CTEQ School on
QCD Analysis and Electroweak Phenomenology

LECTURE 1

Introduction to the Parton Model and Perturbative QCD

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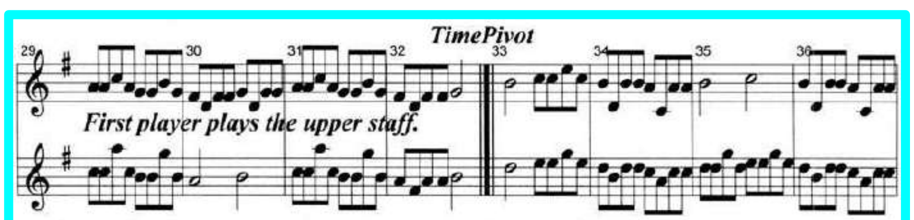
Introduction:

Welcome to QCD:

$$\boxed{\begin{aligned}\mathcal{L}_{\rm QCD} &= \bar{\psi}_i \left(i\gamma^\mu (D_\mu)_{ij} - m\,\delta_{ij}\right)\psi_j - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} \\ &= \bar{\psi}_i (i\gamma^\mu \partial_\mu - m)\psi_i - g G^a_\mu \bar{\psi}_i \gamma^\mu T^a_{ij} \psi_j - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu},\end{aligned}}$$



Mozart: Inverted retrograde canon in G

Patterns, Symmetry (obvious & hidden), interpretation

Notes ⇒ themes ⇒ Melody/Harmony ⇒ interaction/counterpoint ⇒ structure

Welcome to QCD:

$$\boxed{\begin{aligned}\mathcal{L}_{\rm QCD} &= \bar{\psi}_i \left(i\gamma^\mu (D_\mu)_{ij} - m\,\delta_{ij}\right)\psi_j - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} \\ &= \bar{\psi}_i (i\gamma^\mu \partial_\mu - m)\psi_i - g G^a_\mu \bar{\psi}_i \gamma^\mu T^a_{ij} \psi_j - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu},\end{aligned}}$$

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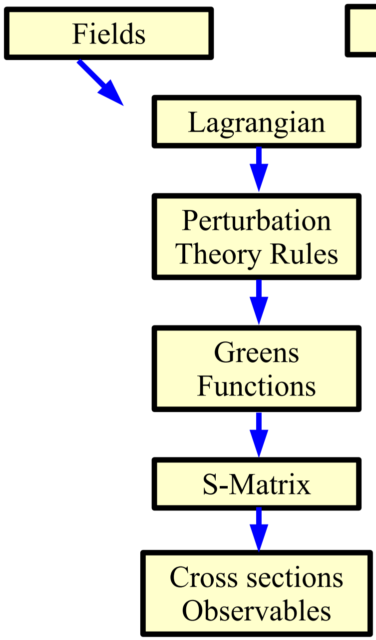
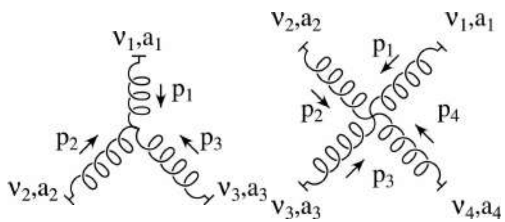
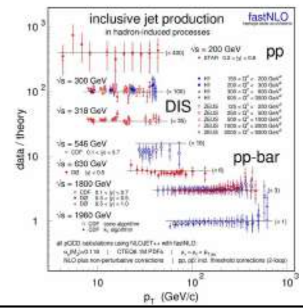
The Game

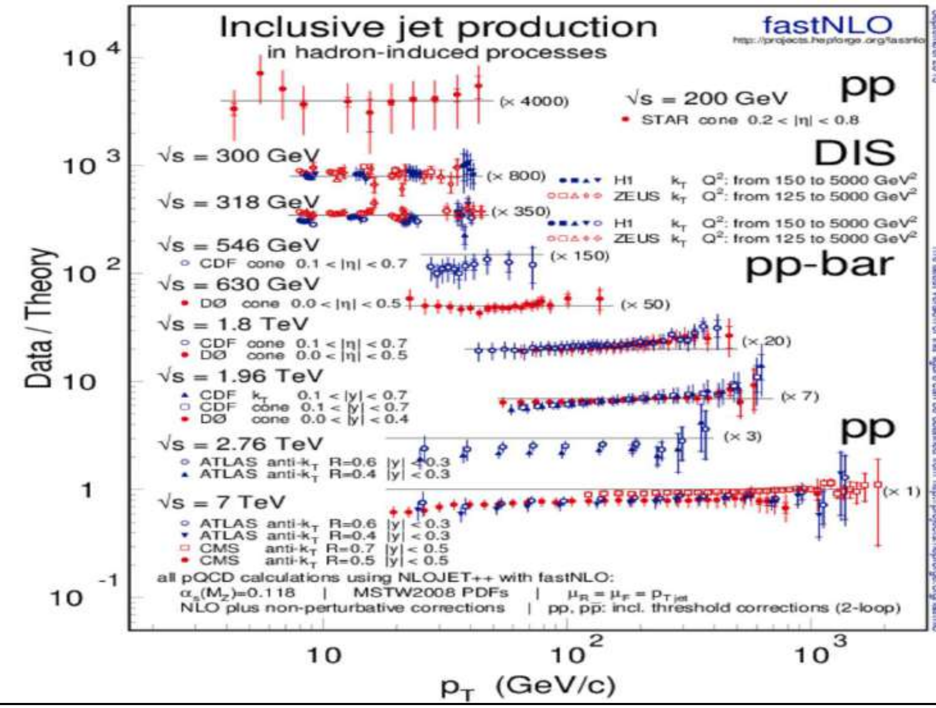


$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i \left(i\gamma^\mu (D_\mu)_{ij} - m\,\delta_{ij}\right)\psi_j - \frac{1}{4}G^a_{\mu\nu}G_a^{\mu\nu}$$





QCD does a remarkable job!!!

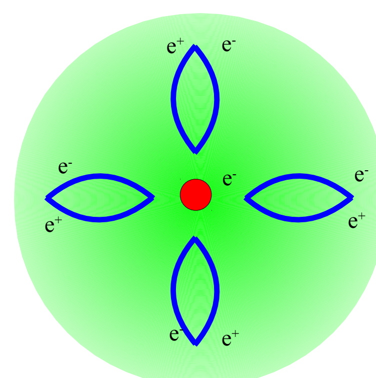


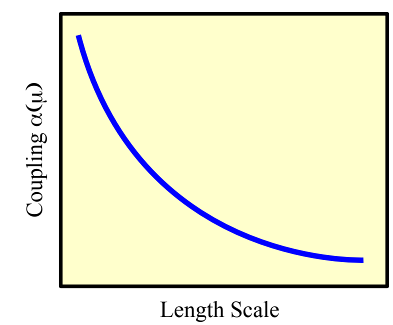
QCD is just like QED,

.... only different

QCD is just like QED, only different ...

QED: Abelian U(1) Symmetry





Perturbation theory at large distance is convergent

$$\alpha(\infty) \sim \frac{1}{137}$$

$$\alpha(M_Z) \sim \frac{1}{128}$$

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\ ,$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - 0$$

Abelian

α is good expansion parameter

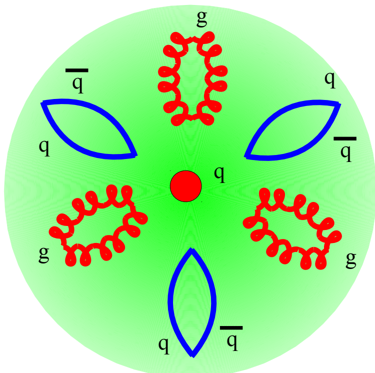
QCD is Non-Abelian SU(3) Symmetry; Quarks are Confined!!! 9

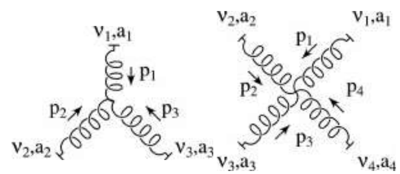
$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}_i \left(i\gamma^\mu (D_\mu)_{ij} - m\,\delta_{ij}\right)\psi_j - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu}$$

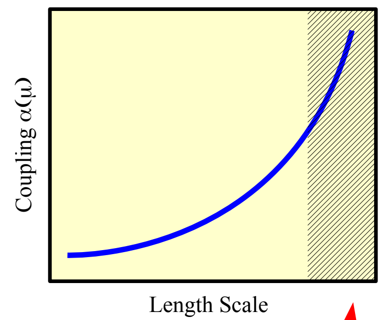
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$$

Non-Abelian

$$[T_a, T_b] = i \sum_{c=1}^{8} f_{abc} T_c$$







$\alpha_s(M_Z) \sim 0.118$

Cannot perform Perturbation theory at large distance

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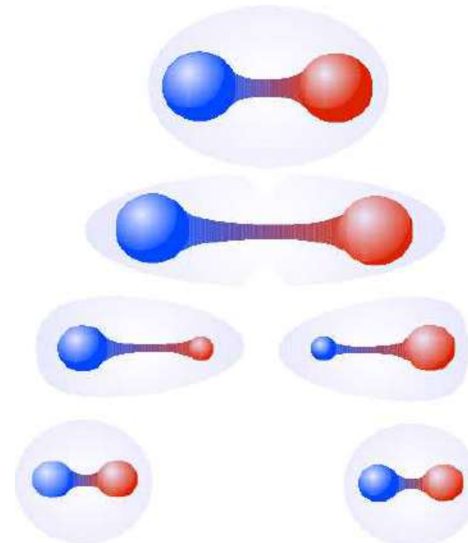
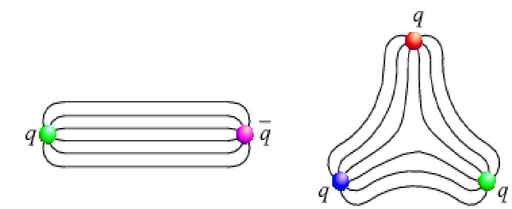
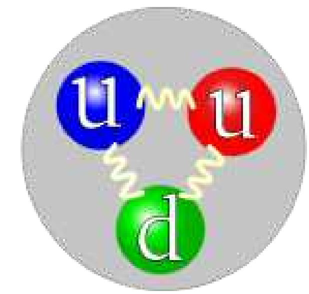
Quark Confinement & String Interpretation 10



Thomas Lippert, NIC-ZAM, Jülich, for the SESAM Collaboration)



http://www.scholarpedia.org/



http://en.wikipedia.org/

Quarks are confined

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Statement of the problem 11

Theorist #1: The universe is completely described by the symmetry group SO(10)

Theorist #2: You're wrong; the correct answer is SuperSymmetric flipped SU(5)xU(1)

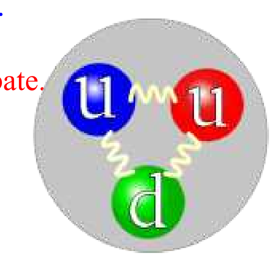
Theorist #3: You've flipped! The only rational choice is E8xE8 dictated by SuperString Theology.

Experimentalist: Enough of this speculative nonsense. I'm going to measure something to settle this question. What can you predict???

Theorist #1: We can predict the interactions between fundamental particles such as quarks and leptons.

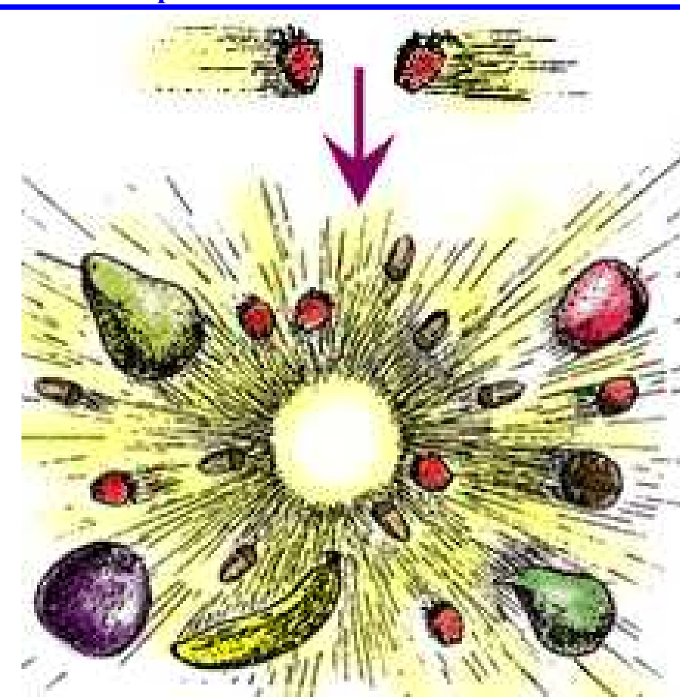
Experimentalist: Great! Give me a beam of quarks and leptons, and I can settle this debate.

Accelerator Operator: Sorry, quarks only come in a 3-pack and we can't break a set!



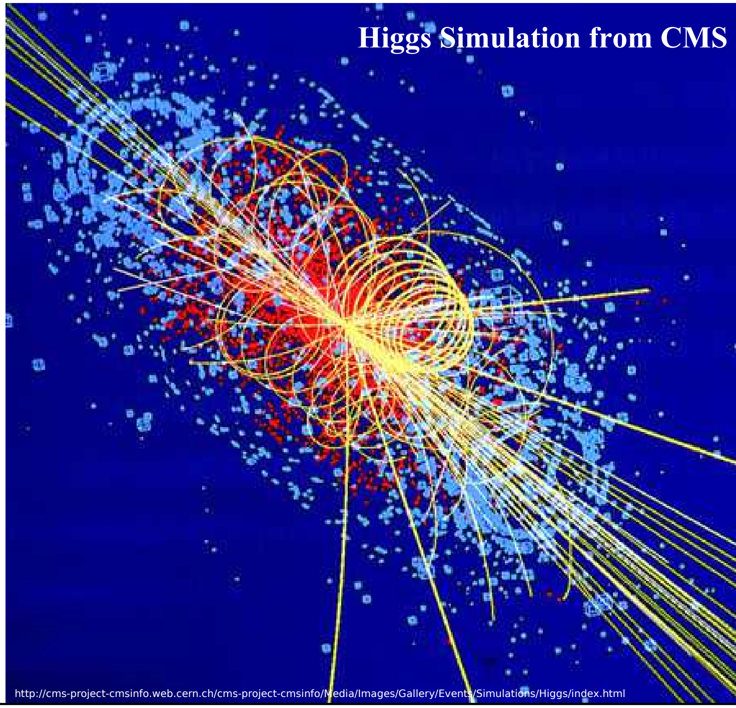
http://en.wikipedia.org/

One interpretation of a hadron-hadron collision 12



Did we find the Higgs?

A bit more realistic interpretation of a hadron-hadron collision



OUTLINE

QCD is a theory with a rich structure

Working in the limit of a spherical horse ...

We are going to look at the essence of what makes QCD so different from the other forces.

As a consequence, we will need to be creative in how we study the properties, now we define observables, and interpret the results.

QCD has a history of more than 40 years, and we are still trying to fully understand its structure.

The goal of these lectures

Provide pictorial/graphical/heuristic explanations for everything that confused me as a student

BEFORE

$\Rightarrow$

AFTER

Overview ... what is to come

Lecture 1:

Overview& essential features
Nature of strong coupling constant
& how it varies with scale
Issues beyond LO and SM
Renormalization Group Equation &
Resummation
Scaling and the proton Structure

Lecture 2:

cf., Simona Malace

The structure of the proton
Deeply Inelastic Scattering (DIS)
The Parton Model
PDF's & Evolution
Scaling and Scale Violation

cf., Pavel Nadolsky

Lecture 3:

Issues at NLO
Collinear and Soft Singularities
Mandelstam Variables
An example from Freshman Physics
Regularized Distributions
Extension to higher orders

Lecture 4:

cf., Radja Boughezal

Drell-Yan and e^+e^- Processes
W/Z/Higgs Production & Kinematics
3-body Phase Space & Dalitz Plots
Sterman-Weinberg Jets
Infrared Safe Observables
Rapidity & Pseudo Rapidity
Jet Definitions

cf., Dave Soper & Andrew Larkoski

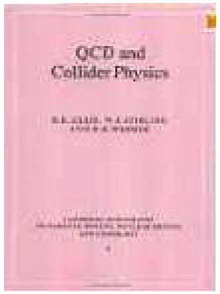
Overview ... what is to come

Homework:

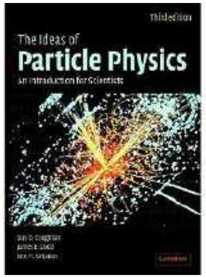
Physics is not a spectator sport

Useful References & Thanks:

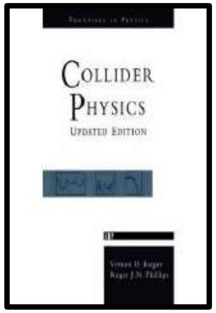
Useful References



Ellis, Stirling, Webber



Coughlan, Dodd, Gripaios



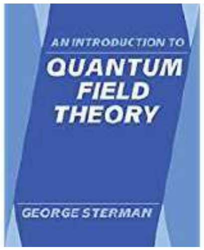
Barger & Phillips

CTEQ Handbook
Reviews of Modern
Physics

An Introduction to QFT
Peskin & Schroeder

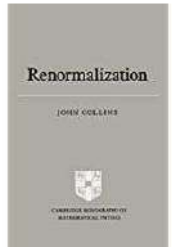
Particle Data Group
http://pdg.lbl.gov

More Useful References



An Introduction to
Quantum Field Theory
George Sterman

Foundations of
Perturbative QCD
John C. Collins



Renormalization:
John C. Collins

Applications of
Perturbative QCD,
Richard D. Field

More Useful References

The CTEQ Pedagogical Page
Linked from **`cteq.org`**

Everything you wanted to know about Lambda-QCD but were afraid to ask
Randall J. Scalise and Fredrick I. Olness

Regularization, Renormalization, and Dimensional Analysis:
Dimensional Regularization meets Freshman E&M
Fredrick Olness & Randall Scalise
e-Print: arXiv:0812.3578

Calculational Techniques in Perturbative QCD: The Drell-Yan Process.
Björn Pötter has prepared a writeup of the lecture given by Jack Smith.
This is a wonderful reference for those learning to do real 1-loop calculations.

Thanks to ...

Thanks to:

Dave Soper, George Sterman,
John Collins, & Jeff Owens for ideas
borrowed from previous CTEQ
introductory lecturers

Thanks to Randy Scalise for the help on
the Dimensional Regularization.

Thanks to my friends at Grenoble who
helped with suggestions and corrections.

Thanks to Jeff Owens for help on
Drell-Yan and Resummation.

To the CTEQ and MCnet folks
for making all this possible.

LibreOffice
The Document Foundation

and the many web pages where I borrowed my figures ...

PowerPoint

The Strong Coupling, Scaling, and Stuff

QCD is Non-Abelian SU(3) Symmetry; Quarks are Confined

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}_i \left(i\gamma^\mu (D_\mu)_{ij} - m\,\delta_{ij}\right)\psi_j - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu}$$

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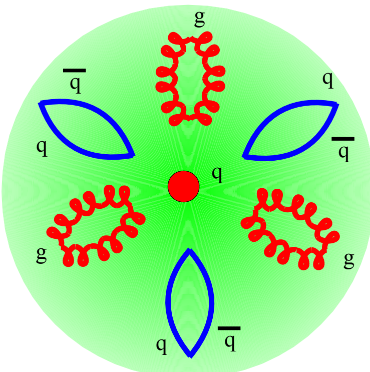
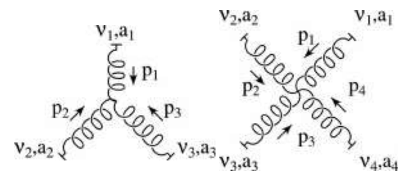
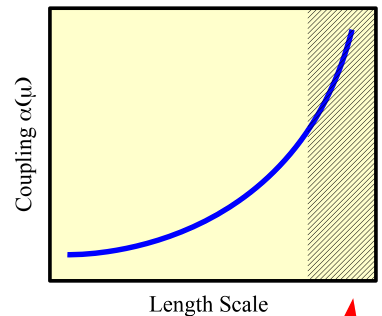






Cannot perform Perturbation theory at large distance

$\Lambda \sim 200\,\mathrm{MeV} \sim 1\,\mathrm{fm}$

Where did that β-function come from ???

