Coop	Unknown
Shafir (1992)	0.97	0.84	0.63
Matthew (2006)	0.91	0.84	0.66
Avg	0.94	0.84	**0.65**

QUANTUM INTERFERENCE

$$p(PD) = \left\|P_{PD}S\right\|^2 = \left\|P_{PD} \cdot I \cdot S\right\|^2$$

$$= \left\|P_{PD} \cdot \left(P_{OD} + P_{OC}\right) \cdot S\right\|^2$$

$$= \left\|P_{PD} \cdot P_{OD} \cdot S + P_{PD} \cdot P_{OC} \cdot S\right\|^2$$

$$= \left\|P_{PD} \cdot P_{OD} \cdot S\right\|^2 + \left\|P_{PD} \cdot P_{OC} \cdot S\right\|^2 + Int$$

$$Int = \left\langle S \mid P_{OC}P_{PD}P_{PD}P_{OD} \mid S\right\rangle + \left\langle S \mid P_{OD}P_{PD}P_{PD}P_{OC} \mid S\right\rangle$$

ATTITUDE QUESTION ORDER EFFECTS

Moore
(2002, *Public Opinion Quarterly*)

Wang, Solloway, Shiffrin, & Busemeyer
(2013, *Proceedings National Academy of Science*)

Question Order Effects: Assimilation

- A Gallup Poll question in 1997, N = 1002, **"Yes"**

- Do you generally think Bill Clinton is honest and trustworthy? **(50%)**

- How about Al Gore?
- **(60%)**

- Do you generally think Al Gore is honest and trustworthy? **(68%)**

- How about Bill Clinton? **(57%)**

Observed proportions in the two question orders