Consider a physical observable: $R(Q^2/\mu^2, \alpha_S)$

Q is the characteristic energy scale of the problem

μ is an artificial scale we introduce to regulate the calculation (more later)

The Renormalization Group Equation (RGE) is: $\dfrac{dR}{d\mu^2} = 0$

$$\left\{\mu^2 \frac{d}{d\mu^2}\right\} R\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) = 0$$

Using the chain rule:

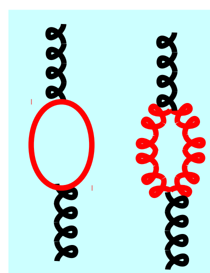
$$\left\{\mu^2 \frac{\partial}{\partial \mu^2} + \underbrace{\left[\mu^2 \frac{\partial \alpha_s(\mu^2)}{\partial \mu^2}\right]}_{\beta(\alpha_s(\mu))} \frac{\partial}{\partial \alpha_s(\mu^2)}\right\} R\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) = 0$$

β function tell us how α_S changes with energy scale!!!

The β-function: $\beta(\alpha_S(\mu))$

β function tell us how α_S changes with energy scale!!!

$$\beta(\alpha_s(\mu)) = \mu^2 \frac{\partial \alpha_s(\mu^2)}{\partial \mu^2} = \frac{\partial \alpha_s(\mu^2)}{\partial \ln \mu^2}$$

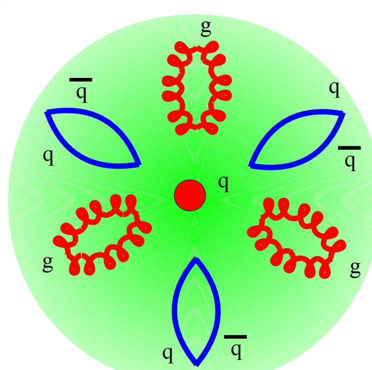


We can calculate this perturbatively

$$\beta(\alpha_s(\mu)) = -\left\{b_0 \alpha_s^2 + b_1 \alpha_s^3 + b_2 \alpha_s^4 + \ldots\right\}$$

$$b_0 = \frac{33 - 2N_F}{12\pi}$$

$$\beta = -\alpha_S^2 \left[\frac{33 - 2N_F}{12\pi}\right] + \ldots$$



Note: b_0 and b_1 are scheme independent.

β is negative; let's find the implications

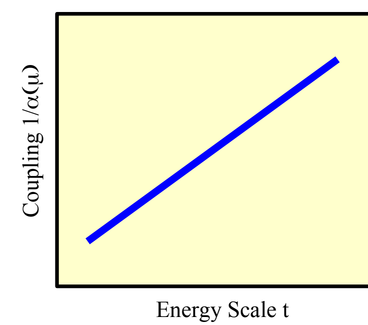
Solve for the running coupling

Let: $t = \ln \mu^2$

$$\beta = \frac{\partial \alpha_s}{\partial t} \simeq -b_0\, \alpha_s^2 + \ldots$$

$$\frac{\partial \alpha_s}{\alpha_s^2} = -b_0\, \partial t$$

$$\frac{1}{\alpha_s} = b_0\, t$$



$$b_0 = \frac{33 - 2N_F}{12\pi} \qquad \beta = -\alpha_S^2 \left[\frac{33 - 2N_F}{12\pi}\right] + \ldots$$

Observe $\beta_{QCD} < 0$ for $N_F < 17$ or for 8 generations or less.

Thus, in general $\beta_{QCD} < 0$ in the QCD theory

Contrast with QED: $\beta_{QED} > 0 = +\alpha^2/3\pi + \ldots$

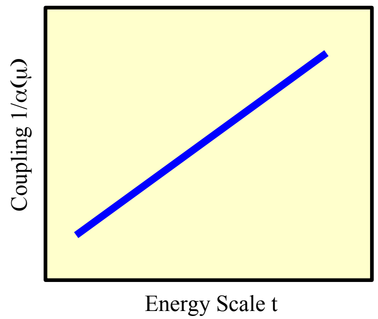
The Nobel Prize in Physics 2004 was awarded jointly to David J. Gross, H. David Politzer and Frank Wilczek "for the discovery of asymptotic freedom

Solve for the running coupling and Λ_{QCD}

Let: $t = \ln \mu^2$

$$\left.\frac{1}{\alpha_S}\right]_{\mu_0}^{\mu_1} = b_0\, t\Big]_{\mu_0}^{\mu_1}$$

$$\frac{1}{\alpha_S(\mu_1)} - \frac{1}{\alpha_S(\mu_0)} = b_0 \ln(\mu_1/\mu_0)$$



β functions gives us running, but we still need a reference

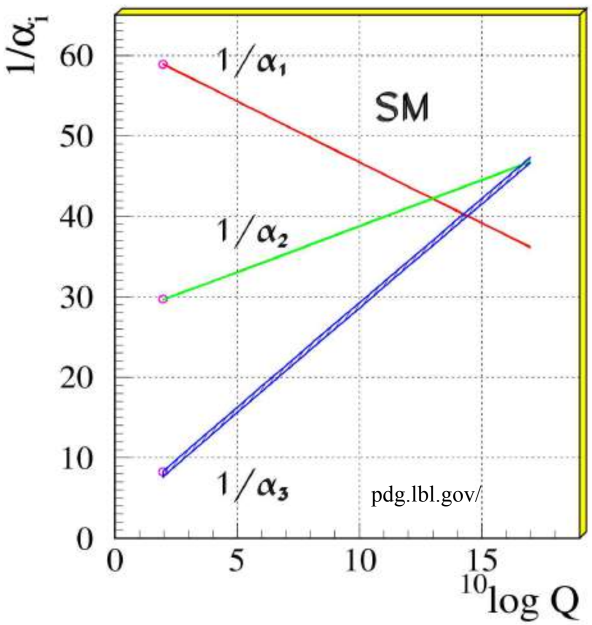
$$\alpha_s(\mu) = \frac{1}{b_0 \ln(\mu/\Lambda_{QCD})}$$

$$\Lambda = \mu\, e^{-1/(b_0\, \alpha_s(\mu))}$$

Landau Pole $\Lambda = \mu$

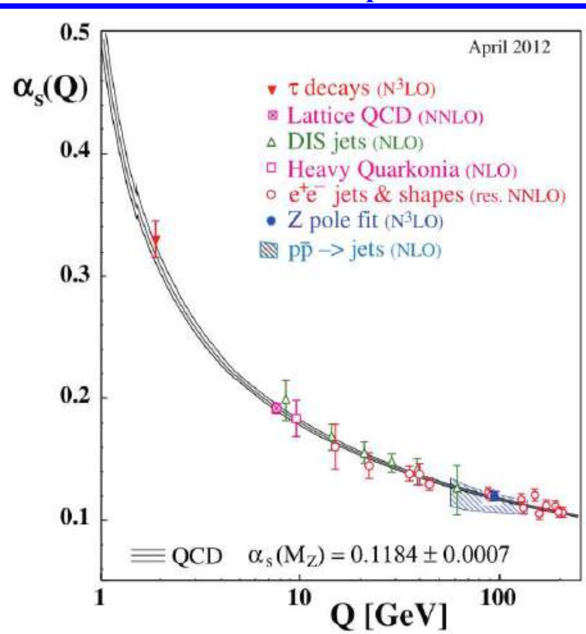
$$\Lambda_{QCD} \sim 200\,\mathrm{MeV} \sim 1\ \mathrm{fm}$$

The Standard Model (SM) Running Couplings: U(1), SU(2), SU(3)



$$\begin{aligned} b_1 &= 0 + (2/3)N_F + \tfrac{1}{10}N_{Higgs} \\ b_2 &= -22/3 + (2/3)N_F + \tfrac{1}{6}N_{Higgs} \\ b_3 &= -11 + (2/3)N_F + 0 \end{aligned}$$

Comparison with data



Siegfried Bethke arXiv:1210.0325 [hep-ex]

$$\alpha_s(M_Z) = 0.118$$

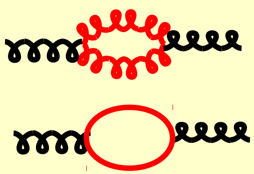
Low Q points have more discriminating power

Caution: α_S is NOT a physical observable

BEYOND NLO

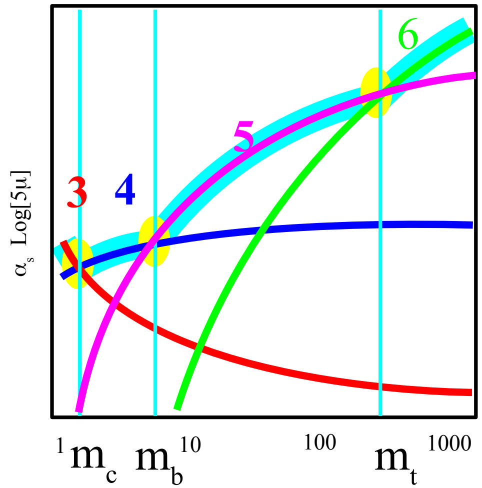
Strong Coupling across mass thresholds

N_F Matters



$$\beta = -\alpha_S^2\left[\frac{33-2N_F}{12\pi}\right]+\ldots$$

At 1-loop and 2-loops, continuous at thresholds



1.3 **4.5** **175**

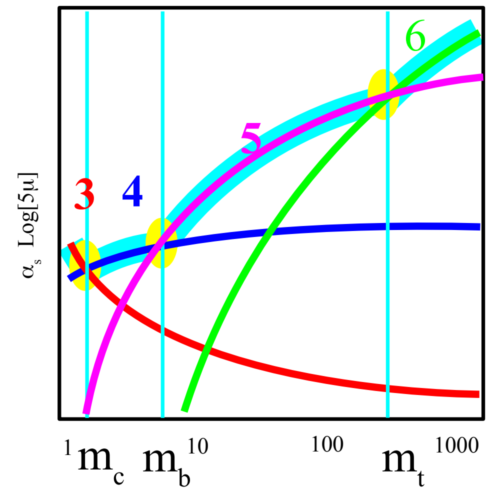
$\Lambda \sim 200\,\mathrm{MeV} \sim 1\ \mathrm{fm}$

Strong Coupling across mass thresholds

$$\alpha_S^{N_F+1}(\mu^2) = \alpha_S^{N_F}(\mu^2)\Big[\,1 + \alpha_s^1(c_{10}+c_{11}L^1) + \alpha_s^2(c_{20}+c_{21}L^1+c_{22}L^2) + \ldots\Big]$$

This is zero

This is non-zero



$L = \ln(\mu^2/m^2)$

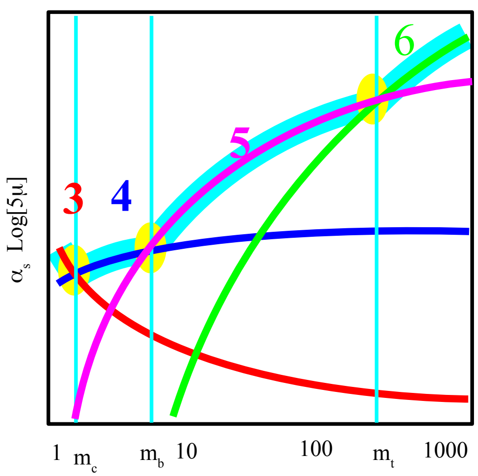
$c_0 = 0$

$c_1 = \dfrac{-11}{72\pi} \neq 0$

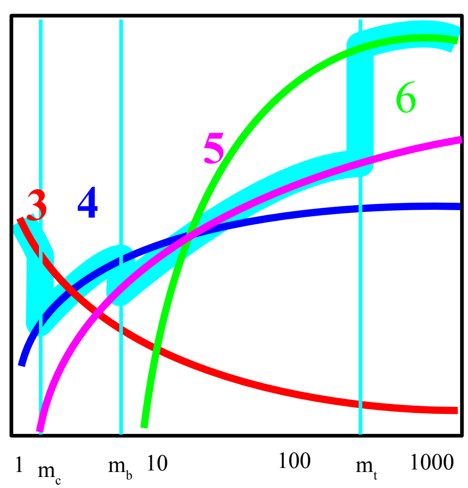
At $\mu = m$ $\qquad \alpha_S^{N_F+1}(\mu^2) = \alpha_S^{N_F}(\mu^2)[1 + 0 + c_{20}\,\alpha_S^2]$

Strong Coupling across mass thresholds

At 1-loop and 2-loops, continuous at thresholds



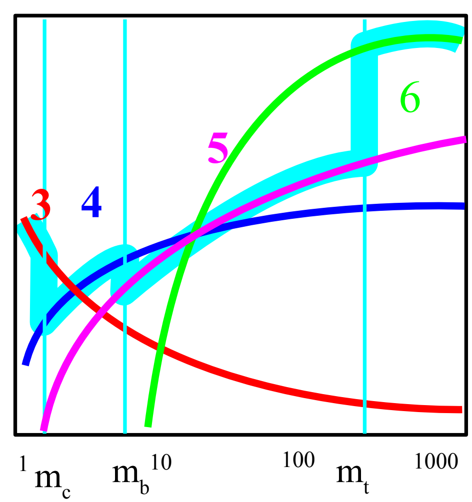
At $O(\alpha_s^3)$, not even continuous at thresholds



$$\alpha_{(n_f)}(M) = \alpha_{(n_f-1)}(M) - \frac{11}{72\pi^2}\,\alpha^3_{(n_f-1)}(M) + \mathcal{O}(\alpha^4_{(n_f-1)})$$

Strong Coupling across mass thresholds

At $O(\alpha_s^3)$, not even continuous at thresholds



Un-physical theoretical constructs:

(E.g., as, PDFs, ...)

Cannot be measured directly

Depends on Schemes

Renormalization Schemes:

MS, MS-Bar, DIS

Renormalization Scale m

Depends on Higher Orders

Physical Observables

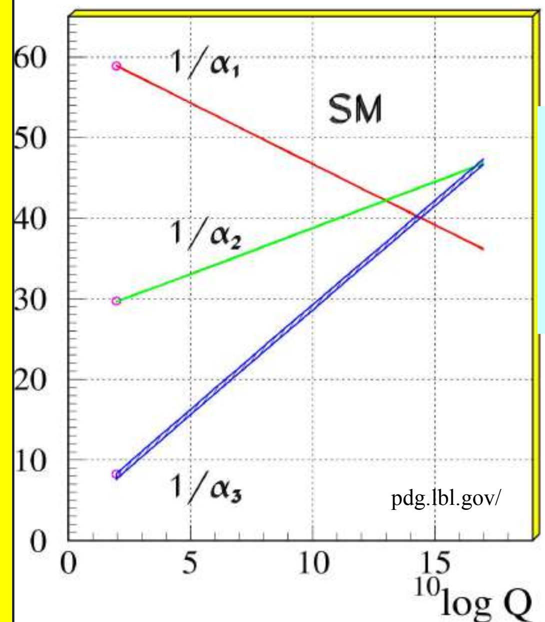
Measure directly

Independent of Schemes/Definitions

Independent of Higher Orders

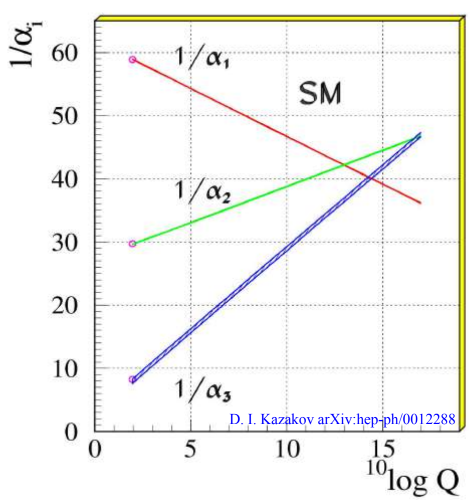
$$\alpha_{(n_f)}(M) = \alpha_{(n_f-1)}(M) - \frac{11}{72\pi^2}\,\alpha^3_{(n_f-1)}(M) + \mathcal{O}(\alpha^4_{(n_f-1)})$$

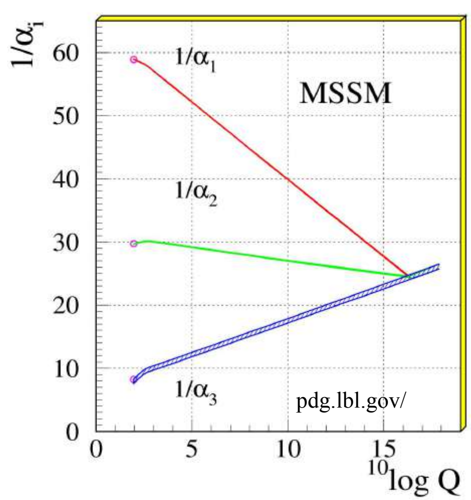
BEYOND SM



$$
\begin{aligned}
b_1 &= \quad 0 + (2/3)N_F + \tfrac{1}{10}N_{Higgs} \\
b_2 &= -22/3 + (2/3)N_F + \tfrac{1}{6}N_{Higgs} \\
b_3 &= \quad -11 + (2/3)N_F + 0
\end{aligned}
$$

Can we do better???



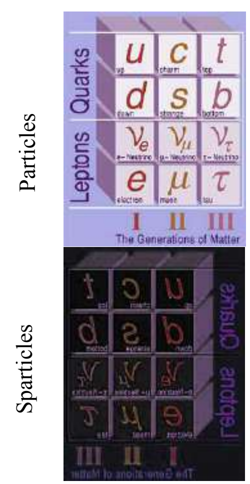


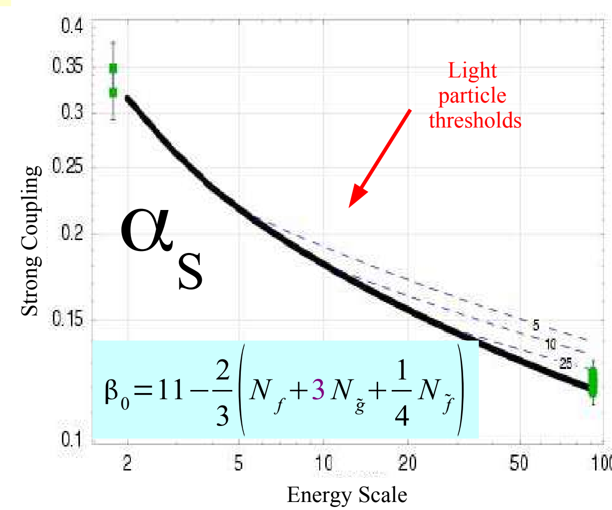
$$
\begin{aligned}
b_1 &= \quad 0 + (2/3)N_F + \tfrac{1}{10}N_{Higgs} \\
b_2 &= -22/3 + (2/3)N_F + \tfrac{1}{6}N_{Higgs} \\
b_3 &= \quad -11 + (2/3)N_F + 0
\end{aligned}
$$

$$\beta_0 = 11 - \frac{2}{3}\left(N_f + 3N_{\tilde{g}} + \frac{1}{4}N_{\tilde{f}}\right)$$

We've only discovered half the particles

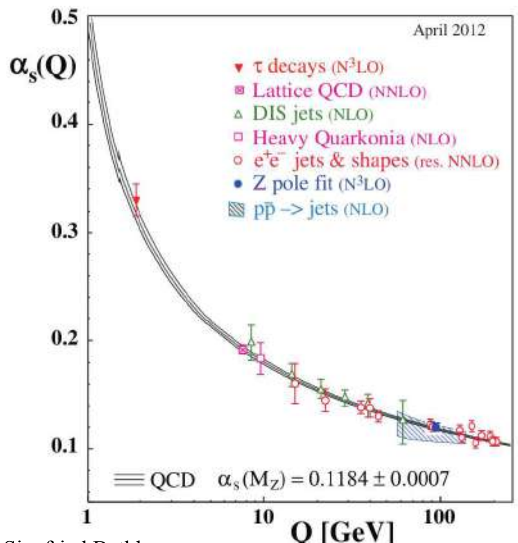
New particles effects evolution of $\alpha_s(\mu)$

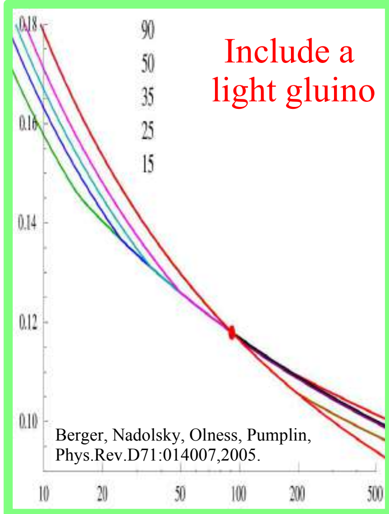




$$\beta_0 = 11 - \frac{2}{3}\left(N_f + 3N_{\tilde{g}} + \frac{1}{4}N_{\tilde{f}}\right)$$

The Standard Model (SM) & SUSY Running Couplings

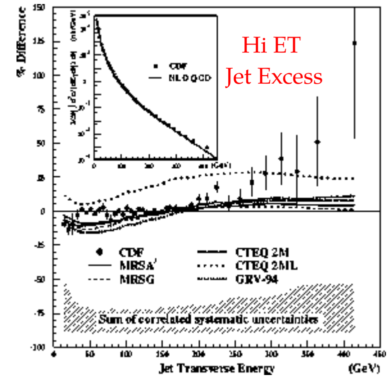




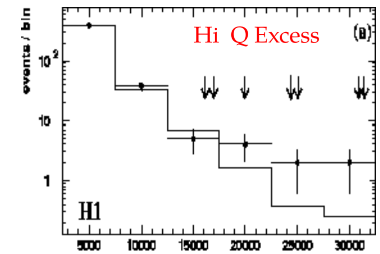
Siegfried Bethke
arXiv:1210.0325 [hep-ex]

$$b_0 = \frac{1}{12\pi}\left\{33 - 2N_F - 6N_{\tilde{g}} - \frac{1}{2}N_{\tilde{F}} \ldots\right\}$$

GOT QCD ???