Clinton-Gore

	Gy	Gn
Cy	.4899	.0447
Cn	.1767	.2886

White-Black

	By	Bn
Wy	.3987	.0174
Wn	.1612	.4227

Gore-Clinton

	Gy	Gn
Cy	.5625	.0255
Cn	.1991	.2130

Black-White

	By	Bn
Wy	.4012	.1379
Wn	.0597	.4012

Context Effects*

	Gy	Gn
Cy	-.0726	.0192
Cn	-.0224	.0756

Context Effects

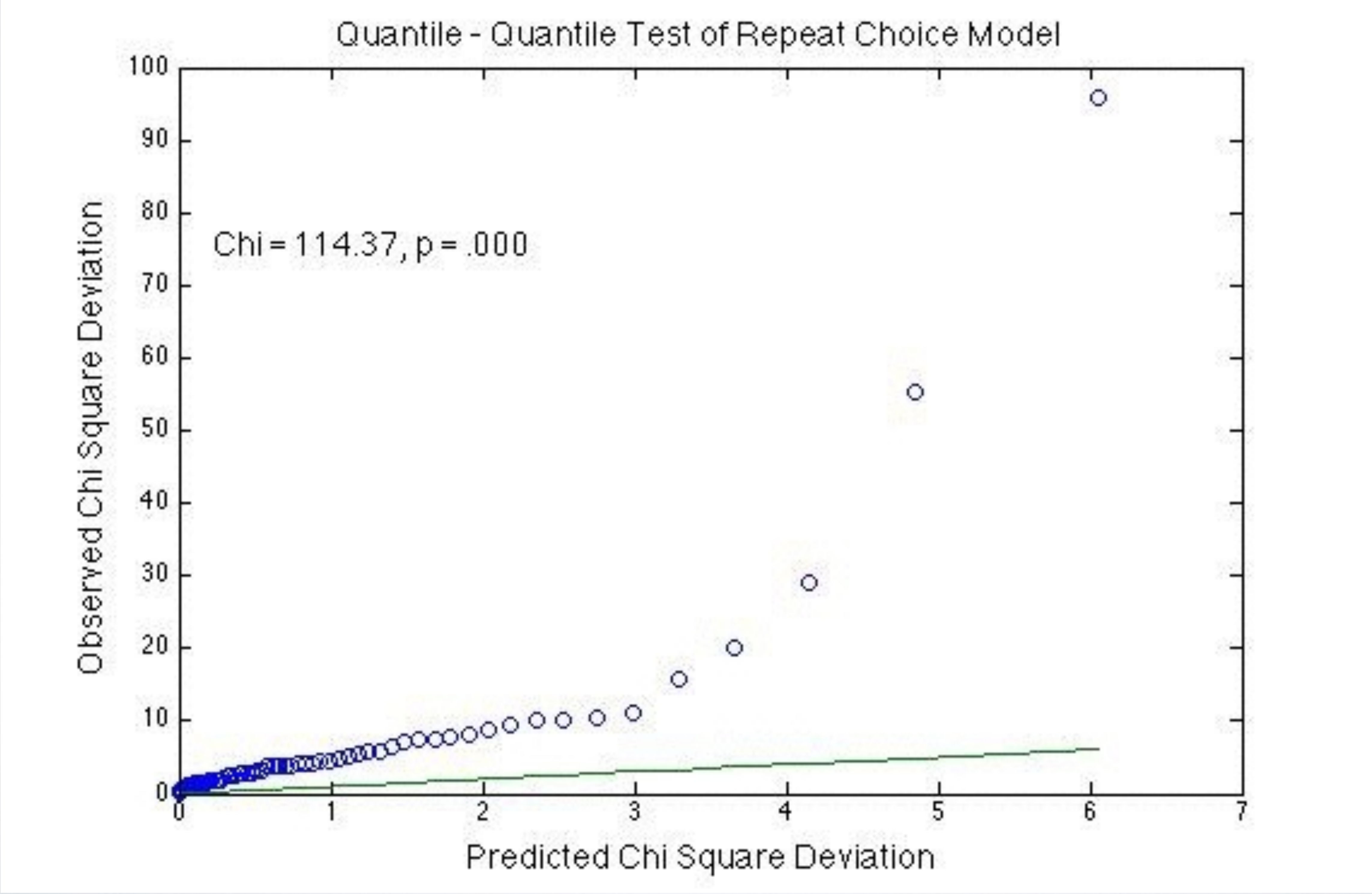
	By	Bn
Wy	-.0025	-.1205
Wn	.1015	.0215

Test order effects:

χ^2 (3) = 10.14, $p < .05$

χ^2 (3) = 73.04, $p < .001$

Results: 72 Pew Surveys over 10 years



QUANTUM MODEL PREDICTION

Assume: One question followed immediately by another with no information in between

$$\Pr[A \text{ yes and then B no}] = p(A_Y B_N) = \left\| P_{\bar{B}} P_A S \right\|^2$$

$$\Pr[B \text{ no and then A yes}] = p(B_N A_Y) = \left\| P_A P_{\bar{B}} S \right\|^2$$

$$\textit{Theorem}: \text{ QQ equality}$$

$$q = \{p(A_Y B_N) + p(A_N B_Y)\} - \{p(B_Y A_N) + p(B_N A_Y)\} = 0$$

Clinton-Gore

	Gy	Gn
Cy	.4899	.0447
Cn	.1767	.2886

White-Black

	By	Bn
Wy	.3987	.0174
Wn	.1612	.4227

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Context Effects

	By	Bn
Wy	-.0025	-.1205
Wn	.1015	.0215

Test order effects:

χ^2 (3) = 10.14, $p < .05$

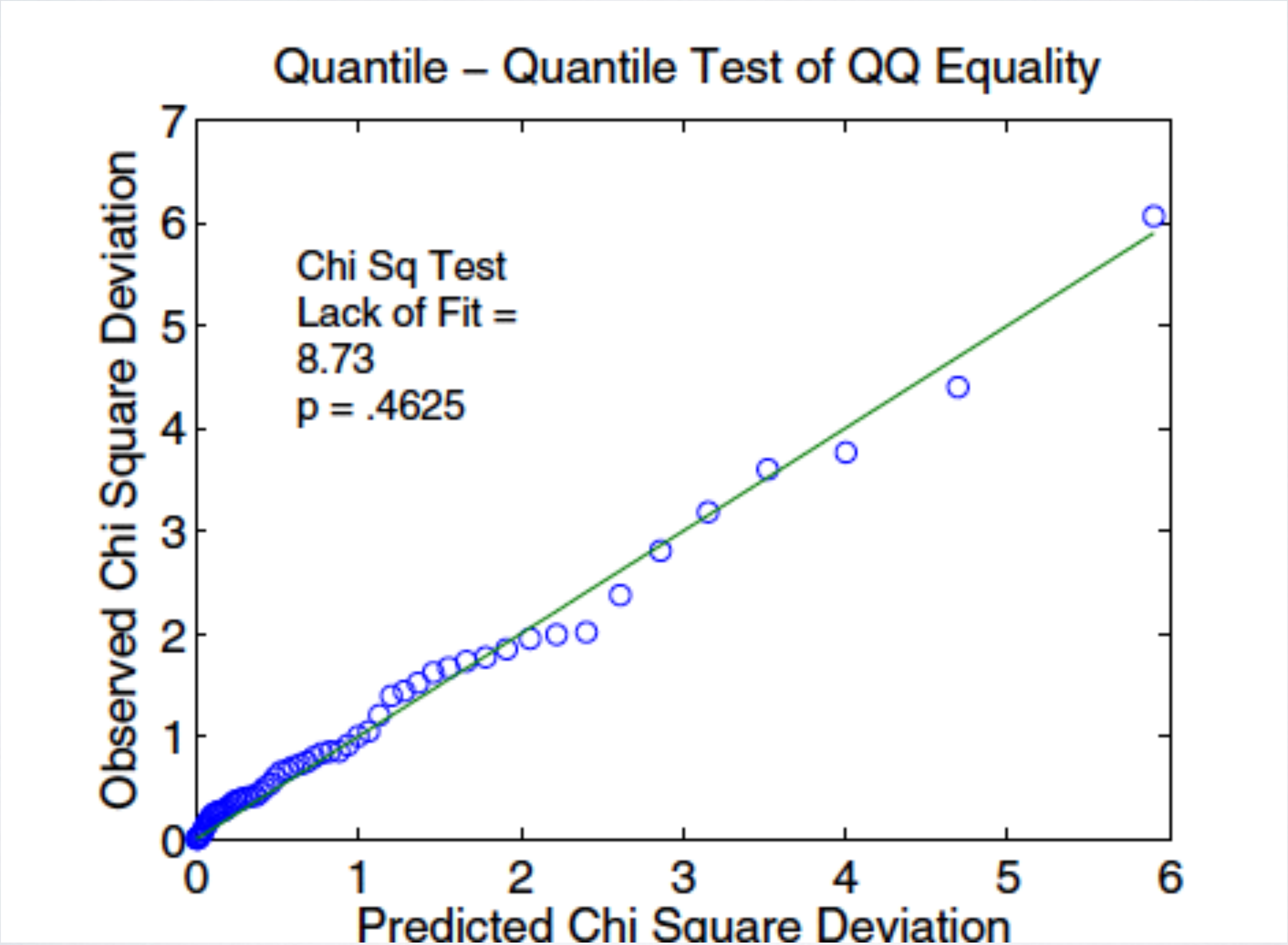
χ^2 (3) = 73.04, $p < .001$

Test QQ equality:

$q = -.003$
χ^2 (1) = .01, $p = .91$

$q = -.02$
χ^2 (1) = .56, $p = .46$

Results: 72 Pew Surveys over 10 years

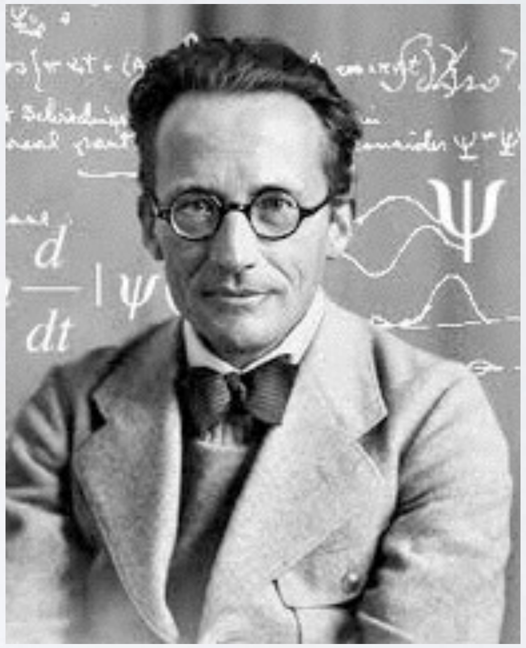


4. DYNAMICS

COMPARISON OF MARKOV AND QUANTUM THEORIES

Markov

Schrödinger



Markov

$N =$ no. Markov states

$p_i =$ prob state i

$\text{Prob[state i]} = p_i$

$p = [p_i] = N \times 1$ vector

$$\sum_i p_i = 1$$

Quantum

$N =$ no. eigen states

$\psi_i =$ amplitude state i

$\text{Prob[state i]} = |\psi_i|^2$

$\psi = [\psi_i] = N \times 1$ vector

$$\sum_i |\psi_i|^2 = 1$$

Markov

$T = N \times N$ matrix

T_{ij} = prob transit j to i

$\sum_i T_{ij} = 1$ (stochastic)

$p(t) = T(t) \cdot p(0)$

Observe state i at time t

(State reduction)

$p(t \mid i) = [0,0,..,1,..0]'$

$p(t+s) = T(s) \cdot p(t \mid i)$

Quantum

$U = N \times N$ matrix

U_{ij} = amp transit j to i

$U^{\dagger}U = I$ (unitary)

$\psi(t) = U(t) \cdot \psi(0)$

Observe state i at time t

(State Reduction)