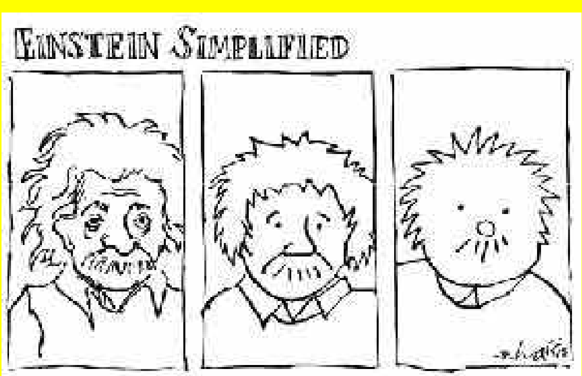
CDF Collaboration, PRL 77, 438 (1996)



H1 Collaboration, ZPC74, 191 (1997)
ZEUS Collaboration, ZPC74, 207 (1997)

Indispensable for discovery of "new physics"

RESUMMATION

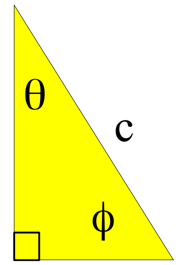


... over simplified

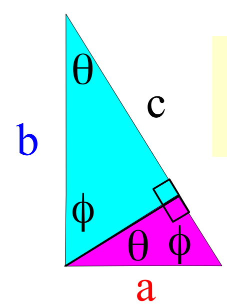
Warm up: Dimensional Analysis: Pythagorean Theorem

GOAL:
Pythagorean Theorem

METHOD:
Dimensional Analysis



$$A_c = c^2 f(\theta,\phi)$$



$$A_a + A_b = a^2 f(\theta,\phi) + b^2 f(\theta,\phi)$$

$$A_a + A_b = A_c$$

$$a^2 + b^2 = c^2$$

Two examples to come: 1) Resummation, and 2) Scaling

Resummation: Over-Simplified

$$\left\{\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s(\mu)) \frac{\partial}{\partial \alpha_s(\mu^2)}\right\} R\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) = 0$$

Logs can be large and spoil perturbation theory

If we expand R in powers of α_s, and we know β, we then know μ dependence of R.

$$R\left(\frac{\mu^2}{Q^2},\alpha_s(\mu^2)\right) = R_0 + \alpha_s(\mu^2)\, R_1\left[\ln(Q^2/\mu^2) + c_1\right]$$

$$+\alpha_s^2(\mu^2)\, R_2\left[\ln^2(Q^2/\mu^2) + \ln(Q^2/\mu^2) + c_2\right] + O(\alpha_s^3(\mu^2))$$

Since μ is arbitrary, choose μ=Q.

$$R\left(\frac{Q^2}{Q^2},\alpha_s(Q^2)\right) = R_0 + \alpha_s(Q^2)\, R_1\left[0 + c_1\right] + \alpha_s^2(Q^2)\, R_2\left[0+0+c_2\right] + \ldots$$

We just summed the logs

QCD is non-abelian SU(3) Symmetry $\Rightarrow$ confinement

More Differential Quantities $\Rightarrow$ More Mass Scales $\Rightarrow$ More Logs!!!

$$\frac{d\sigma}{dQ^2} \sim \ln\left(\frac{Q^2}{\mu^2}\right)$$

$$\frac{d\sigma}{dQ^2} \sim \ln\left(\frac{Q^2}{\mu^2}\right) \quad and \quad \ln\left(\frac{q_T^2}{\mu^2}\right)$$

How do we resum logs? Use the Renormalization Group Equation

For a physical observable R:

$$\mu \frac{dR}{d\mu} = 0$$

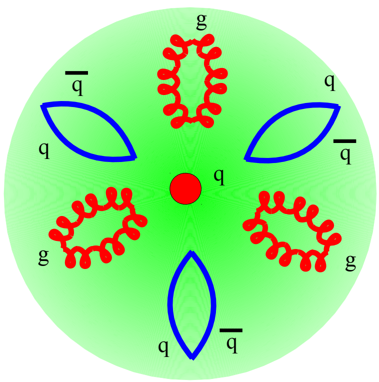
$$\frac{dR}{d\ Gauge} = 0$$

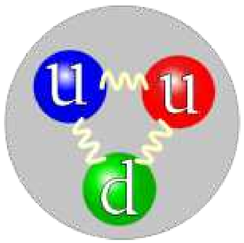
Applied to boson transverse momentum
CSS: Collins, Soper, Sterman
Nucl.Phys.B250:199,1985.

Interesting reference:
Peskin/Schroeder Text
(Renomalization ala Ken Wilson)

Scaling, and the proton structure

How do we determine the proton structure





http://en.wikipedia.org/

Quarks confined, thus we must work with hadrons & mesons

E.g, proton is a "minimal" unit

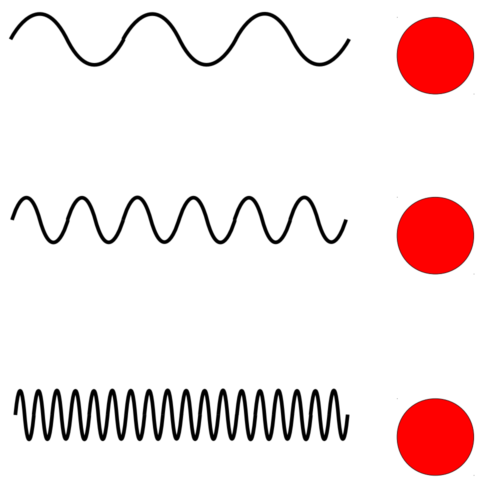
Highest energy (smallest distance) accelerators involve hadrons

E.g., HERA, TEV, LHC

We'd better learn to work with proton

48

What do we expect for a point like particle



$$d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times 1$$

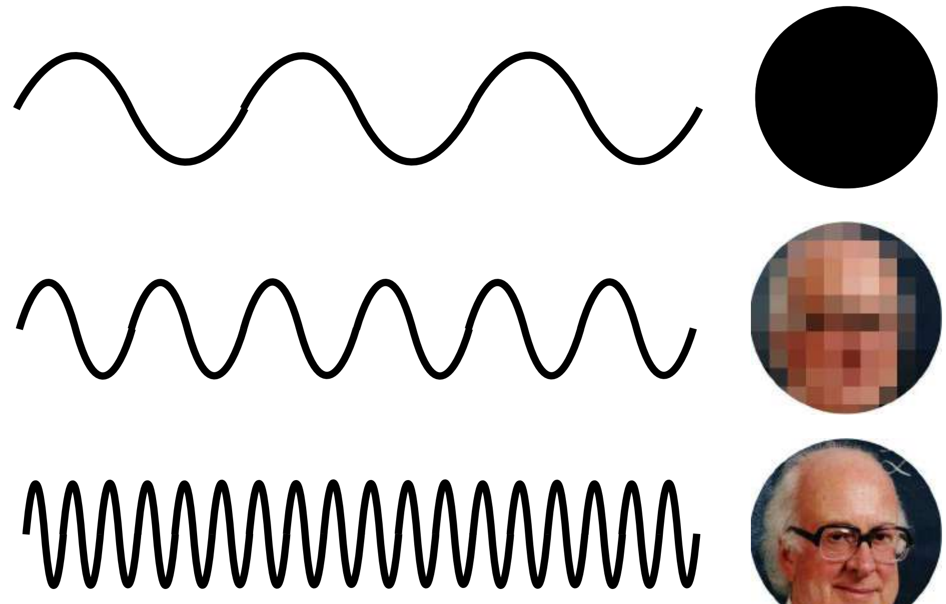
$$d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times 1$$

$$d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times 1$$

Dimensional considerations

Structure Function

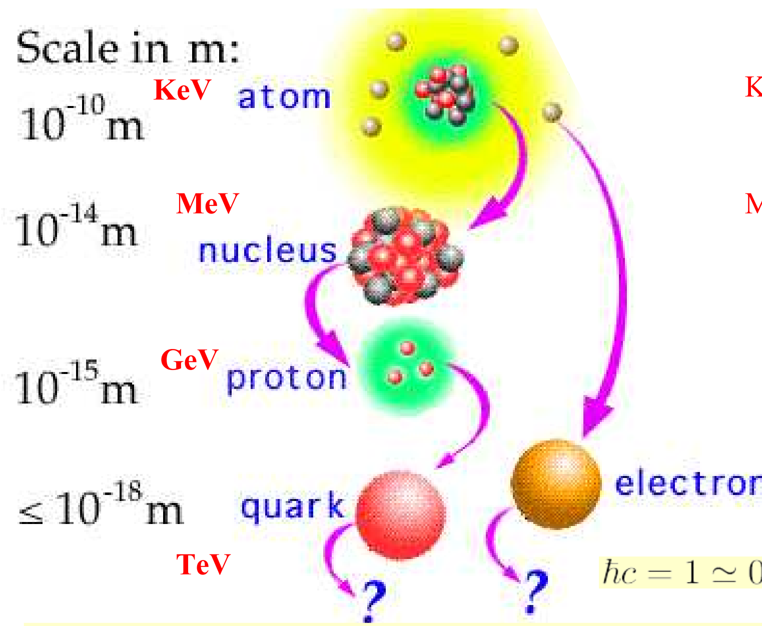
Is this a point like particle ???



We found the Higgs

Scaling, and the proton structure

Relative Sizes



KeV ~10^{-9}

MeV ~10^{-12}

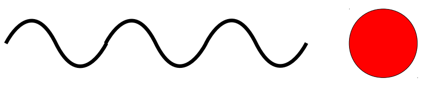
GeV ~10^{-15}

TeV ~10^{-18}

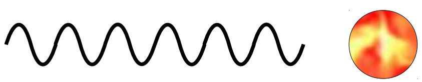
$\hbar c = 1 \simeq 0.2$ GeV fm

Going to smaller scale, we get to simpler, more fundamental objects

Structure of the Proton

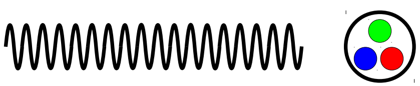


$$d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times 1$$



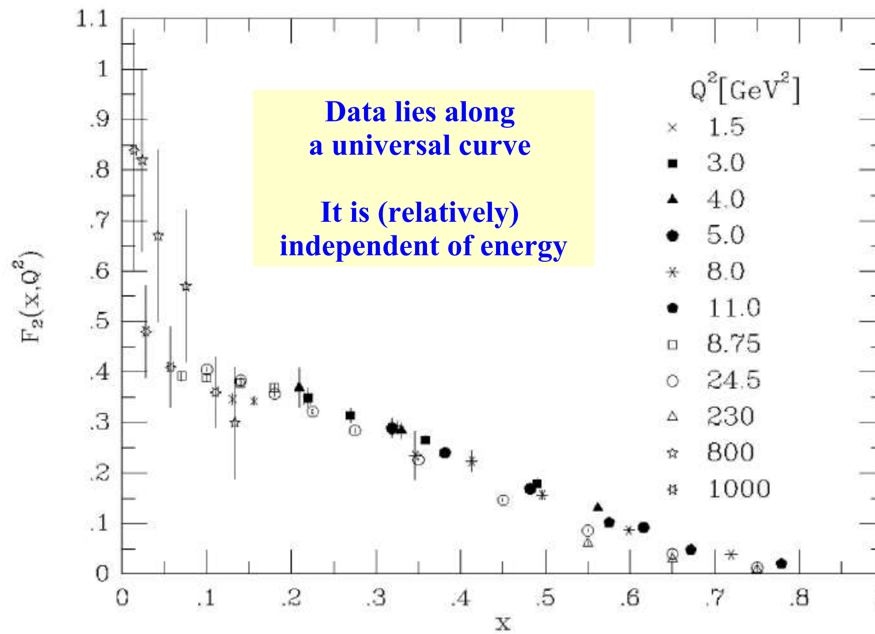
$$d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times F\left(\frac{Q^2}{\Lambda^2}\right)$$

Λ of order of the proton mass scale



$$d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times \sum_i e_i^2$$

The Scaling of the Proton Structure Function



The Scaling of the Proton Structure Function

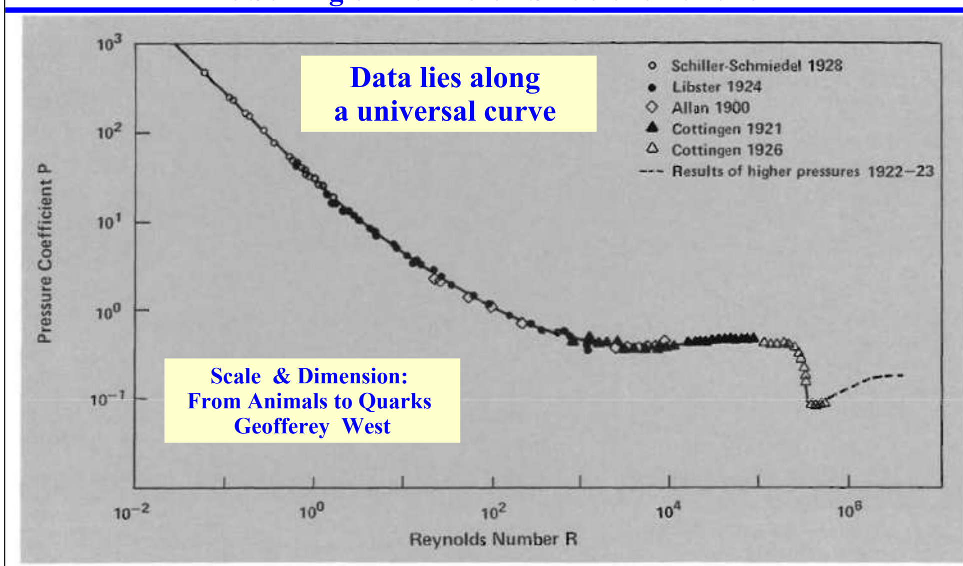


Fig. 5. The scaling curve for the motion of a sphere through a fluid that results when data from a variety of experiments are plotted in terms of two dimensionless variables: the pressure or drag coefficent P versus Reynolds number R. (Figure adapted from AIP Handbook of Physics, 2nd edition (1963):section II, p. 253.)

8 Summer/Fall 1984 LOS ALAMOS SCIENCE

End of lecture 1: Recap

QCD is just like QED, ... only different

- QCD is non-Abelian, Quarks are confined,

Running coupling $\alpha_s(\mu)$ tells how interaction changes with distance

- β-function: logarithmic derivative of $\alpha_s(\mu)$
- We can compute: Negative for QCD, positive for QED

$\alpha_s(\mu)$ is **not** a physical quantity

- Discontinuous at NNLO

New physics can influence $\alpha_s(\mu)$

- Unification of couplings at GUT scale

Running of $\alpha_s(\mu)$ can help us "resum" perturbation theory

Scaling and Dimensional Analysis are useful tools

END OF LECTURE 1

CTEQ School on
QCD Analysis and Electroweak Phenomenology

LECTURE 2

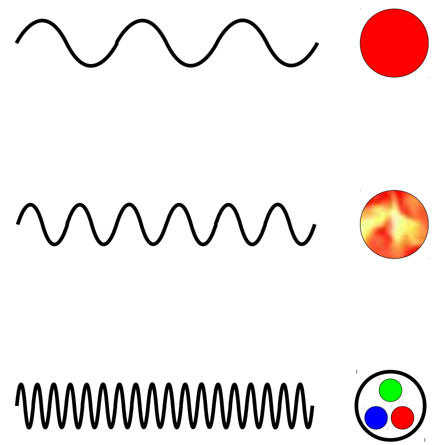
Introduction to the Parton Model and Perturbative QCD

Fred Olness (SMU)

University of Pittsburgh, PA

18-28 July 2017

Structure of the Proton



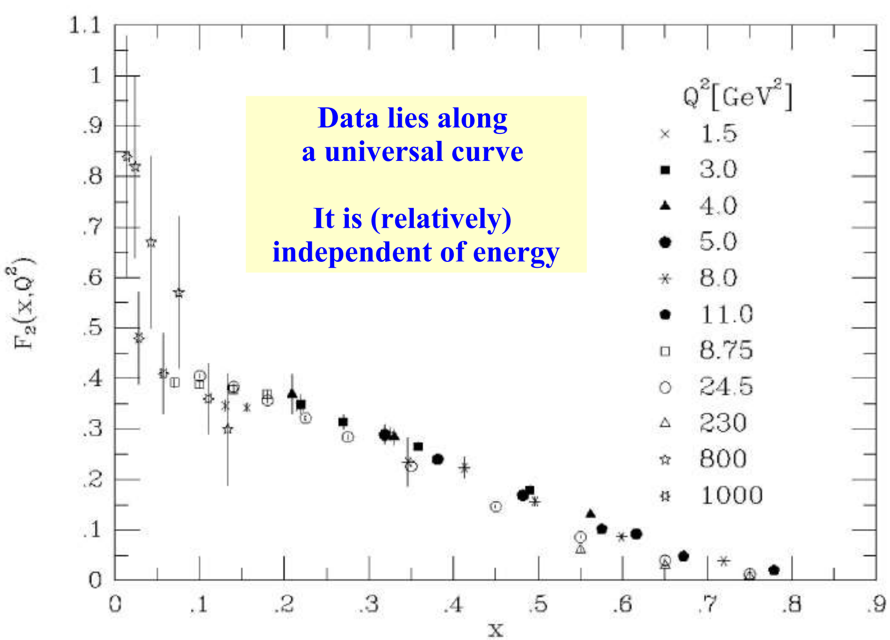
$$d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times 1$$

$$d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times F\left(\frac{Q^2}{\Lambda^2}\right)$$

Λ of order of the proton mass scale

$$d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times \sum_i e_i^2$$

The Scaling of the Proton Structure Function



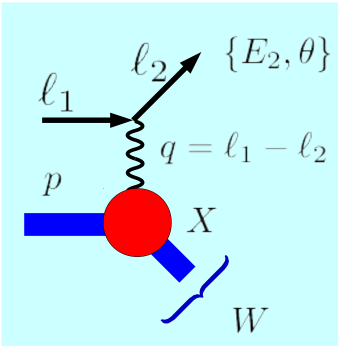
HOW TO CHARACTERIZE THE PROTON

Deeply Inelastic Scattering

(DIS)

Cf. lecture by
Simona Malace

Inclusive Deeply Inelastic Scattering (DIS)

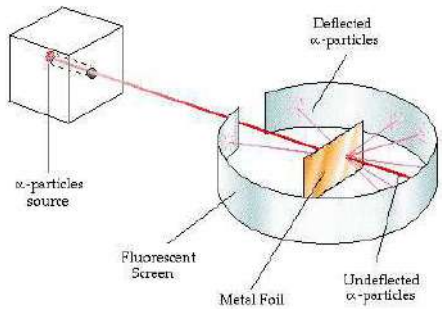


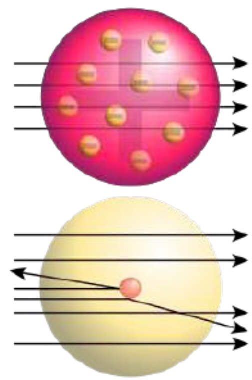
Measure $\{E_2, \theta\} \Leftrightarrow \{x, Q^2\}$ Inclusive

Deep: $Q^2 \geq 1 GeV^2$

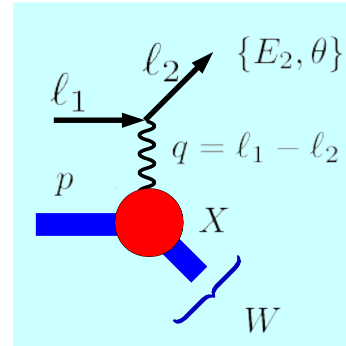
Inelastic: $W^2 \geq M_p^2$

Analogue of Rutherford scattering





Inclusive Deeply Inelastic Scattering (DIS)



Measure $\{E_2, \theta\} \Leftrightarrow \{x, Q^2\}$

$$Q^2 = -q^2 = 4E_1E_2 \sin^2(\theta/2)$$

$$x = \frac{Q^2}{2p \cdot q} = \frac{2E_1E_2 \sin^2(\theta/2)}{M(E_1 - E_2)}$$

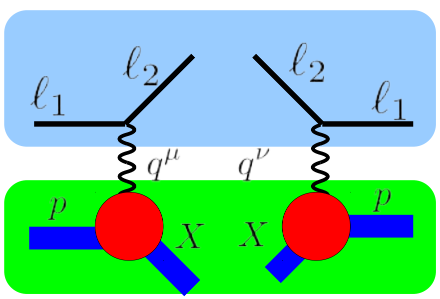
$$d\sigma \sim |A|^2$$

Other common DIS variables

$$\nu = \frac{p \cdot q}{p^2} = E_1 - E_2$$

$$y = \frac{\nu}{E_1} = \frac{Q^2}{2ME_2x}$$

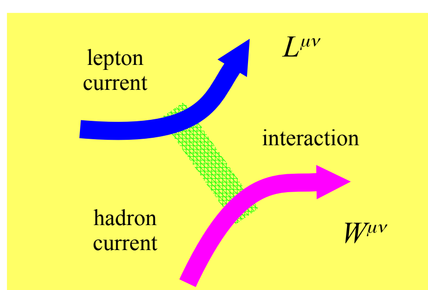
Lepton Tensor (L) and Hadronic Tensor (W)



$L^{\mu\nu}$ Leptonic Tensor

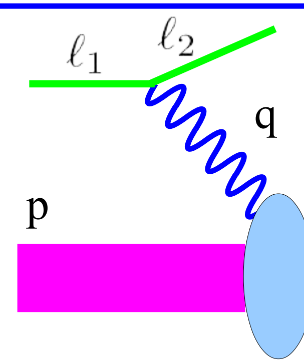
$W_{\mu\nu}$ Hadronic Tensor

$$d\sigma \sim |A|^2 \sim L^{\mu\nu} W_{\mu\nu}$$



Current Interactions

W and F Structure Functions



$$d\sigma \sim |A|^2 \sim L^{\mu\nu} W_{\mu\nu}$$

$$L^{\mu\nu} = L^{\mu\nu}(\ell_1, \ell_2)$$

$$W^{\mu\nu} = W^{\mu\nu}(p, q)$$

Details:
There are also $W_{4,5,6}$
but we neglect these

$$W^{\mu\nu} = -g^{\mu\nu} W_1 + \frac{p^\mu p^\nu}{M^2} W_2 - \frac{i\,\epsilon^{\mu\nu\rho\sigma} p_\rho q_\sigma}{2M^2} W_3 + \ldots$$

Convert to “Scaling” Structure Functions

$$W_1 \to F_1 \qquad W_2 \to \tfrac{M}{\nu} F_2 \qquad W_3 \to \tfrac{M}{\nu} F_3$$

$$\frac{d\sigma}{dx\,dy} = N\left[xy^2F_1 + (1 - y - \tfrac{Mxy}{2E_2})F_2 \pm y(1 - y/2)xF_3\right]$$

Use Helicity Basis

$$\frac{d\sigma}{dx\,dy} = N\left[xy^2F_1 + (1 - y - \tfrac{Mxy}{2E_2})F_2 \pm y(1-y/2)xF_3\right]$$

Taking the limit $M \to 0$ for neutrino DIS

$$\frac{d\sigma^\nu}{dx\,dy} = N\left[(1-y)^2F_+ + 2(1-y)F_0 + F_-\right]$$

For $\bar{\nu}$, $F_+ \Leftrightarrow F_-$

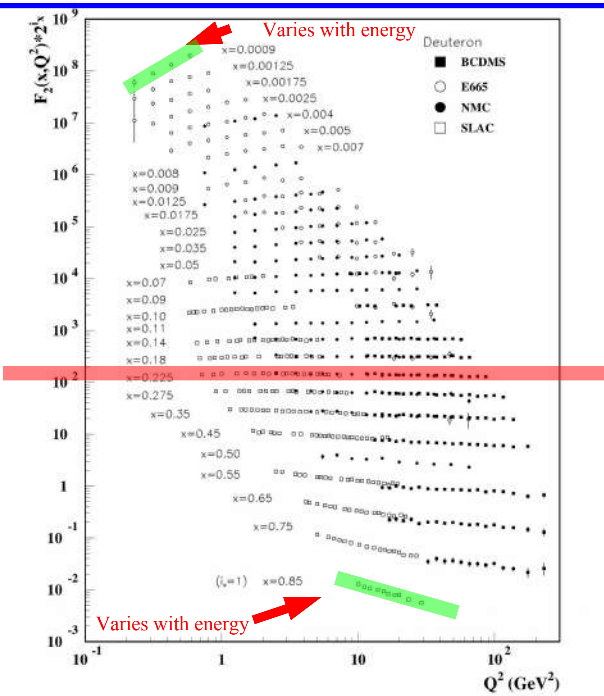
$$\begin{aligned} F_1 &= \tfrac{1}{2}(F_- + F_+) \qquad & F_+ &= F_1 - \tfrac{1}{2}F_3 \\ F_2 &= x(F_- + F_+ + 2F_0) \qquad & F_- &= F_1 + \tfrac{1}{2}F_3 \\ F_3 &= (F_- - F_+) \qquad & F_0 &= \tfrac{1}{2x}F_2 - F_1 \end{aligned}$$

I have not yet mentioned the parton model!!!

A Review of Target Mass Corrections.
Ingo Schienbein et al.
J.Phys.G35:053101,2008.

The Scaling of the Proton Structure Function

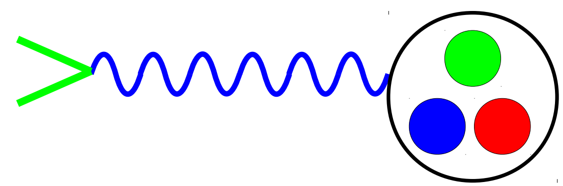
Data is (relatively) independent of energy

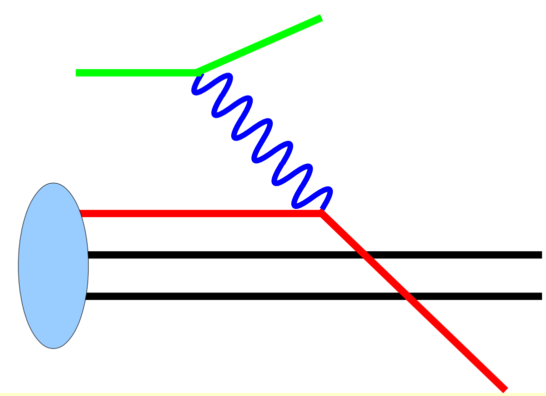


Scaling Violations observed at extreme x values

Parton Model

Proton as a bag of free Quarks

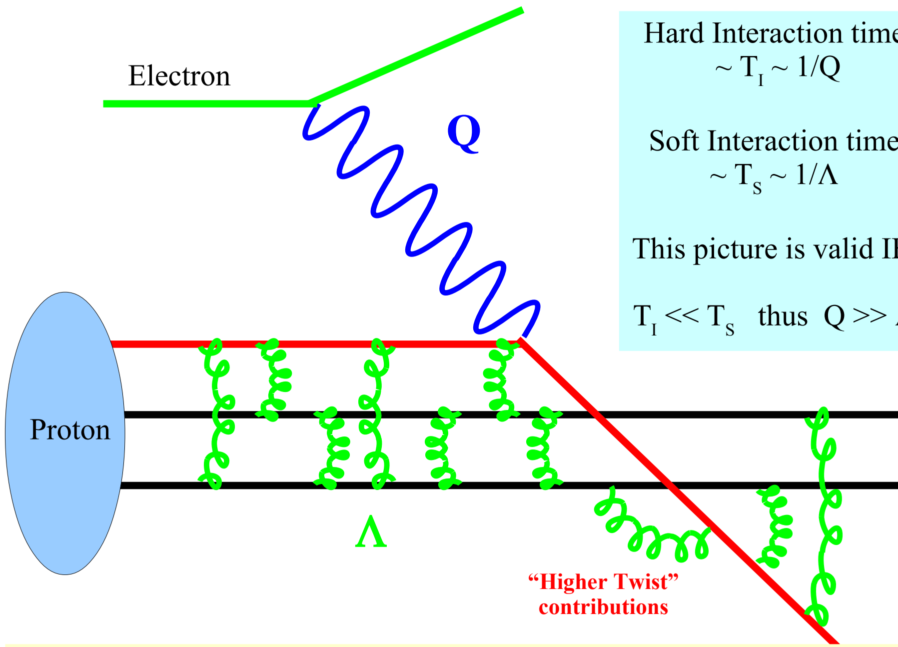




Fred's PDFs

$$f(x,Q) = u(x,Q) + d(x,Q) = 2\,\delta(x - \tfrac{1}{3}) + 1\,\delta(x - \tfrac{1}{3})$$

Quarks are not quite free



Hard Interaction time $\sim T_I \sim 1/Q$

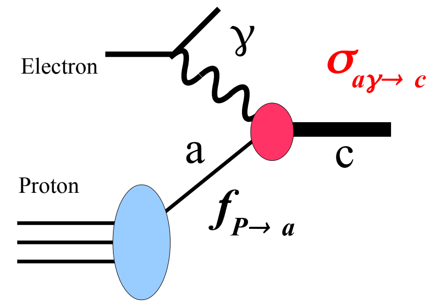
Soft Interaction time $\sim T_S \sim 1/\Lambda$

This picture is valid IF:

$T_I \ll T_S$ thus $Q \gg \Lambda$

Corrections to this picture (non-factorizable/ higher twist) terms are suppressed by powers of Λ/Q

The Parton Model and Factorization



Parton Distribution Functions

(PDFs) $f_{P\to a}$

are the key to calculations

involving hadrons!!!

$$\sigma_{P\gamma\to c} = f_{P\to a} \otimes \hat{\sigma}_{a\gamma\to c}$$

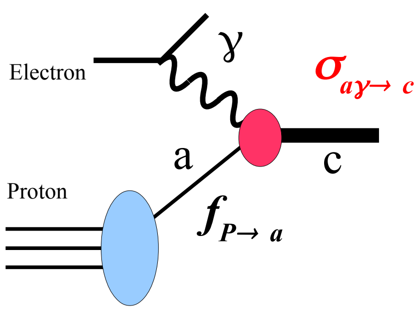
Corrections of order (Λ^2/Q^2)

must extract from experiment

calculable from theoretical model

Cross section is product of independent probabilities!!! *(Homework Assignment)*

The Parton Model and Factorization



Parton Distribution Functions

(PDFs) $f_{P\to a}$

are the key to calculations

involving hadrons!!!

$$\sigma_{P\gamma\to c} = f_{P\to a} \otimes \hat{\sigma}_{a\gamma\to c}$$

Scalar

$$f(x) = \sum q(x) + \bar{q}(x) + \phi(x) + \ldots = u(x) + d(x) + \ldots$$

Homework Problem: Convolutions

Part 1) Show these 3 definitions are equivalent; work out the limits of integration.

$$f\otimes g=\int_0^1\int_0^1 f(x)\,g(y)\,\delta(z-x*y)\,dx\,dy$$

$$f\otimes g=\int f(x)\,g\left(\frac{z}{x}\right)\frac{dx}{x}$$

$$f\otimes g=\int f\left(\frac{z}{y}\right)g(y)\frac{dy}{y}$$

Part 2) Show convolutions are the "natural" way to multiply probabilities.

If f represents the heads/tails probability distribution for a single coin flip, show that the distribution of 2 coins is $f\oplus f$ and 3 coins is: $f\oplus f\oplus f$.

$$f\oplus g=\int f(x)\,g(y)\,\delta(z-(x+y))\,dx\,dy$$

$$f(x)=\frac{1}{2}(\delta(1-x)+\delta(1+x))$$

Careful: convolutions involve + and *

BONUS*: How many processes can you think of that don't factorize?*

Structure Function & PDF Correspondence at Leading Order

$$\frac{d\sigma^\nu}{dx\,dy} = N\left[(1-y)^2 F_+ + 2(1-y)F_0 + F_-\right]$$

Compute with Hadronic Tensor

$$\frac{d\sigma^\nu}{dx\,dy} = N\left[(1-y)^2(2\bar{q}) + 2(1-y)(\phi) + (2q)\right]$$

Compute in Parton Model

Scalar

$$\begin{aligned} F_+ &= 2\bar{q} \\ F_- &= 2q \\ F_0 &= \phi \end{aligned} \qquad \begin{aligned} F_+ &= F_1 - \tfrac{1}{2}F_3 \\ F_- &= F_1 + \tfrac{1}{2}F_3 \\ F_0 &= \tfrac{1}{2x}F_2 - F_1 \end{aligned}$$

Scalar

$$F_L = 0 = F_0$$

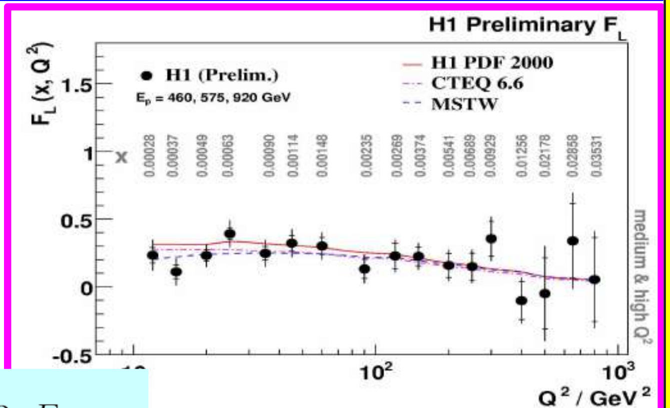
$$F_2 = 2xF_1$$

Callan-Gross Relation

$$F_L = 2xF_0$$

F_L

Why is F_L special ???



H1 Collaboration and ZEUS Collaboration (S. Glazov for the collaboration). Nucl.Phys.Proc.Suppl.191:16-24,2009.

$$F_L = 2xF_0 = F_2 - 2xF_1$$

$$F_L = 0 \quad \Longrightarrow \quad F_2 = 2xF_1$$

Callan-Gross

$$F_L \sim \frac{m^2}{Q^2}\,q(x) + \alpha_S\left\{c_g \otimes g(x) + c_q \otimes q(x)\right\}$$

Masses are important

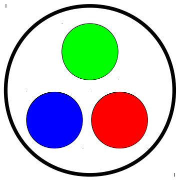
Higher orders are important

TOY

PDFs

Proton as a bag of free Quarks: Part 2
21

$$f(x,Q) = u(x,Q) + d(x,Q) = 2\ \delta(x-\tfrac{1}{3}) + 1\ \delta(x-\tfrac{1}{3})$$



$$u(x,Q) = 2\ \delta(x-\tfrac{1}{3})$$

$$d(x,Q) = 1\ \delta(x-\tfrac{1}{3})$$

Perfect Scaling PDFs
Q independent

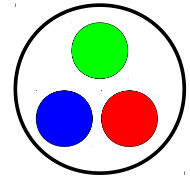
Quark Number Sum Rule

$$\langle q\rangle = \int_0^1 dx\, q(x) \qquad \langle u\rangle = 2 \quad \langle d\rangle = 1 \quad \langle s\rangle = 0$$

Quark Momentum Sum Rule

$$\langle x\, q\rangle = \int_0^1 dx\, x\, q(x) \qquad \langle x\, u\rangle = \tfrac{2}{3} \quad \langle x\, d\rangle = \tfrac{1}{3}$$

Problem #1: The proton does not add up???
22



$$F_+ = 2\bar{q}$$
$$F_- = 2q$$
$$F_L = \phi$$

$$q + \bar{q} = \frac{F_+ + F_-}{2}$$

Momentum Sum Rule

$$\sum_i \langle x\, q_i\rangle = \int_0^1 dx \sum x\,[q_i(x) + \bar{q}_i(x)] = 50\% \neq 100\%$$

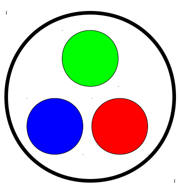
Substitute F

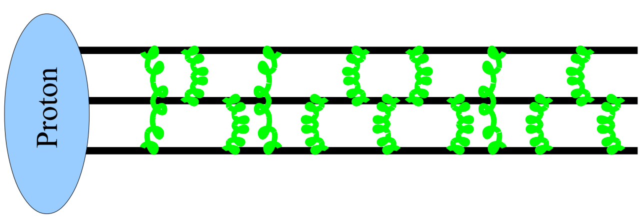
SOLUTION:

Gluons carry half the momentum,

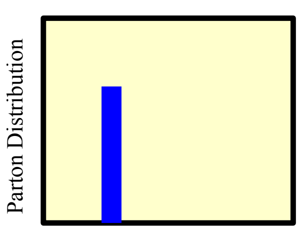
but don't couple to the photons

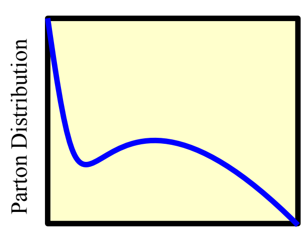
Gluons smear out PDF momentum
23





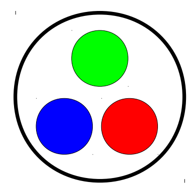
Gluons allow partons to exchange momentum fraction





α_s is large at low Q, so it is easy to emit soft gluons

Problem #2: Infinitely many quarks
24

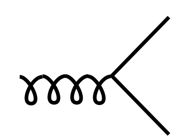


Reconsider the Quark Number Sum Rule

$$\langle u, d\rangle = \infty \qquad \langle q\rangle = \int_0^1 dx\, q(x)$$

Quark Number Sum Rule:
More Precisely

$$q(x) \sim 1/x^{1.5}$$



$u\ d\ s\ \ldots$

$\bar{u}\ \bar{d}\ \bar{s}\ \ldots$

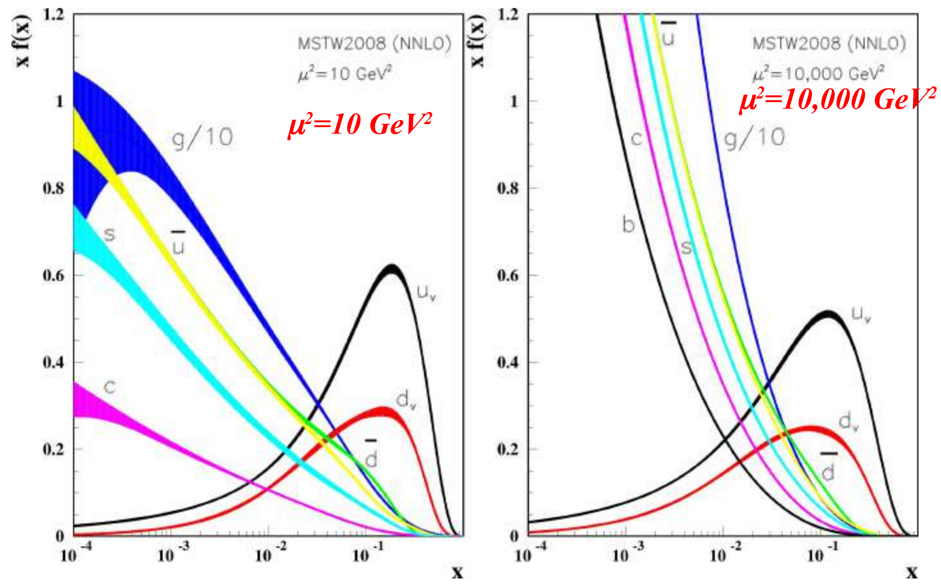
$$\langle u-\bar{u}\rangle = 2 \qquad \langle d-\bar{d}\rangle = 1 \qquad \langle s-\bar{s}\rangle = 0$$

SOLUTION: Infinite number of u quarks in proton, because they can be pair produced:
(We neglect saturation ....)

PDFs

cf., lectures by Pavel Nadolsky

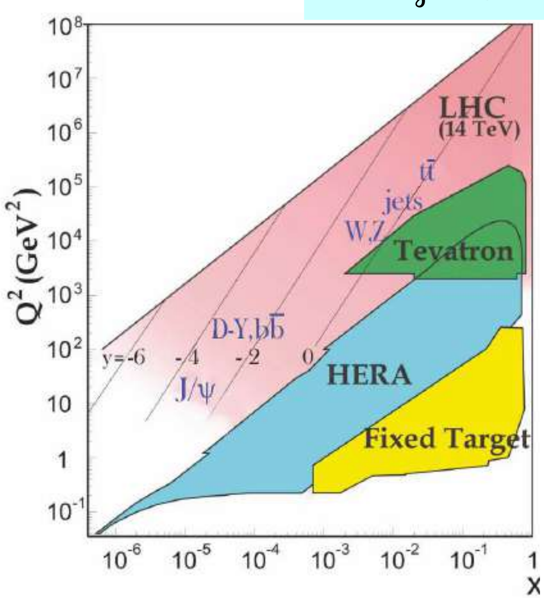
Sample PDFs: The rich structure of the proton



Scaling violations are essential feature of PDFs

Where do PDFs come from???? Universality!!!

$$\sigma_{P\gamma \to c} = f_{P\to a} \otimes \sigma_{a\gamma\to c}$$



Calculable from theoretical model

Must extract from experiment

Note we can combine different experiments.
FACTORIZATION!!!