$\psi(t \mid i) = [0,0,..,1,..0]'$

$\psi(t+s) = U(s) \cdot \psi(t \mid i)$

Markov

Kolmogorov Eq

$$\frac{d}{dt}T(t) = K \cdot T(t)$$

Intensity Matrix

$$K = [k_{ij}]$$

$$k_{ij} > 0, i \neq j,$$

$$\sum_j k_{ij} = 0$$

Quantum

Schrödinger Eq

$$i\frac{d}{dt}U(t) = H \cdot U(t)$$

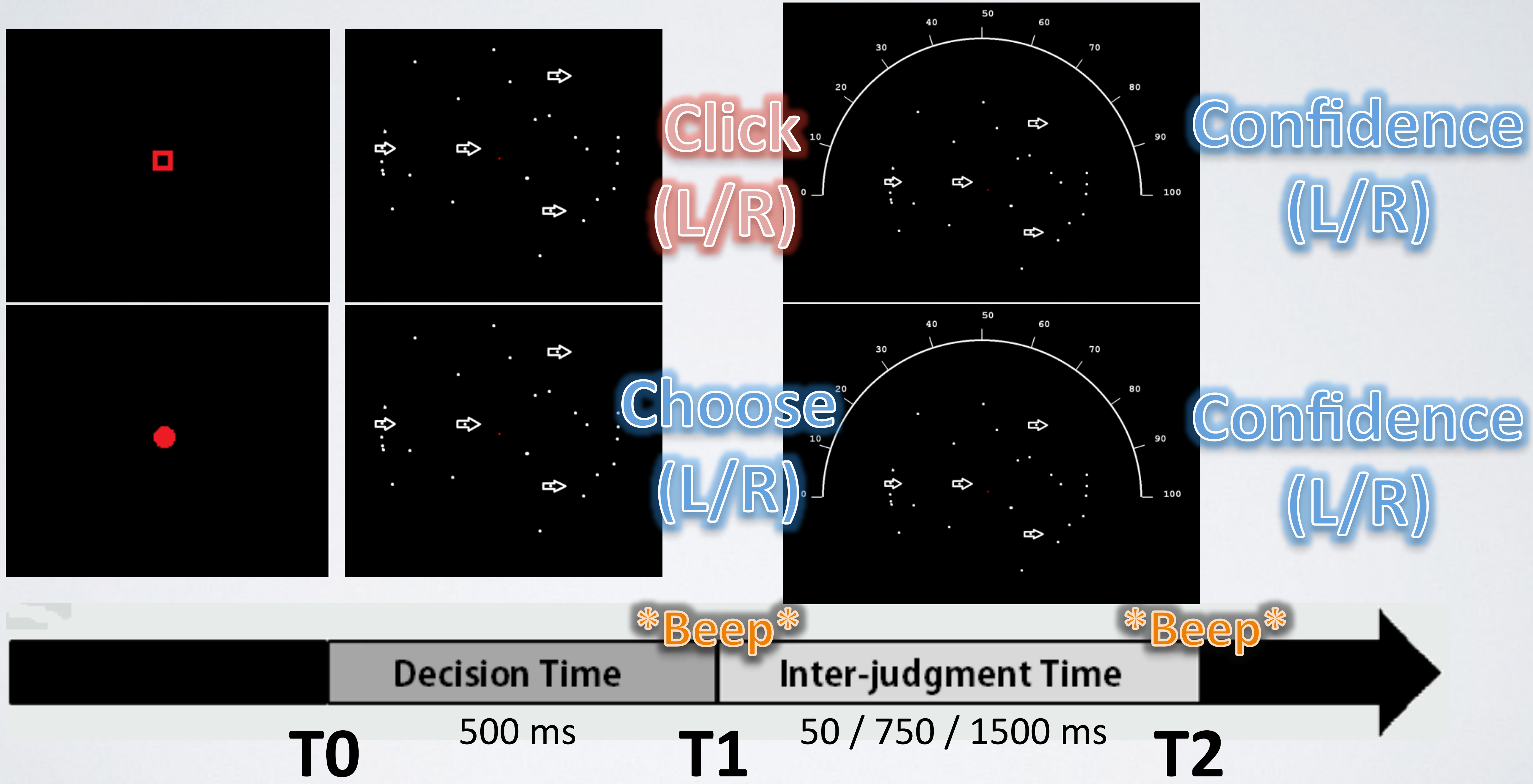
Hamiltonian Matrix

$H = H^{\dagger}$, Hermitian

RANDOM WALK MODELS OF DECISION MAKING