HOMEWORK

Sum Rules
&
Structure Functions

$$
\begin{aligned}
F_2^{ep} &= \tfrac{4}{9}x\left[u+\bar{u}+c+\bar{c}\right] \\
&+ \tfrac{1}{9}x\left[d+\bar{d}+s+\bar{s}\right] \\
F_2^{en} &= \tfrac{4}{9}x\left[d+\bar{d}+c+\bar{c}\right] \\
&+ \tfrac{1}{9}x\left[u+\bar{u}+s+\bar{s}\right] \\
F_2^{\nu p} &= 2x\left[d+s+\bar{u}+\bar{c}\right] \\
F_2^{\nu n} &= 2x\left[u+s+\bar{d}+\bar{c}\right] \\
F_2^{\bar{\nu} p} &= 2x\left[u+c+\bar{d}+\bar{s}\right] \\
F_2^{\bar{\nu} n} &= 2x\left[d+c+\bar{u}+\bar{s}\right] \\
F_3^{\nu p} &= 2\left[d+s-\bar{u}-\bar{c}\right] \\
F_3^{\nu n} &= 2\left[u+s-\bar{d}-\bar{c}\right] \\
F_3^{\bar{\nu} p} &= 2\left[u+c-\bar{d}-\bar{s}\right] \\
F_3^{\bar{\nu} n} &= 2\left[d+c-\bar{u}-\bar{s}\right]
\end{aligned}
$$

Verify:

i.e., Check for typos ...

We use these different observables to dis-entangle the flavor structure of the PDFs

See talks by
Stephen Parke
&
Jonathan Paley
(Neutrinos)
&
Pavel Nadolsky (PDFs)

In the limit

$\theta_{Cabibbo} = 0$

$m_c = 0$

Verify:

i.e., Check for typos ...

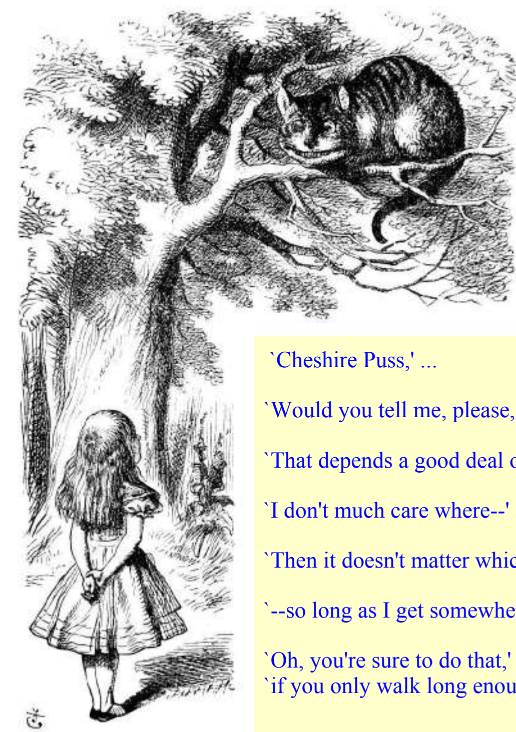
Adler (1966)		$\int_0^1 \frac{dx}{2x}\left[F_2^{\nu n} - F_2^{\nu p}\right] = 1$
Bjorken (1967)		$\int_0^1 \frac{dx}{2x}\left[F_2^{\bar{\nu} p} - F_2^{\nu p}\right] = 1$
Gross Llewellyn-Smith (1969)		$\int_0^1 dx\left[F_3^{\nu p} + F_3^{\bar{\nu} p}\right] = 6$
Gottfried (1967)	if $\bar{u} = \bar{d}$	$\int_0^1 dx\left[F_2^{ep} - F_2^{en}\right] = \frac{1}{3}$
Homework (19??)		$\frac{5}{18}F_2^{\nu N} - F_2^{eN} = ?$

Before the parton model was invented, these relations were observed. Can you understand them in the context of the parton model?

This one has been particularly important/controversial

Evolution

What does the proton look like???



The answer is dependent upon the question

\`Cheshire Puss,' ...

\`Would you tell me, please, which way I ought to go from here?'

\`That depends a good deal on where you want to get to,' said the Cat.

\`I don't much care where--' said Alice.

\`Then it doesn't matter which way you go,' said the Cat.

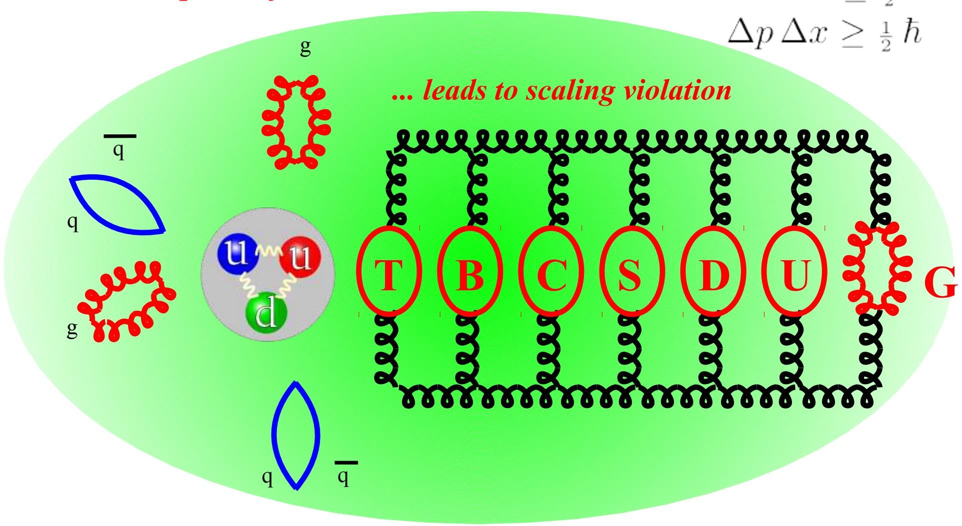
\`--so long as I get somewhere,' Alice added as an explanation.

\`Oh, you're sure to do that,' said the Cat,
\`if you only walk long enough.'

Evolution: What you see depends upon what you ask

Proton is a complex object

$$\Delta E\,\Delta t \geq \tfrac{1}{2}\hbar$$
$$\Delta p\,\Delta x \geq \tfrac{1}{2}\hbar$$



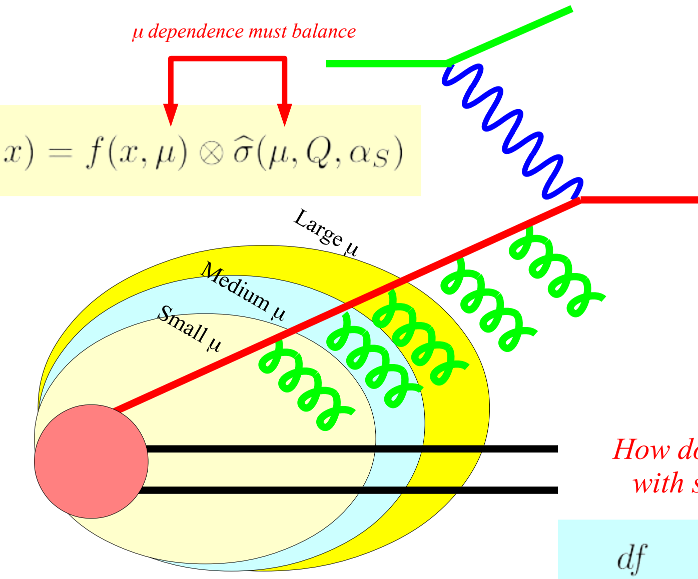
$\Lambda_{QCD} \sim 200$ MeV

m_t	m_b	m_c	m_s	m_d	m_u	m_g
175	4.5	1.3	0.3	0.00?	0.00?	0

Evolution of the PDFs

μ dependence must balance

$$\sigma(Q,x) = f(x,\mu) \otimes \hat{\sigma}(\mu, Q, \alpha_S)$$



How does f change with scale μ???

$$\frac{df}{d\,\ln[\mu]} = ???$$

Homework: Mellin Transform

$$\tilde{f}(n) = \int_0^1 dx\, x^{n-1}\, f(x)$$

$$\sigma = f \otimes \omega$$

$$f(x) = \frac{1}{2\pi i}\int_C dn\, x^{-n}\, \tilde{f}(n)$$

$$\tilde{\sigma} = \tilde{f}\,\tilde{\omega}$$

C is parallel to the imaginary axis, and to the right of all singularities

1) Take the Mellin transform of $f(x) = \sum_{m=1}^{\infty} a_m x^m$, and verify the inverse transform of $\tilde{f}$ regenerates $f(x)$

2) Take the Mellin transform of $\sigma = f \otimes \omega$ to demonstrate that the Mellin transform separates a convolution yields $\tilde{\sigma} = \tilde{f}\,\tilde{\omega}$.

A useful reference:
Courant, Richard and Hilbert, David. Methods of Mathematical Physics, Vol. 1. New York: Wiley, 1989. 561 p.

Renormalization Group Equation

Parton Model $\quad \sigma = f \otimes \omega$

ω or $\hat{\sigma}$
Not physical!
Poor notation

Renormalization Group Equation $\quad \dfrac{d\sigma}{d\mu} = 0 = \dfrac{d\tilde{f}}{d\mu}\,\tilde{\omega} + \tilde{f}\,\dfrac{d\tilde{\omega}}{d\mu}$

Take Mellin Transform

Separation of variables $\quad \dfrac{1}{\tilde{f}}\,\dfrac{d\tilde{f}}{d\ln[\mu]} = -\gamma = -\dfrac{1}{\tilde{\omega}}\,\dfrac{d\tilde{\omega}}{d\ln[\mu]}$

DGLAP Equation

DGLAP $\quad \dfrac{d\tilde{f}}{d\ln[\mu]} = -\tilde{f}\;\gamma \qquad \dfrac{df}{d\ln[\mu]} = P \otimes f$

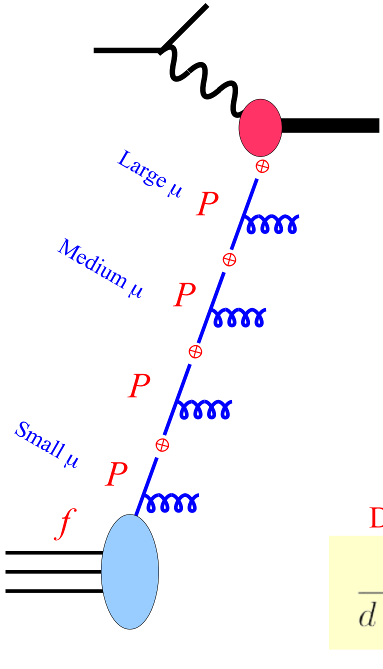
$\tilde{f} \sim \mu^{-\gamma}$

Anomalous Dimension

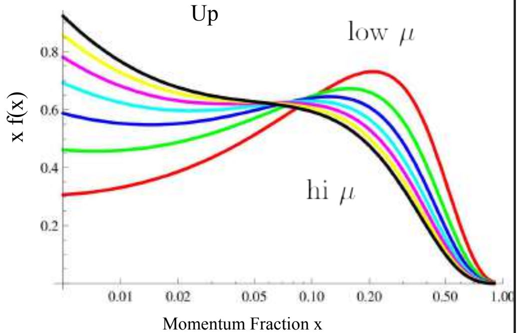
If "f" scaled,
γ would vanish

It is the dimension of the mass scaling

Evolution of the PDFs

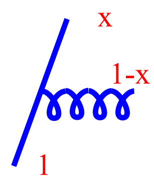


Evolution (generally) shifts partons from hi-x to low-x

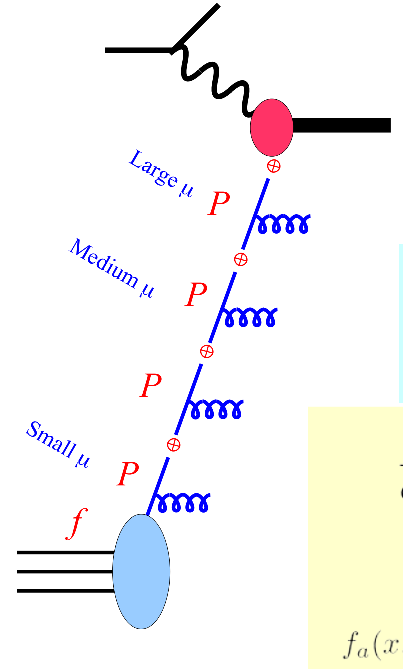


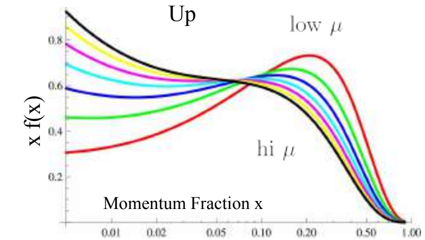
DGLAP Equation

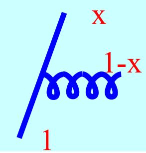
$$\frac{d\tilde{f}}{d\ln[\mu]} = P \otimes f$$



Evolution of the PDFs







$$P^{(1)}_{qq}(x) = C_F \left[\frac{1+x^2}{1-x}\right]_+$$

$$\frac{df}{d\ln[\mu]} = P \otimes f \simeq \frac{\alpha_S}{2\pi} P^{(1)} \otimes f$$

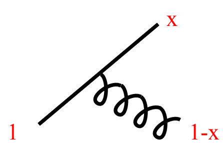
$$P \simeq \delta + \frac{\alpha_s}{2\pi} P^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 P^{(2)} + \dots$$

$$f_a(x,\mu_1) \sim f_a(x,\mu_0) + \frac{\alpha_S}{2\pi} P^{(1)}_{ab} \otimes f_b \ln\left(\frac{\mu_1^2}{\mu_2^0}\right)$$

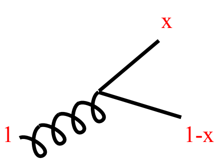
The Splitting Functions:

Read backwards

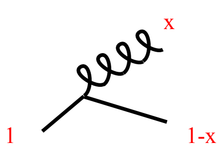
Note singularities



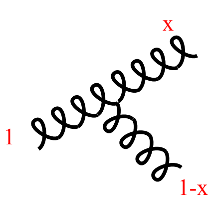
$$P^{(1)}_{qq}(x) = C_F \left[\frac{1+x^2}{1-x}\right]_+$$



$$P^{(1)}_{qg}(x) = T_F \left[(1-x)^2 + x^2\right]$$



$$P^{(1)}_{gq}(x) = C_F \left[\frac{(1-x)^2+1}{x}\right]$$



$$P^{(1)}_{gg}(x) = 2C_A \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x)\right]$$

$$+ \left[\frac{11}{6}C_A - \frac{2}{3}T_F N_F\right] \delta(1-x)$$

Homework: Part 1 The Plus Function

Definition of the Plus prescription:

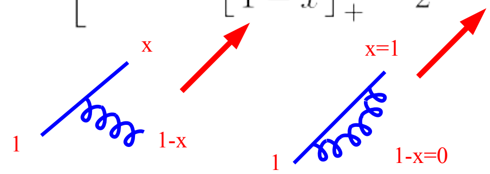
$$\int_0^1 dx \, \frac{f(x)}{(1-x)_+} = \int_0^1 dx \, \frac{f(x)-f(1)}{(1-x)}$$

1) Compute: $$\int_a^1 dx \, \frac{f(x)}{(1-x)_+} = ???$$

Homework: Part 2 P(q←q)

2) Verify:

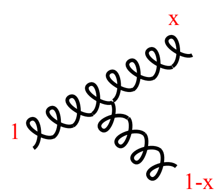
$$P^{(1)}_{qq}(x) = C_F\left[\frac{1+x^2}{1-x}\right]_+ \equiv C_F\left[(1+x^2)\left[\frac{1}{1-x}\right]_+ + \frac{3}{2}\,\delta(1-x)\right]$$

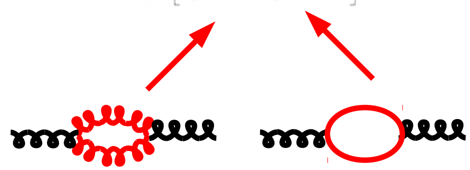


Observe

$$P^{(1)}_{gg}(x) = 2C_F\left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x)\right]$$

$$+\left[\frac{11}{6}C_A - \frac{2}{3}T_F N_F\right]\ \delta(1-x)$$



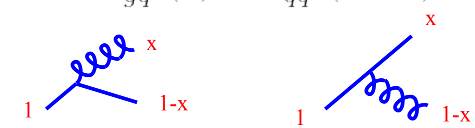


HOMEWORK: Part 3: Symmetries & Limits

Verify the following relation among the regular parts (from the real graphs)

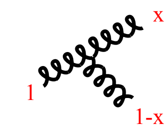
For the regular part show:

$$P^{(1)}_{gq}(x) = P^{(1)}_{qq}(1-x)$$



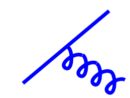
For the regular part show:

$$P^{(1)}_{gg}(x) = P^{(1)}_{gg}(1-x)$$

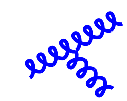


Verify, in the soft limit:

$$P^{(1)}_{qq}(x) \underset{x\to 1}{\longrightarrow} 2C_F\,\frac{1}{(1-x)_+}$$



$$P^{(1)}_{gg}(x) \underset{x\to 1}{\longrightarrow} 2C_F\,\frac{1}{(1-x)_+}$$

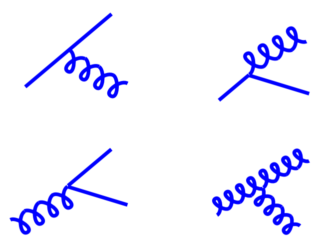


HOMEWORK: Part 4: Conservation Rules

Verify conservation of momentum fraction

$$\int_0^1 dx\,x\ [P_{qq}(x) + P_{gq}(x)] = 0$$

$$\int_0^1 dx\,x\ [P_{qg}(x) + P_{gg}(x)] = 0$$



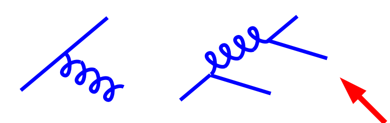
Verify conservation of fermion number

$$\int_0^1 dx\ \ [P_{qq}(x) - P_{q\bar{q}}(x)] = 0$$

Homework: Part 5: Using the Real to guess the Virtual

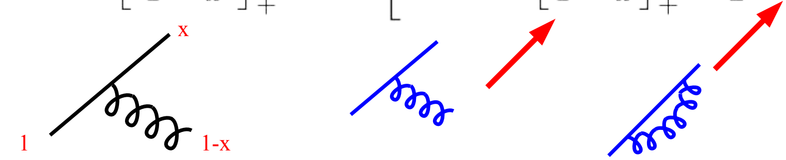
Use conservation of fermion number to compute the delta function term in P(q←q)

$$\int_0^1 dx\ \ [P_{qq}(x) - P_{q\bar{q}}(x)] = 0$$



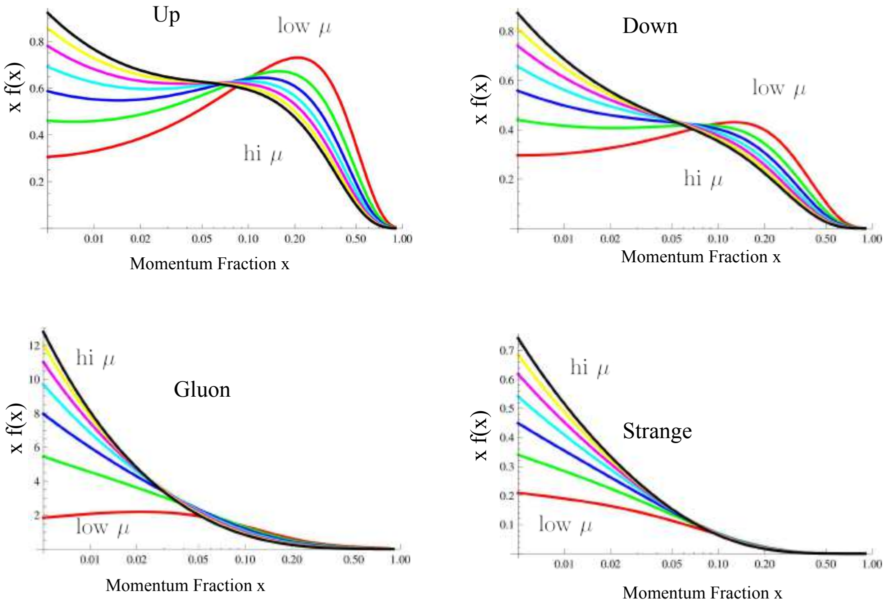
This term only starts at NNLO

$$P^{(1)}_{qq}(x) = C_F\left[\frac{1+x^2}{1-x}\right]_+ \equiv C_F\left[(1+x^2)\left[\frac{1}{1-x}\right]_+ + \frac{3}{2}\,\delta(1-x)\right]$$



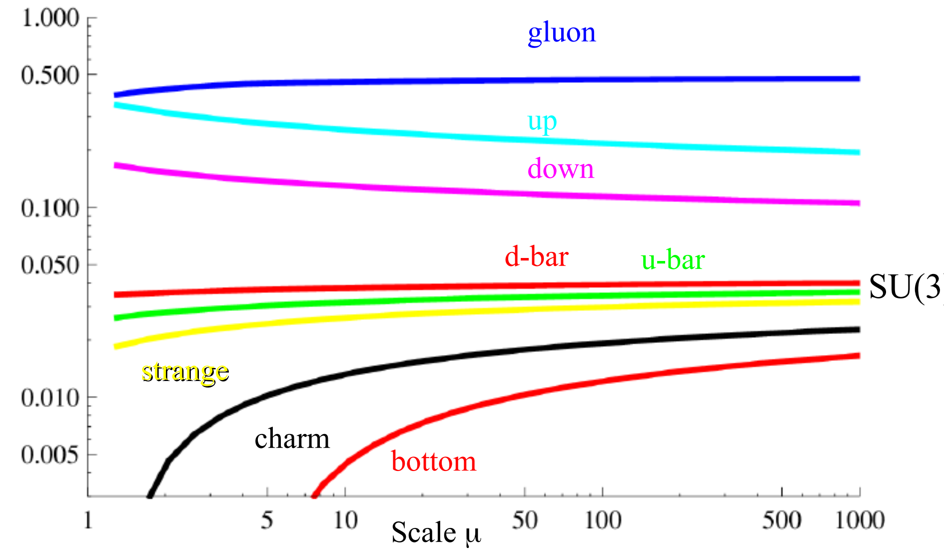
Powerful tool: Since we know real and virtual must balance, we can use to our advantage!!!