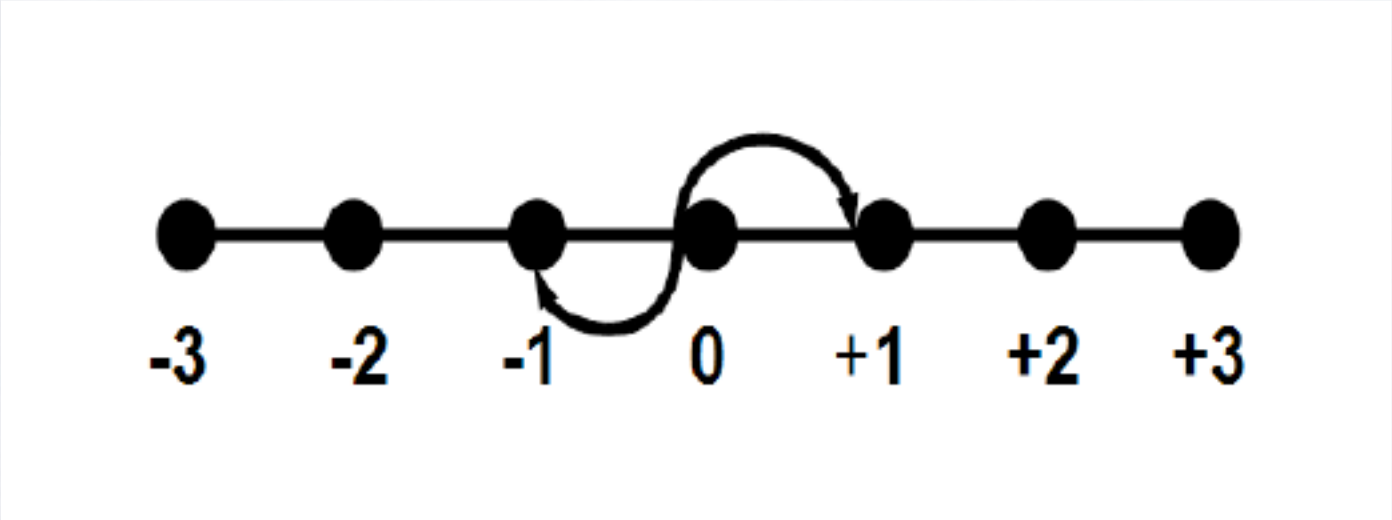
Kvam, Pleskac, Busemeyer
(Proceedings of the National Academy of Science)

Random Dot Motion Task



N=7 state Random Walk Model of Confidence

(Toy example, actual model uses N=101 states)



Confident Signal Not Present

Uncertain

Confident Signal Present

CRITICAL TEST OF MODELS

Condition 1: Measure confidence only at t_2
Condition 2: Measure choice at t_1 and confidence at t_2