Evolution of the PDFs 45



PDF Momentum Fractions vs. scale μ 46

Momentum Fraction



Scaling violations are essential feature of PDFs

End of lecture 2: Recap 47

Rutherford Scattering ⇒ Deeply Inelastic Scattering (DIS)

- Works for protons as well as nuclei

Compute Lepton-Hadron Scattering 2 ways

- Use Leptonic/Hadronic Tensors to extract Structure Functions
- Use Parton Model; relate PDFs to F_{123}

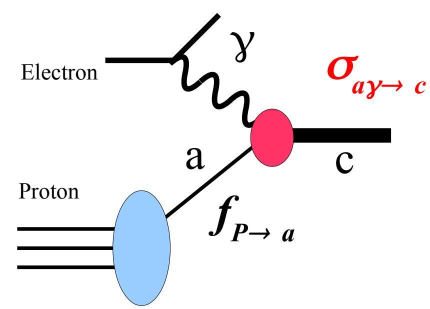
Parton Model Factorizes Problem:

- PDFs are independent of process
- Thus, we can combine different experiments. ESSENTIAL!!!

PDFs are not truly scale invariant; they evolve

- We use evolution to “resum” an important set of graphs

The Parton Model and Factorization 48



Parton Distribution Functions (PDFs) $f_{P\to a}$ are the key to calculations involving hadrons!!!

$$\sigma_{P\gamma\to c} = f_{P\to a} \otimes \hat{\sigma}_{a\gamma\to c}$$

Corrections of order (Λ^2/Q^2)

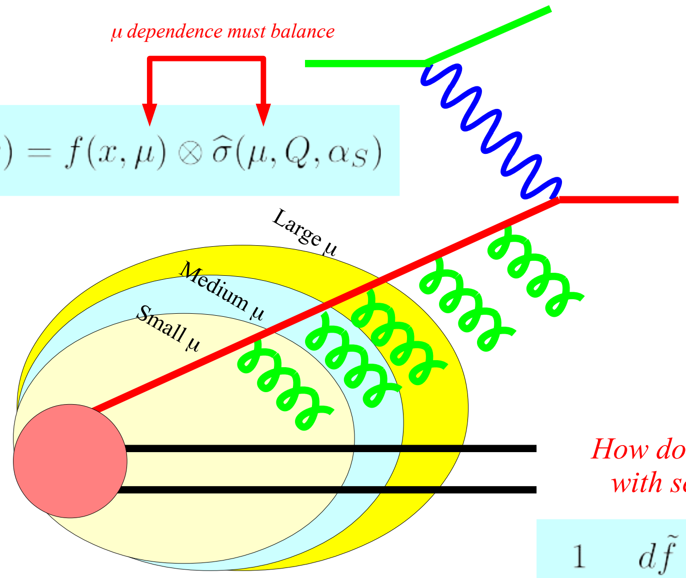
must extract from experiment

calculable from theoretical model

Cross section is product of independent probabilities!!! *(Homework Assignment)*

END OF LECTURE 2

Recap: Parton Model, Factorization, Evolution



$$\sigma(Q,x) = f(x,\mu) \otimes \hat{\sigma}(\mu,Q,\alpha_S)$$

How does f change with scale μ???

$$\frac{1}{\tilde{f}} \frac{d\tilde{f}}{d\ln[\mu]} = -\gamma$$

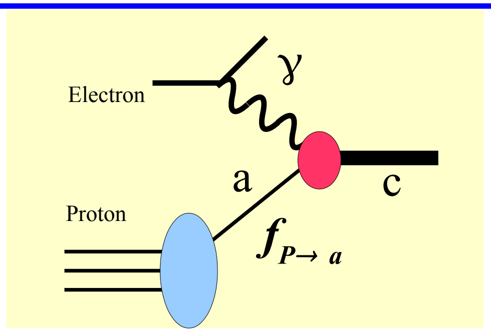
DGLAP Evolution Equation

DIS

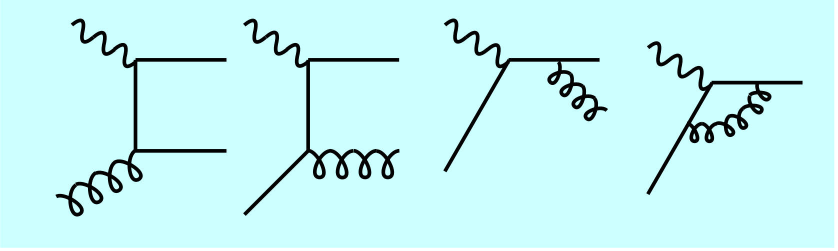
AT

NLO

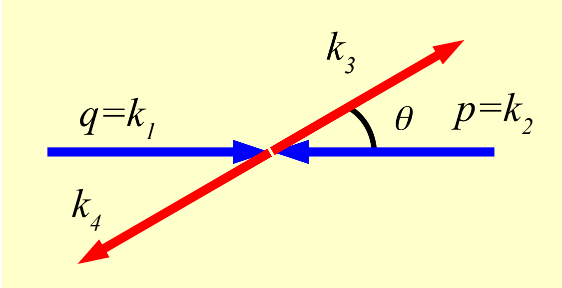
DIS at NLO



Sample NLO contributions to DIS

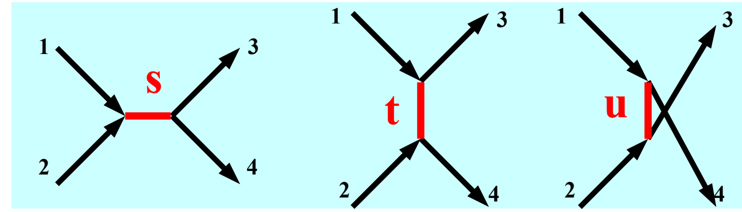


DIS NLO Kinematics



$$
\begin{aligned}
k_1 \equiv q^\mu &= \left(\frac{s-Q^2}{2\sqrt{s}}, 0, 0, \frac{(s+Q^2)}{2\sqrt{s}}\right) & -q^2 = Q^2 > 0 \\
k_2 \equiv p^\mu &= \left(\frac{s+Q^2}{2\sqrt{s}}, 0, 0, \frac{-(s+Q^2)}{2\sqrt{s}}\right) & p^2 = 0 \\
k_3^\mu &= \tfrac{\sqrt{s}}{2}\,(1, +\sin\theta, 0, +\cos\theta) & k_3^2 = 0 \\
k_4^\mu &= \tfrac{\sqrt{s}}{2}\,(1, -\sin\theta, 0, -\cos\theta) & k_4^2 = 0
\end{aligned}
$$

Mandelstam Variables {s,t,u}



$$
\begin{aligned}
s &= (k_1+k_2)^2 \equiv (k_3+k_4)^2 \\
t &= (k_1-k_3)^2 \equiv (k_2-k_4)^2 \\
u &= (k_1-k_4)^2 \equiv (k_2-k_3)^2 \\
s+t+u &= m_1^2+m_2^2+m_3^2+m_4^2
\end{aligned}
$$

Exercise

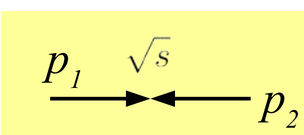
{s,t,u} are partonic

$$
s = +Q^2\frac{(1-x)}{x} \qquad t = -Q^2\frac{(1-z)}{2x} \qquad u = -Q^2\frac{(1+z)}{2x}
$$

$$
x = \frac{Q^2}{2p\cdot q} \qquad x \subset [0,1] \qquad\qquad z \equiv \cos\theta \qquad z \subset [-1,1]
$$

Homework

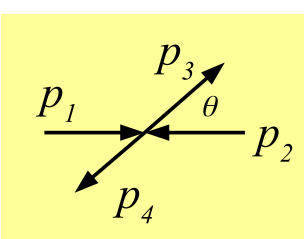
1) Let's work out the general 2→2 kinematics for general masses.



a) Start with the incoming particles.
Show that these can be written in the general form:

$$
\begin{aligned}
p_1 &= (E_1, 0, 0, +p) & p_1^2 = m_1^2 \\
p_2 &= (E_2, 0, 0, -p) & p_2^2 = m_2^2
\end{aligned}
$$

... with the following definitions:



$$
E_{1,2} = \frac{\hat{s} \pm m_1^2 \mp m_2^2}{2\sqrt{\hat{s}}} \qquad p = \frac{\Delta(\hat{s}, m_1^2, m_2^2)}{2\sqrt{\hat{s}}}
$$

$$
\Delta(a,b,c) = \sqrt{a^2+b^2+c^2-2(ab+bc+ca)}
$$

Note that $\Delta(a,b,c)$ is symmetric with respect to its arguments, and involves the only invariants of the initial state: s, m_1^2, m_2^2.

b) Next, compute the general form for the final state particles, p_3 and p_4. Do this by first aligning p_3 and p_4 along the z-axis (as p_1 and p_2 are), and then rotate about the y-axis by angle θ.

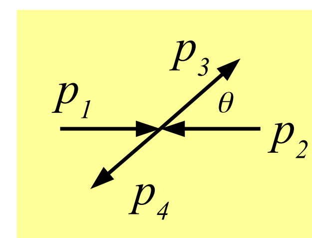
Homework Part 2

PROBLEM #2: Consider the reaction: $pp \to pp$ $(12 \to 34)$ with **CMS** scattering angle θ. The CMS energy is $\sqrt{s} = 2\,TeV$.

a) Compute the boost from the CMS frame to the rest frame of #2 (lab frame)

b) Compute the energy of #1 in the lab frame.

c) Compute the scattering angle θ_{lab} as a function of the CMS θ and invariants.



Hint: by using invariants you can keep it simple.
I.e., don't do it the way Goldstein does.

The power of invariants

Matrix element: NLO DIS

$$|\mathcal{M}|^2 = \frac{s}{-t} + \frac{-t}{s} + \frac{2uQ^2}{st}$$

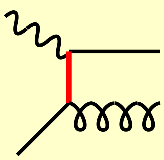
For the real $2 \to 2$ graphs

$$\simeq \frac{2(1-x)}{(1-z)} + \frac{2(1-z)}{(1-x)} + \frac{2x(1+z)}{(1-x)(1-z)}$$

Singular at z=1

$z \to 1, \quad \cos\theta \to 1$

$\theta \to 0, \quad t \to 0$

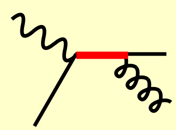


Collinear Singularity

Separate infinity, and subtract

Singular at x=1

$x \to 1, \quad s \to 0$



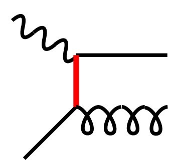
Soft Singularity

Separate infinity, cancel with virtual graphs

The Plan

Collinear Divergences

$$|\mathcal{M}|^2 \underset{z\to 1}{\longrightarrow} \frac{2}{(1-z)} \frac{(1+x^2)}{(1-x)}$$



Looks like a PDF splitting function

Plan

1) Separate ∞ at z=1

2) Subtract … *(should be part of PDF)*

Method

Need to regulate ∞

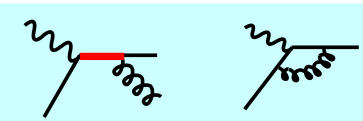
Choices

1) Dimensional Regularization

2) Quark Mass

3) θ Cut

Soft Singularities



Plan

1) Separate ∞ at x=1

2) Cancel between Real and Virtual graphs

Method

Need to regulate ∞

Choices

1) Dimensional Regularization

2) Gluon Mass

3) ...

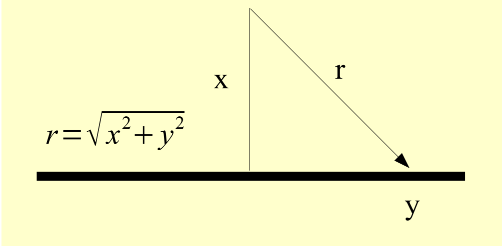
We'll use a simple example to illustrate the key points: 13

Dimensional Regularization
meets
Freshman E&M

M. Hans, Am.J.Phys. 51 (8) August (1983). p.694
C. Kaufman, Am.J.Phys. 37 (5), May (1969) p.560
B. Delamotte, Am.J.Phys. 72 (2) February (2004) p.170

Regularization, Renormalization, and Dimensional Analysis:
Dimensional Regularization meets Freshman E&M.
Olness & Scalise, arXiv:0812.3578 [hep-ph]

Infinite Line of Charge 14

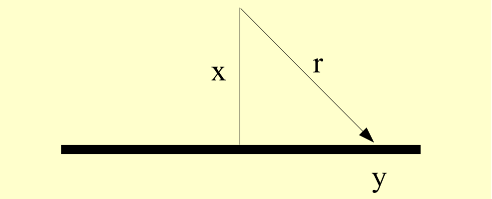


$$dV=\frac{1}{4\pi\epsilon_0}\frac{dQ}{r} \qquad \lambda=Q/y$$

$$V=\frac{\lambda}{4\pi\epsilon_0}\int_{-\infty}^{+\infty} dy\,\frac{1}{\sqrt{x^2+y^2}}=\infty$$

Note: ∞ can be very useful

Scale Invariance 15



$$
\begin{aligned}
V(kx) &= \\
&=\frac{\lambda}{4\pi\epsilon_0}\int_{-\infty}^{+\infty} dy\,\frac{1}{\sqrt{(kx)^2+y^2}}\\
&=\frac{\lambda}{4\pi\epsilon_0}\int_{-\infty}^{+\infty} d\left(\frac{y}{k}\right)\frac{1}{\sqrt{x^2+(y/k)^2}}\\
&=\frac{\lambda}{4\pi\epsilon_0}\int_{-\infty}^{+\infty} dz\,\frac{1}{\sqrt{x^2+z^2}}\\
&=V(x)
\end{aligned}
$$

$$V(kx)=V(x)$$

Naively Implies:
$V(kx) - V(x) = 0$

Note: $\infty + c = \infty$

$\therefore\quad \infty - \infty = c$

How do we distinguish this from

$\infty - \infty = c+17$

Cutoff Method 16

$$V=\frac{\lambda}{4\pi\epsilon_0}\int_{-L}^{+L} dy\,\frac{1}{\sqrt{x^2+y^2}}$$

$$V=\frac{\lambda}{4\pi\epsilon_0}\log\left[\frac{+L+\sqrt{L^2+x^2}}{-L+\sqrt{L^2+x^2}}\right]$$

V(x) depends on artificial regulator L

We cannot remove the regulator L

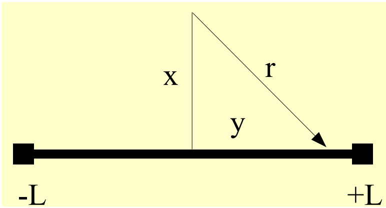
All physical quantities are independent of the regulator:

Electric Field $\quad E(x)=\frac{-dV}{dx}=\frac{\lambda}{2\pi\epsilon_0\, x}\frac{L}{\sqrt{L^2+x^2}}\rightarrow\frac{\lambda}{2\pi\epsilon_0\, x}$

Energy $\quad \delta V=V(x_1)-V(x_2)\xrightarrow[L\to\infty]{}\frac{\lambda}{4\pi\epsilon_0}\log\left[\frac{x_2^2}{x_1^2}\right]$

Problem solved at the expense of an extra scale L
<u>AND</u> we have a broken symmetry: translation invariance

Broken Translational Symmetry



Shift: $y \rightarrow y' = y - c$

y=[+L+c, -L+c]

$$V=\frac{\lambda}{4\pi\epsilon_0}\int_{-L+c}^{+L+c} dy\ \frac{1}{\sqrt{x^2+y^2}}$$

$$V=\frac{\lambda}{4\pi\epsilon_0}\ \log\left[\frac{+(L+c)+\sqrt{(L+c)^2+x^2}}{-(L-c)+\sqrt{(L-c)^2+x^2}}\right]$$

V(r) depends on "y" coordinate!!!

In QFT, gauge symmetries are important. E.g., Ward identies

Dimensional Regularization

Compute in n-dimensions

$$dy \rightarrow d^n y = \frac{d\Omega_n}{2}\ y^{n-1} dy$$

$$\Omega_n = \int d\Omega_n = \frac{2\pi^{n/2}}{\Gamma(n/2)} \qquad \Omega_{1,2,3,4} = \{2, 2\pi, 4\pi, 2\pi^2\}$$

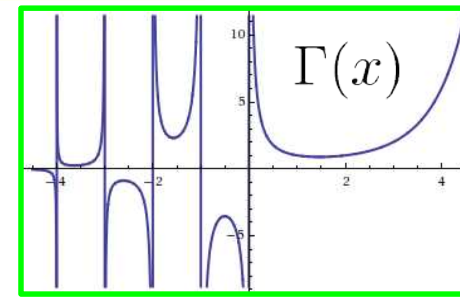
$$V=\frac{\lambda}{4\pi\epsilon_0}\int_0^{+\infty} d\Omega_n\ \frac{y^{n-1}}{\mu^{n-1}}\ \frac{dy}{\sqrt{x^2+y^2}}$$

Each term is individually dimensionaless

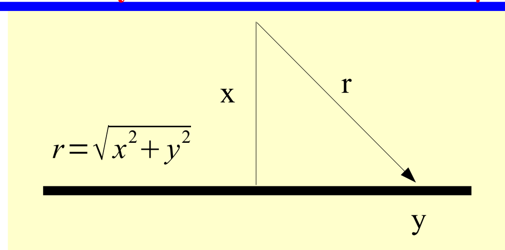
New scale μ

$n = 1 - 2\epsilon$

$$V=\frac{\lambda}{4\pi\epsilon_0}\left(\frac{\mu^{2\epsilon}}{x^{2\epsilon}}\ \frac{\Gamma[\epsilon]}{\pi^{\epsilon}}\right)$$



Why do we need an extra scale μ ???



$$dV=\frac{1}{4\pi\epsilon_0}\ \frac{dQ}{r} \qquad \lambda = Q/y$$

$$V=\frac{\lambda}{4\pi\epsilon_0}\ f(x)$$

Dimensional Regularization

All physical quantities are independent of the regulators:

Electric Field
$$E(x) = \frac{-dV}{dx} = \frac{\lambda}{4\pi\epsilon_0}\left[\frac{2\epsilon\mu^{2\epsilon}\Gamma[\epsilon]}{\pi^{\epsilon}x^{1+2\epsilon}}\right] \underset{\epsilon\rightarrow 0}{\rightarrow} \frac{\lambda}{2\pi\epsilon_0}\ \frac{1}{x}$$

Energy
$$\delta V = V(x_1) - V(x_2) \underset{\epsilon\rightarrow 0}{\rightarrow} \frac{\lambda}{4\pi\epsilon_0}\ \log\left[\frac{x_2^2}{x_1^2}\right]$$

Problem solved at the expense of an extra scale μ **AND** regulator ϵ

Translation invariance is preserved!!!

Dimensional Regularization respects symmetries

Renormalization

$$V \to \frac{\lambda}{4\pi\,\epsilon_0}\left[\frac{1}{\epsilon}+\ln\left[\frac{e^{-\gamma_E}}{\pi}\right]+\ln\left[\frac{\mu^2}{x^2}\right]\right]$$ Original

$$V \to \frac{\lambda}{4\pi\,\epsilon_0}\left[\frac{1}{\epsilon}+\ln\left[\frac{e^{-\gamma_E}}{\pi}\right]+\ln\left[\frac{\mu^2}{x^2}\right]\right]$$ MS

$$V \to \frac{\lambda}{4\pi\,\epsilon_0}\left[\frac{1}{\epsilon}+\ln\left[\frac{e^{-\gamma_E}}{\pi}\right]+\ln\left[\frac{\mu^2}{x^2}\right]\right]$$ MS-Bar

Physical quantities are independent of renormalization scheme!

$$V_{\overline{MS}}(x_1)-V_{\overline{MS}}(x_2)=\delta V=V_{MS}(x_1)-V_{MS}(x_2)$$

But only if performed consistently:

$$V_{\overline{MS}}(x_1)-V_{MS}(x_2)\neq\delta V\neq V_{MS}(x_1)-V_{\overline{MS}}(x_2)$$

Connection to QFT

This was the potential from our "Toy" calculation:

$$V \to \frac{\lambda}{4\pi\,\epsilon_0}\left[\frac{1}{\epsilon}+\ln\left[\frac{e^{-\gamma_E}}{1\;\pi}\right]+\ln\left[\frac{\mu^2}{x^2}\right]\right]$$

This is a partial result from
a real NLO Drell-Yan Calculation:
Cf., B. Potter

$$\frac{D(\epsilon)}{\epsilon}=\left(\frac{4\pi\,\mu^2}{Q^2}\right)\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\to\left[\frac{1}{\epsilon}+\ln\left[\frac{e^{-\gamma_E}}{4\pi}\right]+\ln\left[\frac{\mu^2}{Q^2}\right]\right]$$

Recap

Regulator provides unique definition of V, f, ω

- Cutoff regulator L:
  - simple, but does NOT respect symmetries
- Dimensional regulator ε:
  - respects symmetries: translation, Lorentz, Gauge invariance
  - introduces new scale μ

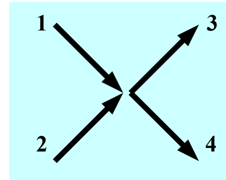
All physical quantities (E, dV, σ) are independent of the regulator
- AND the new scale μ
- Renormalization group equation: dσ/dμ=0

We can define renormalized quantities (V,f,ω)
- Renormalized (V,f,ω) are scheme dependent and arbitrary
- Physical quantities (E,dV, σ) are unique and scheme independent
  - if we apply the scheme consistently

Apply Dimensional Regularization to QFT

D-Dimensional Phase Space

$$d\sigma = \frac{1}{2s}\,|\mathcal{M}|^2\,d\Gamma$$



$$d\Gamma_i = \frac{d^D k_i}{(2\pi)^D}\,(2\pi)\,\delta(k_i^2)$$ 1-particle

$$d\Gamma = d\Gamma_3\,d\Gamma_4\,(2\pi)^D\,\delta^D(k_1+k_2-k_3-k_4)$$ Final state

$$d\Gamma = \frac{1}{16\pi}\,\left(\frac{s}{16\pi}\right)^{-\epsilon}\,\frac{(1-z^2)^{-\epsilon}}{\Gamma[1-\epsilon]}\,dz$$ Final state

$$g \to g\,\mu^{\epsilon}$$ Enter, μ scale

All the pieces

$$d\Gamma = \frac{1}{16\pi}\,\left(\frac{16\pi\mu^2}{Q^2}\right)^{+\epsilon}\,\frac{1}{\Gamma[1-\epsilon]}\,\frac{x^{\epsilon}}{(1-x)^{\epsilon}}\,(1-z^2)^{-\epsilon}\,dz$$

Homework: Part 1

#1) Show:

$$\frac{d^3 p}{(2\pi)^3\,2E} = \frac{d^4 p}{(2\pi)^4}\,(2\pi)\,\delta^+(p^2-m^2)$$

This relation is often useful as the RHS is manifestly Lorentz invariant

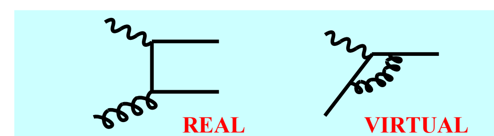
#2) Show that the 2-body phase space can be expressed as:

$$d\Gamma = \frac{d^3 p_3}{(2\pi)^3\,2E_3}\,\frac{d^3 p_4}{(2\pi)^3\,2E_4}\,(2\pi)^4\,\delta(p_1+p_2-p_3-p_4) = \frac{d\cos(\theta)}{16\pi}$$

Note, we are working with massless partons, and θ is in the partonic CMS frame

Soft Singularities

Soft Singularities



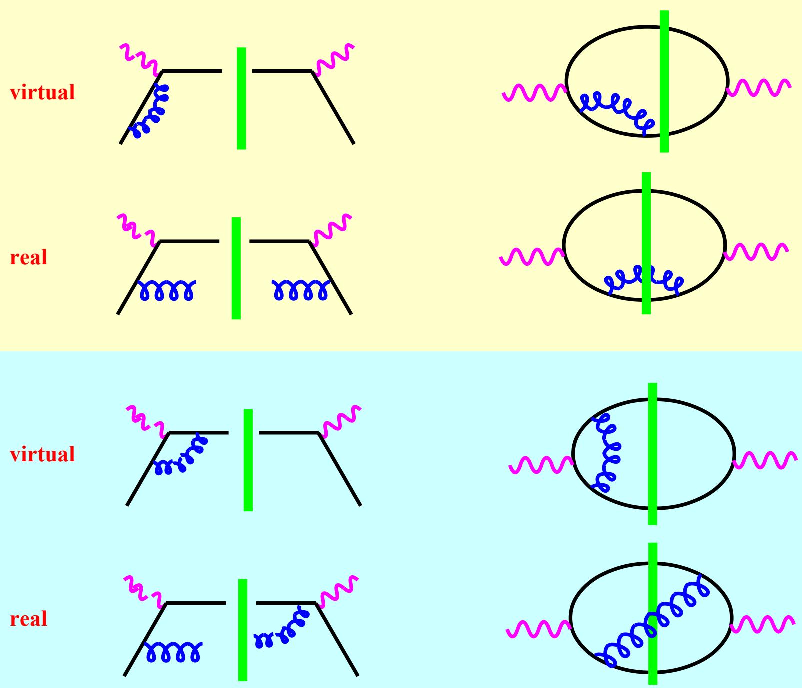
$$\underbrace{\frac{x^{\epsilon}}{(1-x)^{\epsilon}}}_{\text{From phase space}}\;\underbrace{\frac{1}{(1-x)}}_{\text{Soft Singularity}} = \underbrace{\frac{1}{(1-x)_+}}_{\text{Finite remainder}} - \underbrace{\frac{1}{\epsilon}\,\delta(1-x)}_{\text{To be canceled by virtual diagram}}$$

This only makes sense under the integral

$$\frac{f(x)}{(1-x)_+} = \frac{f(x)-f(1)}{(1-x)}$$

$$\int_0^1 dx\,f(x)\,\frac{x^{\epsilon}}{(1-x)^{1+\epsilon}} = \int_0^1 dx\,\frac{f(x)-f(1)}{(1-x)} - \frac{1}{\epsilon}\int_0^1 dx\,\delta(1-x)\,f(x)$$

KLN (Kinoshita, Lee, Nauenberg) Theorem



Collinear Singularities

Collinear Singularity

$$\int_{-1}^{1} dz\,(1-z^2)^{-\epsilon}\,|\mathcal{M}|^2 \simeq -\underbrace{\frac{1}{\epsilon}\,\frac{(1+x^2)}{(1-x)}}_{\text{This looks like part of the PDF}} + \underbrace{\frac{1-4x+4(1+x^2)\,\ln 2}{2(1-x)}}_{\text{This is finite for z=[-1,1]}}$$

... looks like a splitting kernel

Key Points

1) Subtract

2) This is defined by the scheme

3) Need to match schemes of ω and PDF
   ... MS, MS-Bar, DIS, ...

4) Note we have regulator ε and extra scale μ

How do we know

what goes in ω and PDFs ???

Compute NLO Subtractions

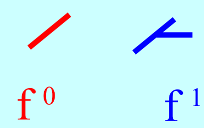
for a <u>partonic</u> target

Basic Factorization Formula $\sigma = f \otimes \omega + \mathcal{O}(\Lambda^2/Q^2)$

Higher Twist

At Zeroth Order:

$$\sigma^0 = f^0 \otimes \omega^0 \;\; + O(\Lambda^2/Q^2)$$



Use: $f^0 = \delta$ for a parton target.

Therefore: $\sigma^0 = f^0 \otimes \omega^0 = \delta \otimes \omega^0 = \omega^0$

$$\boxed{\sigma^0 = \omega^0}$$

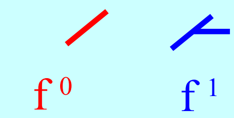
Warning: *This trivial result leads to many misconceptions at higher orders*

Basic Factorization Formula $\sigma = f \otimes \omega + \mathcal{O}(\Lambda^2/Q^2)$

At First Order:

$$\sigma^1 = f^1 \otimes \omega^0 + f^0 \otimes \omega^1$$

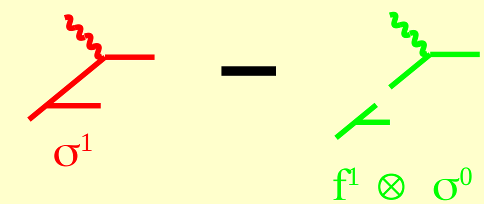
$$\sigma^1 = \;\; f^1 \otimes \sigma^0 + \omega^1$$



We used: $f^0 = \delta$ for a parton target.

Therefore: $\omega^1 = \sigma^1 - f^1 \otimes \sigma^0$

$\omega^1 =$



$f^1 \sim \frac{\alpha_s}{2\pi} P^{(1)}$

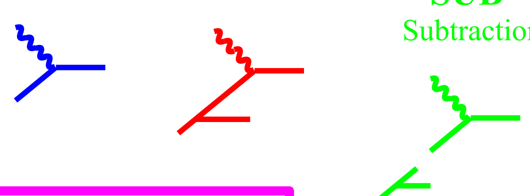
$P^{(1)}$ defined by scheme choice

Combined Result:

Complete NLO Term: ω^1

$$\underbrace{\omega^0 + \omega^1}_{\text{TOT}} = \sigma^0 + \sigma^1 - \underbrace{f^1 \otimes \sigma^0}_{\text{SUB}}$$

TOT **LO** **NLO** **SUB** Subtraction



TOT = LO + NLO - SUB

Use the Basic Factorization Formula $\sigma = f \otimes \omega \otimes d + \mathcal{O}(\Lambda^2/Q^2)$

At Second Order *(NNLO):*

Therefore:

$$\sigma^2 = f^2 \otimes \omega^0 \otimes d^0 + \ldots$$
$$+ f^1 \otimes \omega^1 \otimes d^0 + \ldots$$

$\omega^2 = ???$

Include Fragmentation Functions d

Compute ω^2 at second order.
Make a diagrammatic representation of each term.

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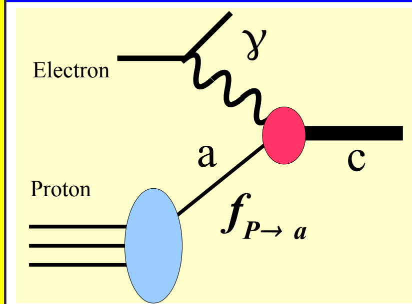
Do we get different answers with different schemes???

Pictorial Demonstration of Scheme Consistency

38

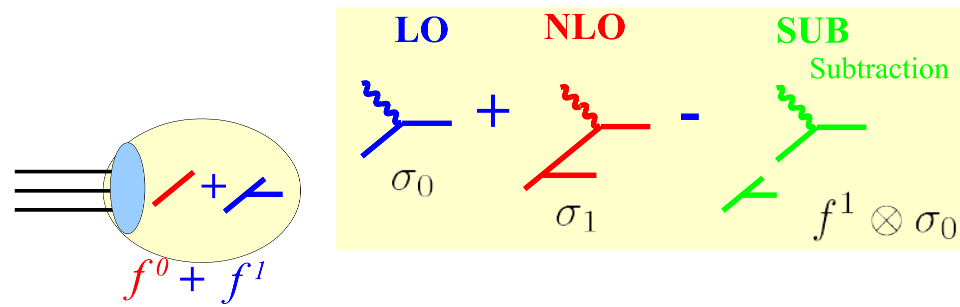


Parton Model

$$\sigma(Q^2) = f(\mu, \alpha_s) \otimes \widehat{\omega}(Q^2, \mu^2, \alpha_s)$$

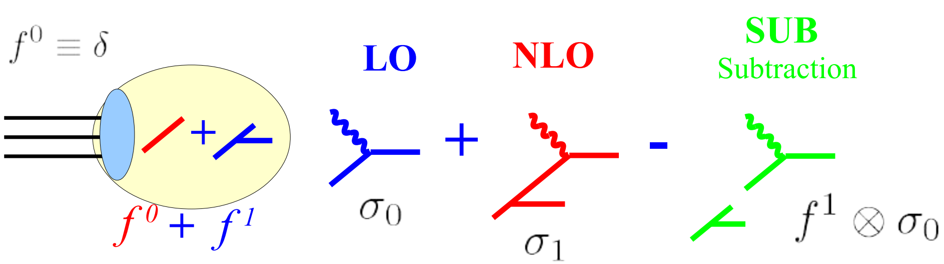
Evolution Equation

$$\frac{df}{\ln[\mu^2]} = P \otimes f$$



Pictorial Demonstration of Scheme Consistency

39



$$[\delta + f^1] \otimes [\overbrace{\sigma_0 + \sigma_1 - f^1 \otimes \sigma_0}^{\text{Complete NLO Term}}]$$

$$f^1 \sim \frac{\alpha_s}{2\pi} P^{(1)}$$

$P^{(1)}$ defined by scheme choice

$$\sigma_0 + \sigma_1 + f^1 \otimes \sigma_0 - f^1 \otimes \sigma_0 + \mathcal{O}(\alpha_s^2)$$

From NLO Subtraction

From PDF Evolution

Contains BOTH collinear and non-collinear region

QCD is Bullet-proof

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Do we get different answers with different schemes???

NO !!!

End of lecture 3: Recap

NLO Theoretical Calculations:

- Essential for accurate comparison with experiments

We encounter singularities:

- Soft singularities: cancel between real and virtual diagrams
- Collinear singularities: "absorb" into PDF

Regularization and Renormalization:

- Regularize & Renormalize intermediate quantities
- Physical results independent of regulators (e.g., L, or μ and ε)
- Renormalization introduces scheme dependence (MS-bar, DIS)

Factorization works:

- Hard cross section $\widehat{\sigma}$ or ω is not the same as σ
- Scheme dependence cancels out (if performed consistently)

END OF LECTURE 3

We already studies

DIS

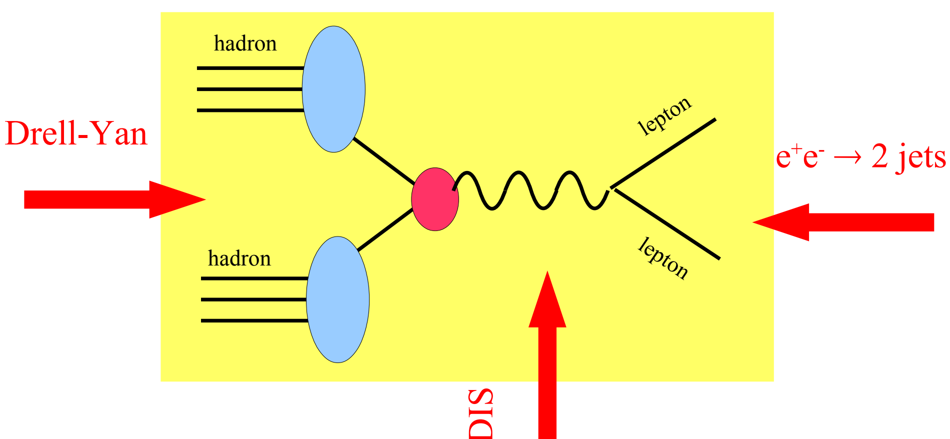
Now we consider

Drell-Yan Process

e^+e^-

Important for Tevatron and LHC

What is the Explanation

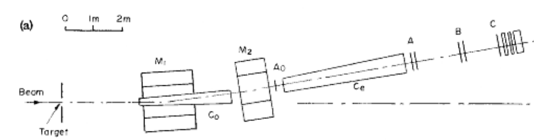


Drell-Yan and e^+e^- have an interesting historical relation

A Drell-Yan Example: Discovery of J/Psi

The Process: $p + Be \to e^+ e^- X$

very narrow width
$\Rightarrow$ long lifetime



at BNL AGS

VOLUME 33, NUMBER 23 PHYSICAL REVIEW LETTERS 2 DECE

Experimental Observation of a Heavy Particle J†

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(Received 12 November 1974)

We report the observation of a heavy particle J, with mass $m = 3.1$ GeV and width approximately zero. The observation was made from the reaction $p + \mathrm{Be} \to e^+ + e^- + x$ by measuring the e^+e^- mass spectrum with a precise pair spectrometer at the Brookhaven National Laboratory's 30-GeV alternating-gradient synchrotron.

This experiment is part of a large program to daily with a thin Al foil. The beam spot

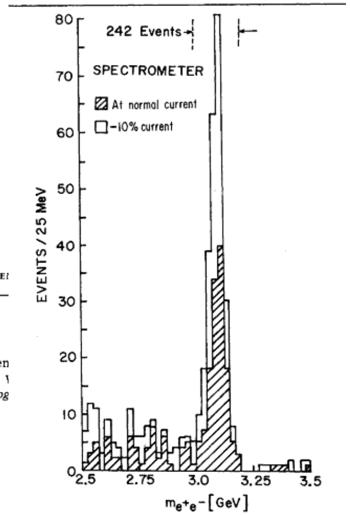
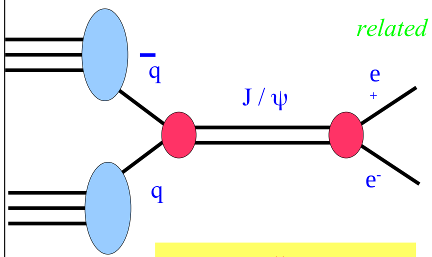
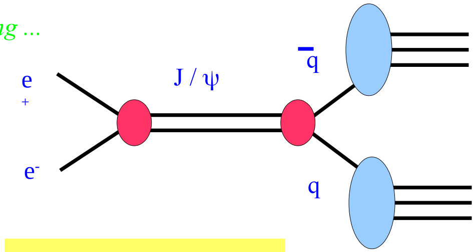


FIG. 2. Mass spectrum showing the existence of J. Results from two spectrometer settings are plotted showing that the peak is independent of spectrometer currents. The run at reduced current was taken two months later than the normal run.

The November Revolution: 1973

related by crossing ...





Drell-Yan
Brookhaven AGS

e^+e^- Production
SLAC SPEAR
Frascati ADONE

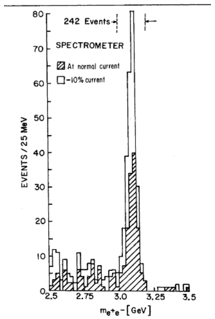
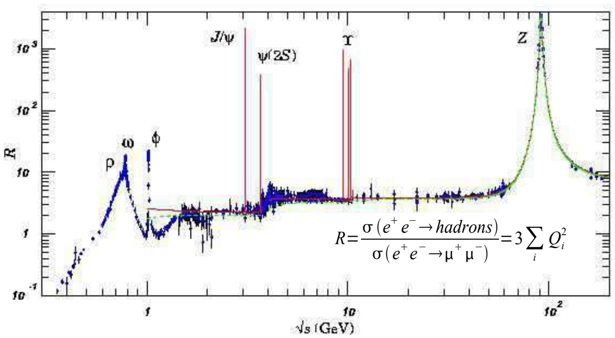


FIG. 2. Mass spectrum showing the existence of J. Results from two spectrometer settings are plotted showing that the peak is independent of spectrometer currents. The run at reduced current was taken two months later than the normal run.