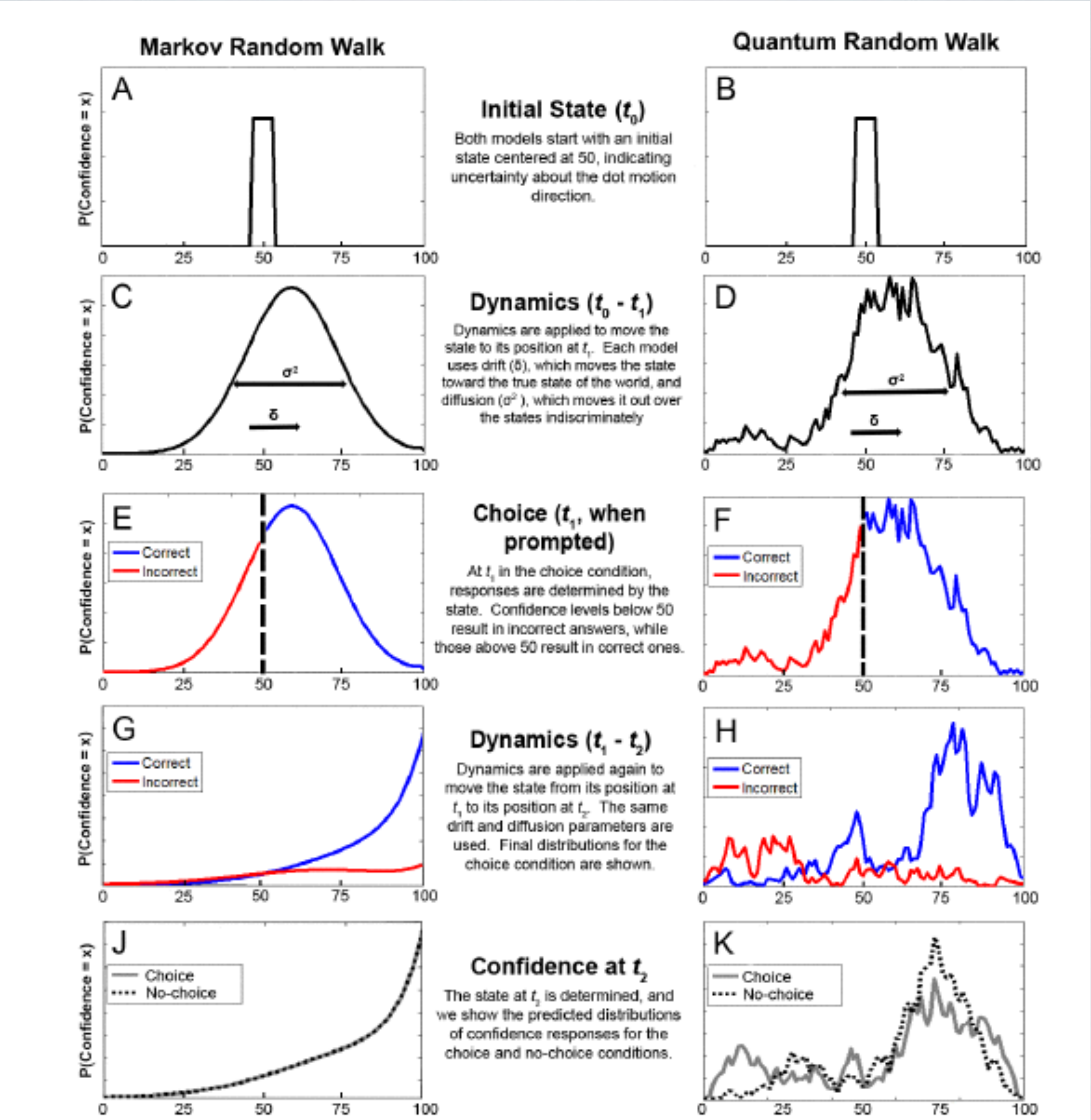
Markov

$$p(C(t_2) = k \mid Cond2) = p(C(t_2) = k \mid Cond1)$$

Quantum

$$p(C(t_2) = k \mid Cond2) \neq p(C(t_2) = k \mid Cond1)$$

Statistically test distribution differences using Kolmogorov- Smirnov Statistic


Markov Random Walk
Quantum Random Walk
Initial State (t_0)
Both models start with an initial state centered at 50, indicating uncertainty about the dot motion direction.
Dynamics (t_0 - t_1)
Dynamics are applied to move the state to its position at t_1. Each model uses drift (δ), which moves the state toward the true state of the world, and diffusion (σ^2), which moves it out over the states indiscriminately
Choice (t_1, when prompted)
At t_1 in the choice condition, responses are determined by the state. Confidence levels below 50 result in incorrect answers, while those above 50 result in correct ones.
Dynamics (t_1 - t_2)
Dynamics are applied again to move the state from its position at t_1 to its position at t_2. The same drift and diffusion parameters are used. Final distributions for the choice condition are shown.
Confidence at t_2
The state at t_2 is determined, and we show the predicted distributions of confidence responses for the choice and no-choice conditions.
P(Confidence = x)
A
B
C
D
E
F
G
H
J
K
σ^2
δ
Correct
Incorrect
Choice
No-choice
0
25
50
75
100

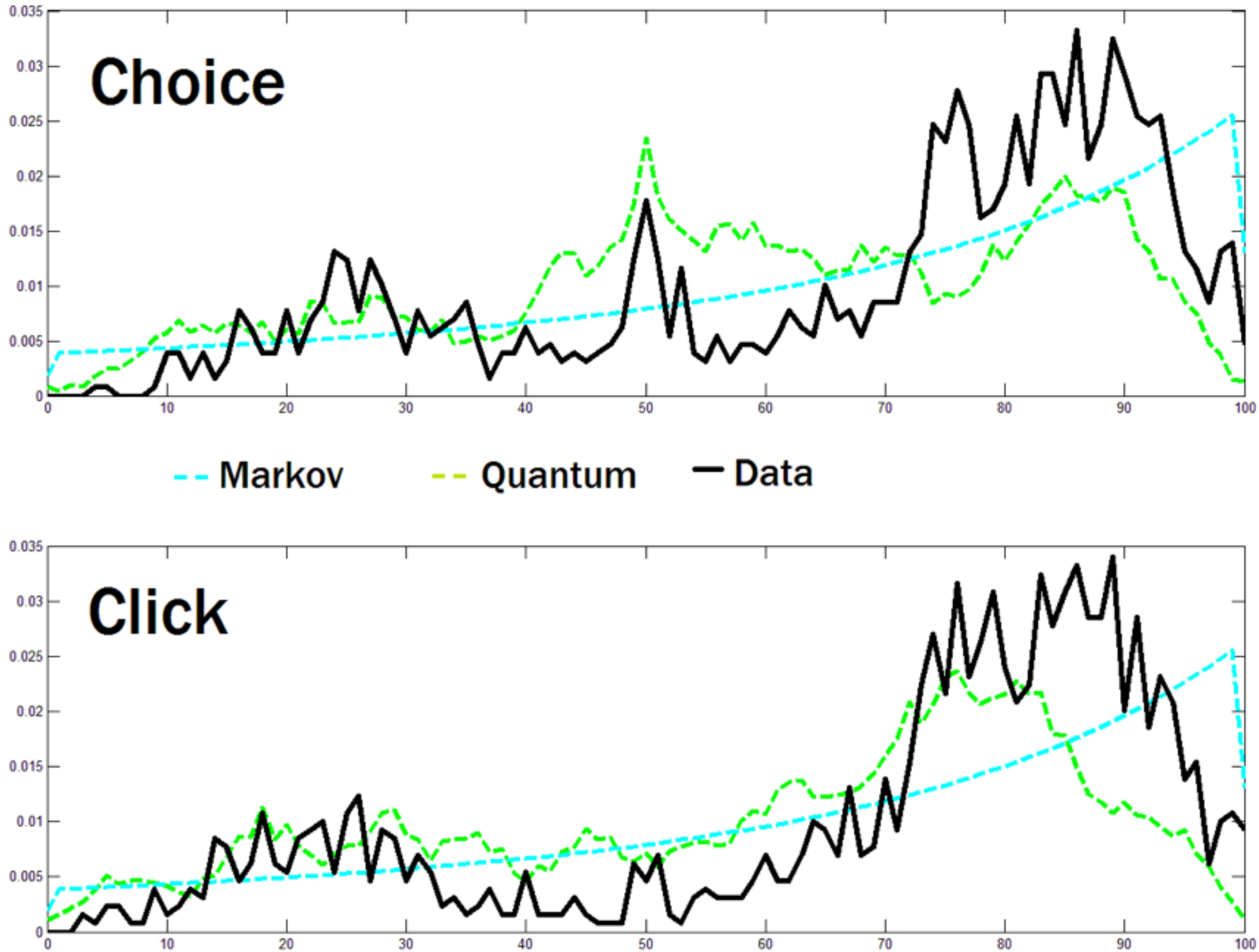

Choice
Markov
Quantum
Data
Click
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0
10
20
30
40
50
60
70
80
90
100

Table 1. Summary of model comparison and statistical effects.

Participant	Interference*	Second-Stage Processing†	Log Bayes Factor
1	-0.18 [-0.26, -0.11]‡	0.12 [0.08, 0.18]‡	212
2	-0.15 [-0.23, -0.07]‡	0.08 [0.03, 0.14]‡	41
3	-0.15 [-0.22, -0.07]‡	0.01 [-0.04, 0.06]	-131
4	-0.14 [-0.23, -0.07]‡	0.10 [0.04, 0.15]‡	190
5	-0.11 [-0.19, -0.04]‡	0.07 [0.02, 0.13]‡	837
6	-0.08 [-0.16, -0.01]‡	0.13 [0.07, 0.18]‡	223
7	-0.07 [-0.15, 0.01]	-0.01 [-0.07, 0.05]	-148
8	-0.05 [-0.14, 0.02]	0.04 [-0.08, 0.10]	339
9	-0.01 [-0.09, 0.07]	-0.02 [-0.06, 0.04]	150
Group Level	-0.11 [-0.18, -0.04]‡	0.06 [0.01, 0.12]‡	1713

*The mean posterior coefficient and 95% HDI for the main effect of the choice / click manipulation on half-scale confidence

†The mean posterior coefficient and 95% HDI for the interaction between dot coherence and second stage processing time on full-scale confidence.

‡95% highest density interval for the estimate of the corresponding parameter excluded zero.

CONCLUSIONS

- Quantum theory provides an alternative framework for developing probabilistic and dynamic models of decision making
- Provides a coherent account for puzzling violations of law of total probability found in a variety of decision making studies
- Forms a new foundation for understanding widely different phenomena in decision making using a common set of axiomatic principles

"Mathematical models of cognition so often seem like mere formal exercises. Quantum theory is a rare exception. Without sacrificing formal rigor, it captures deep insights about the workings of the mind with elegant simplicity. This book promises to revolutionize the way we think about thinking."

Steven Sloman
Cognitive, Linguistic, and Psychological Sciences, Brown University

"This book is about why and how formal structures of quantum theory are essential for psychology - a breakthrough resolving long-standing problems and suggesting novel routes for future research, convincingly presented by two main experts in the field."

Harald Atmanspacher
Department of Theory and Data Analysis, Institut fuer Grenzgebiete der Psychologie und Psychohygiene e.V.

<FURTHER ENDORSEMENT TO FOLLOW>

Much of our understanding of human thinking is based on probabilistic models. This innovative book by Jerome R. Busemeyer and Peter D. Bruza argues that, actually, the underlying mathematical structures from quantum theory provide a much better account of human thinking than traditional models. They introduce the foundations for modeling probabilistic-dynamic systems using two aspects of quantum theory. The first, "contextuality," is a way to understand interference effects found with inferences and decisions under conditions of uncertainty. The second, "quantum entanglement," allows cognitive phenomena to be modeled in non-reductionist way. Employing these principles drawn from quantum theory allows us to view human cognition and decision in a totally new light. Introducing the basic principles in an easy-to-follow way, this book does not assume a physics background or a quantum brain and comes complete with a tutorial and fully worked-out applications in important areas of cognition and decision.

Jerome R. Busemeyer is a Professor in the Department of Psychological and Brain Sciences at Indiana University, Bloomington, USA.

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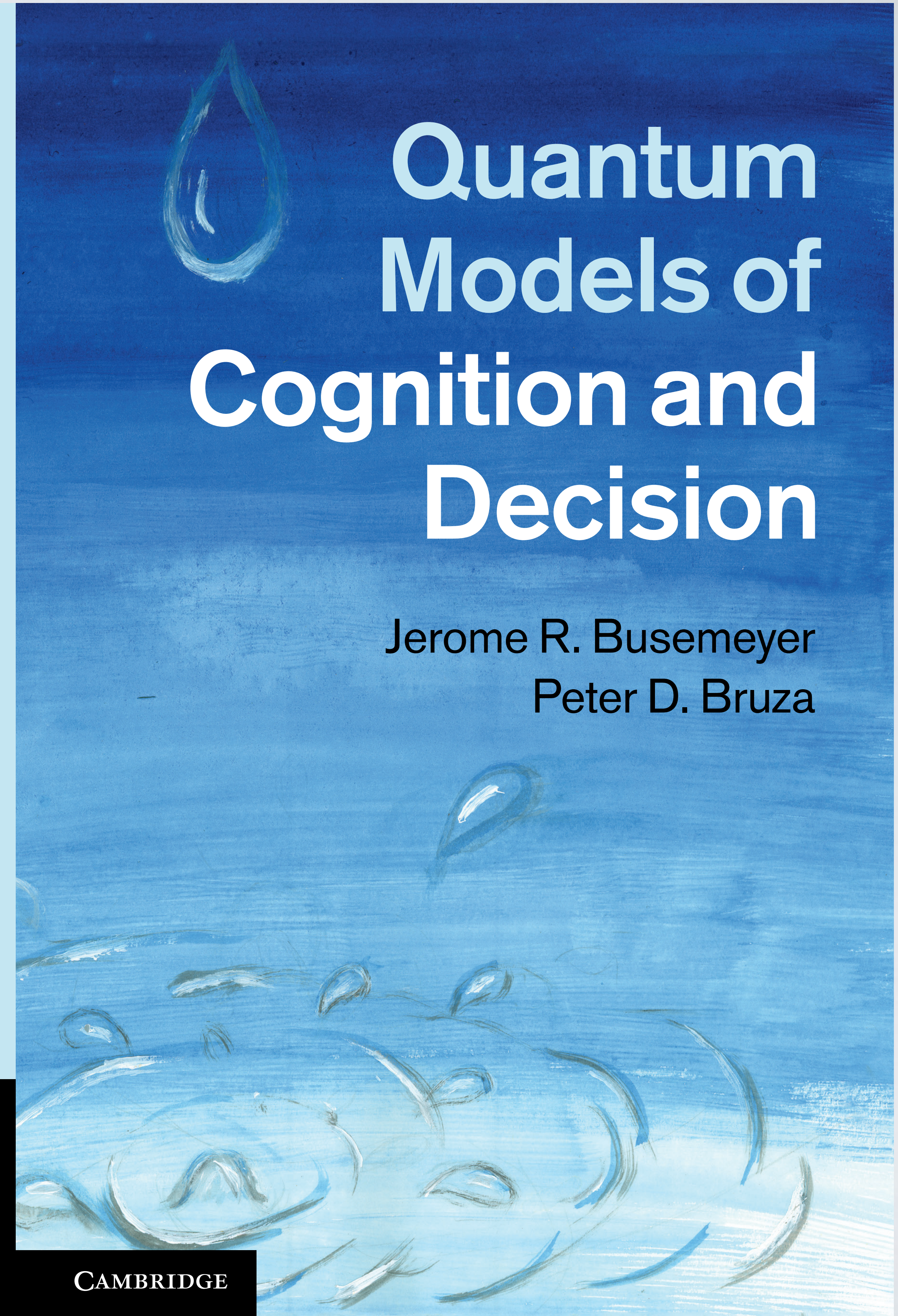

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