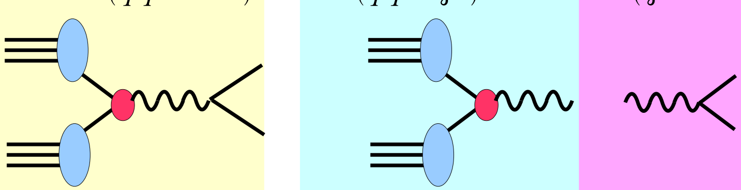
We'll look at Drell-Yan

Specifically W/Z production

Side Note: From $pp\to\gamma$ / Z / W, we can obtain $pp\to\gamma$ / Z / $W\to l^+l^-$

Schematically:

$$d\sigma(q\bar{q}\to l^+l^-) = d\sigma(q\bar{q}\to\gamma^*) \times d\sigma(\gamma^*\to l^+l^-)$$

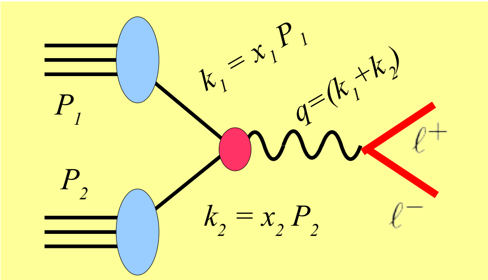


For example:

$$\frac{d\sigma}{dQ^2\,d\hat{t}}(q\bar{q}\to l^+l^-) = \frac{d\sigma}{d\hat{t}}(q\bar{q}\to\gamma^*) \times \frac{\alpha}{3\pi Q^2}$$

Kinematics in the hadronic CMS

Kinematics for Drell-Yan



$$P_1 = \frac{\sqrt{s}}{2}(1,0,0,+1) \qquad P_1^2=0$$

$$P_2 = \frac{\sqrt{s}}{2}(1,0,0,-1) \qquad P_2^2=0$$

$$k_1 = x_1 P_1 \qquad k_1^2=0$$

$$k_2 = x_2 P_2 \qquad k_2^2=0$$

$$\frac{d\sigma}{dx_1\, dx_2} = \sum_{q,\bar{q}} \{q(x_1)\bar{q}(x_2)+q(x_2)\bar{q}(x_1)\}\ \hat{\sigma}$$

Hadronic cross section

Parton distribution functions

Partonic cross section

Kinematics for Drell-Yan

Trade $\{x_1, x_2\}$ variables for $\{\tau, y\}$

$$x_{1,2} = \sqrt{\tau}\, e^{\pm y} \qquad y = \frac{1}{2}\ln\left(\frac{x_1}{x_2}\right) \qquad \tau = x_1\, x_2$$

$$s = (P_1+P_2)^2 = \frac{\hat{s}}{x_1 x_2} = \frac{\hat{s}}{\tau}$$

Therefore $\tau = x_1 x_2 = \frac{\hat{s}}{s} \equiv \frac{Q^2}{s}$

Fractional energy[2] between partonic and hadronic system

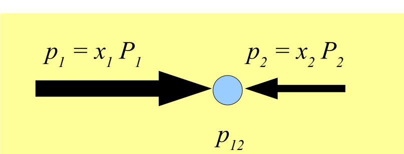
Using: $d x_1 d x_2 = d\tau\, dy$

$$\frac{d\sigma}{d\tau\, dy} = \sum_{q,\bar{q}} \{q(x_1)\bar{q}(x_2)+q(x_2)\bar{q}(x_1)\}\ \hat{\sigma}$$

Rapidity & Longitudinal Momentum Distributions

The rapidity is defined as: $y = \frac{1}{2}\ln\left\{\frac{E_{12}+p_L}{E_{12}-p_L}\right\}$

Partonic CMS has longitudinal momentum w.r.t. the hadron frame



$$p_{12}=(p_1+p_2)=(E_{12},0,0,p_L)$$

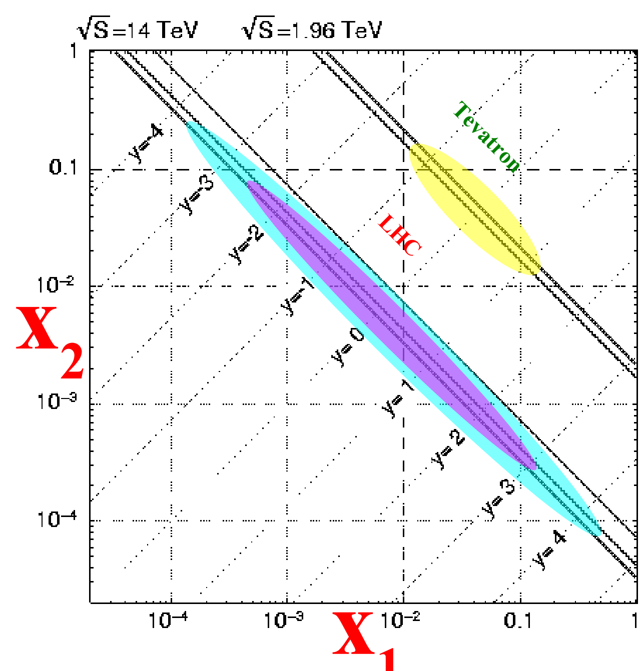
$$E_{12}=\frac{\sqrt{s}}{2}(x_1+x_2)$$

$$p_L=\frac{\sqrt{s}}{2}(x_1-x_2) \equiv \frac{\sqrt{s}}{2}x_F$$

x_F is a measure of the longitudinal momentum

$$y = \frac{1}{2}\ln\left\{\frac{E_{12}+p_L}{E_{12}-p_L}\right\} = \frac{1}{2}\ln\left\{\frac{x_1}{x_2}\right\}$$

Kinematics for W / Z / Higgs Production



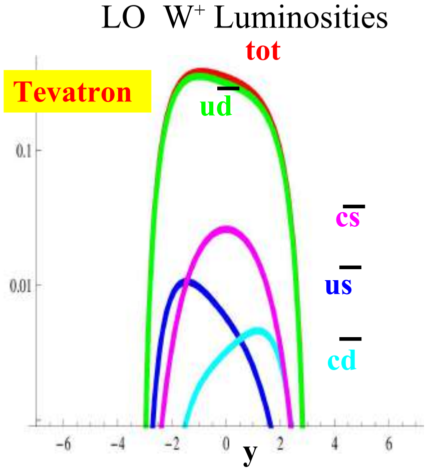
$$y = \frac{1}{2}\ln\left(\frac{x_1}{x_2}\right) \qquad \tau = x_1\, x_2$$

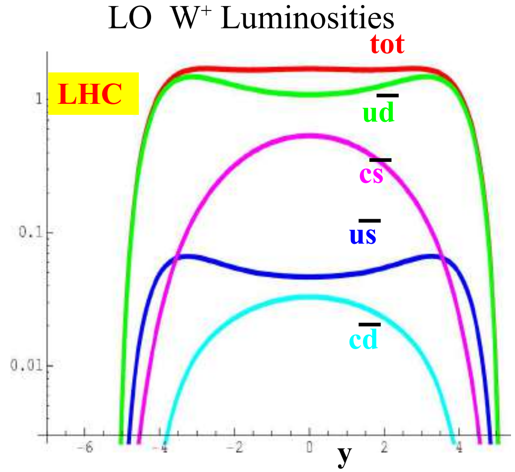
$$x_{1,2} = \sqrt{\tau}\, e^{\pm y}$$

Z
W

H
Z
W

Kinematics for W production at Tevatron & LHC





$$d\sigma = \int dx_1 \int dx_2 \int d\tau \, \{q(x_1)\bar{q}(x_2)+q(x_2)\bar{q}(x_1)\} \;\; \hat{\sigma}\; \delta(\tau - \frac{M^2}{S})$$

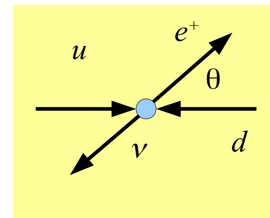
$$\frac{d\sigma}{d\tau} = \frac{dL}{d\tau} \;\; \hat{\sigma}(\tau) \qquad\qquad \frac{dL}{d\tau} = f \otimes f$$

Kinematics in a Hadron-Hadron Interaction:

The CMS of the parton-parton system is moving
longitudinally relative to the hadron-hadron system

How do we measure the W-boson mass?

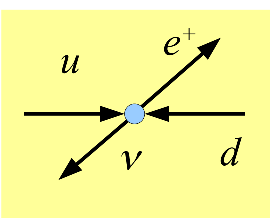
$$u+\bar{d} \rightarrow W^+ \rightarrow e^+ \nu$$



Can't measure W directly
Can't measure ν directly
Can't measure longitudinal momentum

We can measure the P_T of the lepton

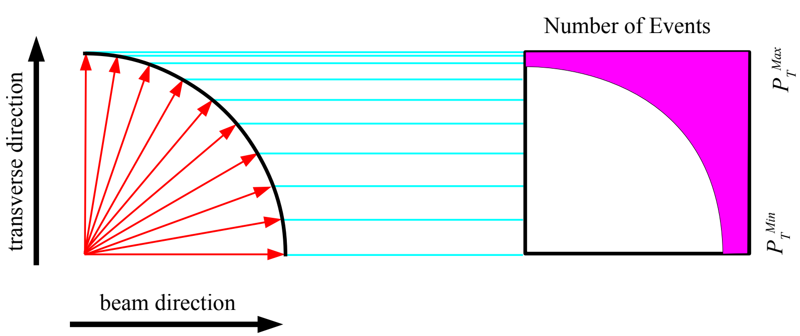
The Jacobian Peak



Suppose lepton distribution is uniform in θ

The dependence is actually $(1+\cos\theta)^2$, but we'll worry about that later

What is the distribution in P_T?



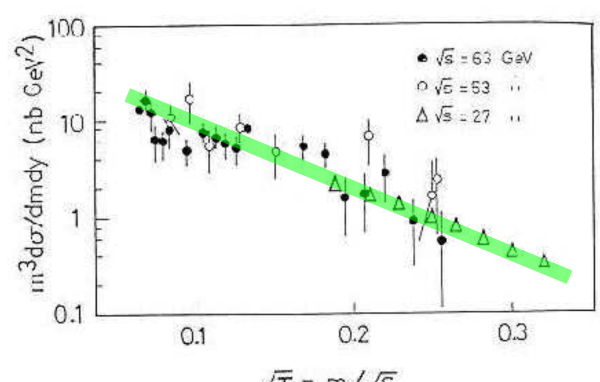
We find a peak at $P_T^{max} \approx M_W/2$

Drell-Yan Cross Section and the Scaling Form

Using: $\hat{\sigma}_0 = \dfrac{4\pi\alpha^2}{9\hat{s}} Q_i^2$ and $\delta(Q^2-\hat{s}) = \dfrac{1}{s\,x_1}\,\delta(x_2 - \dfrac{\tau}{x_1})$

we can write the cross section in the scaling form:

$$Q^4 \frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{9} \sum_{q,\bar{q}} Q_i^2 \int_\tau^1 \frac{dx_1}{x_1} \, \tau \, \{q(x_1)\bar{q}(\tau/x_1) + \bar{q}(x_1) q(\tau/x_1)\}$$



Notice the RHS is a function of only τ, not Q.

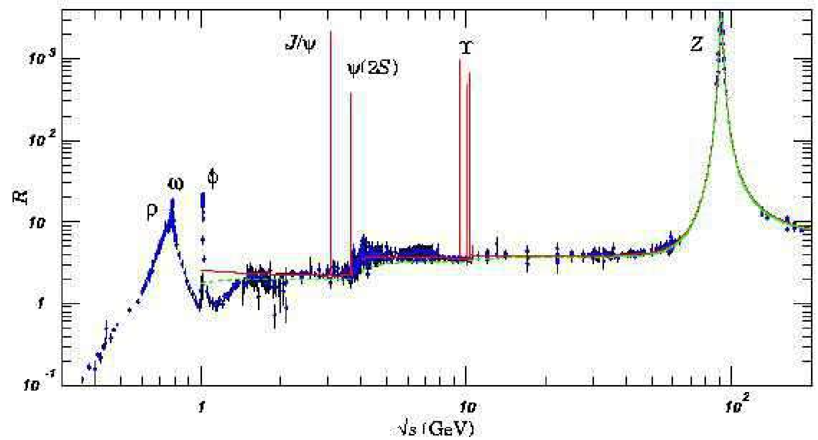
This quantity should lie on a universal scaling curve.

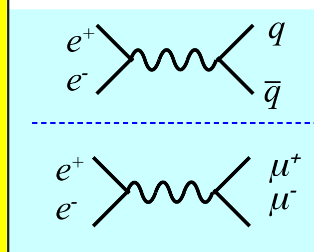
Cf., DIS case, & scattering of point-like constituents

e^+e^- R ratio

$$R=\frac{\sigma(e^+e^-\rightarrow hadrons)}{\sigma(e^+e^-\rightarrow\mu^+\mu^-)}$$

e^+e^- Ratio of hadrons to muons





3 quark colors

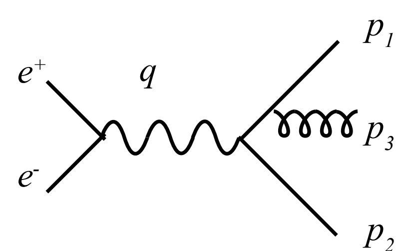
$$R=\frac{\sigma(e^+e^-\rightarrow hadrons)}{\sigma(e^+e^-\rightarrow\mu^+\mu^-)}=3\sum_i Q_i^2\left[1+\frac{\alpha_s}{\pi}\right]$$

NLO correction

e^+e^-

NLO corrections

e^+e^- to 3 particles final state



Define the energy fractions E_i:

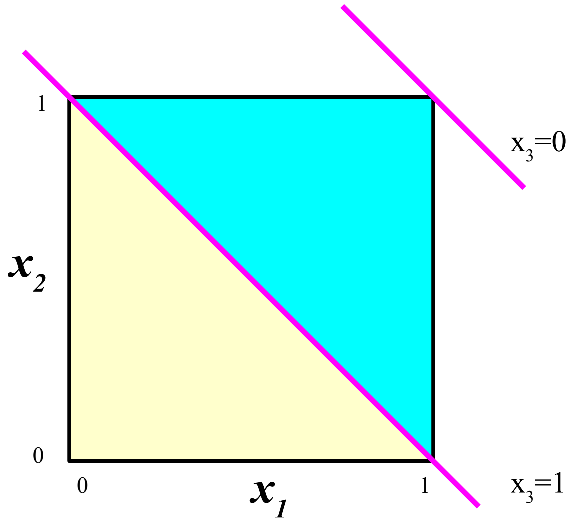
$$x_i=\frac{E_i}{\sqrt{s}/2}=\frac{2p_i\cdot q}{s}$$

Energy Conservation: $\sum_i x_i=2$

Range of x: $x_i\subset[0,1]$

Exercise: show 3-body phase space is flat in dx_1dx_2

3-Particle Phase Space

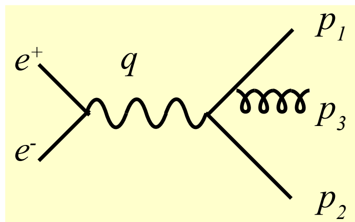


$$x_i = \frac{E_i}{\sqrt{s}/2}$$

$$x_i \subset [0,1]$$

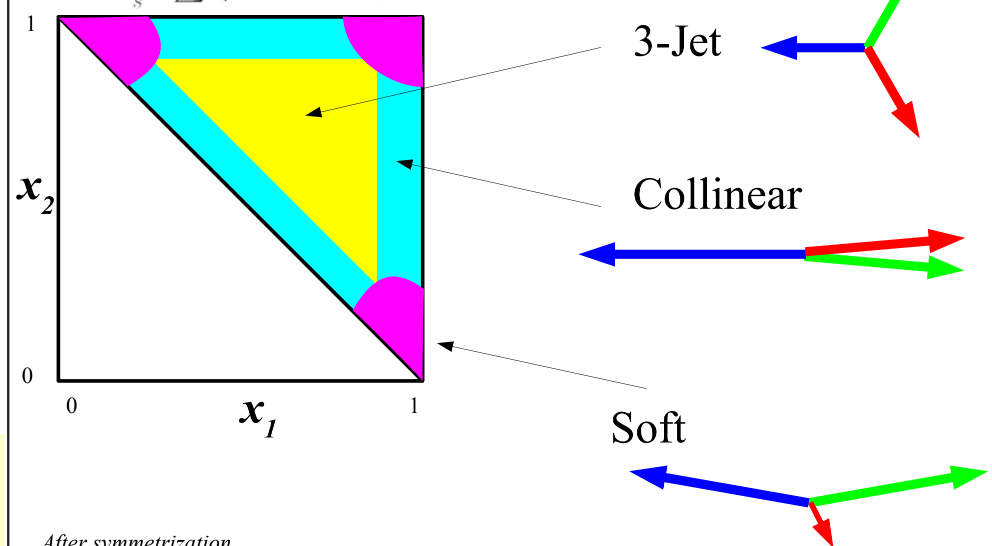
$$x_1 + x_2 + x_3 = 2$$

$$d\Gamma \sim dx_1\, dx_2$$



3-Particle Configurations

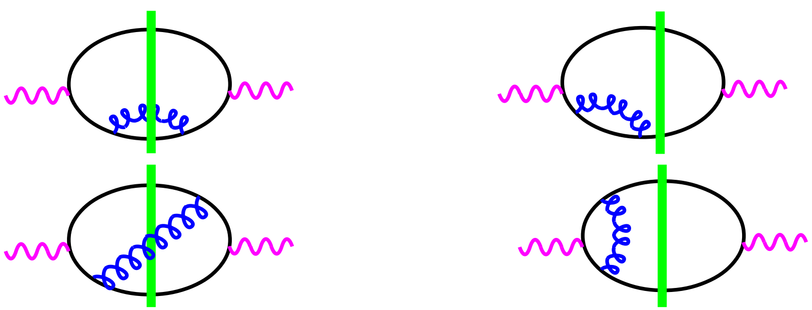
$\sigma_0 = \frac{4\pi\alpha^2}{s} \sum e_q^2 \qquad C_F = 4/3$



After symmetrization

$$\frac{1}{\sigma_0}\frac{d\sigma}{dx_1\, dx_2} = \frac{\alpha_s}{2\pi} C_F \frac{x_1^2 + x_2^2 + x_3^2}{(1-x_1)(1-x_2)(1-x_3)}$$

Singularities cancel between 2-particle and 3-particle graphs



$$\sigma_2^{(\epsilon)} = \sigma_0\, C_F \frac{\alpha_s}{\pi} (\ldots) \left[+ \frac{-1}{\epsilon^2} + \frac{-3}{2\epsilon} + \frac{\pi^2}{2} - 4 \right]$$

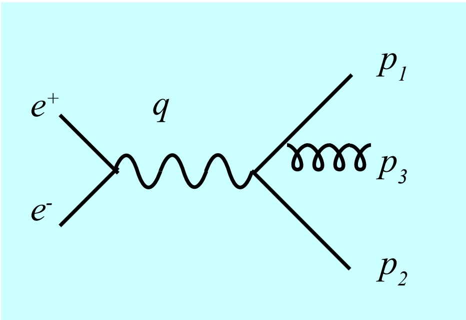
$$\sigma_3^{(\epsilon)} = \sigma_0\, C_F \frac{\alpha_s}{\pi} (\ldots) \left[+ \frac{+1}{\epsilon^2} + \frac{+3}{2\epsilon} + \frac{-\pi^2}{2} - \frac{19}{4} \right]$$

$$\sigma_2^{(\epsilon)} + \sigma_3^{(\epsilon)} = \sigma_0\, C_F \frac{\alpha_s}{\pi} (\ldots) \left[0 + 0 + 0 + -\frac{35}{4} \right]$$

Same result with gluon mass regularization

e^+e^- Differential Cross Sections

Differential Cross Section



What do we do about soft and collinear singularities????

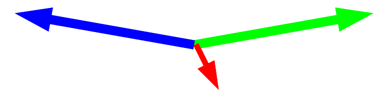
Introduce the concept of "Infrared Safe Observable"

The soft and collinear singularities will cancel
ONLY
if the physical observables are appropriately defined.

Infrared Safe Observables

Observables must satisfy the following requirements:

Soft



if $p_s \to 0$

$$\mathcal{O}_{n+1}(p_1, ..., p_n, p_s) \longrightarrow \mathcal{O}_n(p_1, ..., p_n)$$

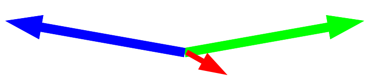
Collinear

if $p_a \parallel p_b$

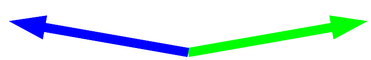
$$\mathcal{O}_{n+1}(p_1, ..., p_a, p_b, ..., p_n) \longrightarrow \mathcal{O}_n(p_1, ..., p_a + p_b, ..., p_n)$$

Infrared Safe Observables

Soft

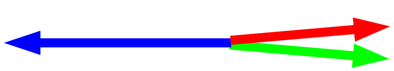


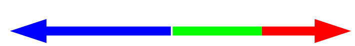
if $p_s \to 0$



$$\mathcal{O}_{n+1}(p_1, ..., p_n, p_s) \longrightarrow \mathcal{O}_n(p_1, ..., p_n)$$

Collinear





if $p_a \parallel p_b$

$$\mathcal{O}_{n+1}(p_1, ..., p_a, p_b, ..., p_n) \longrightarrow \mathcal{O}_n(p_1, ..., p_a + p_b, ..., p_n)$$

Examples: Infrared Safe Observables

Infrared Safe Observables:

Event shape distributions
Jet Cross sections

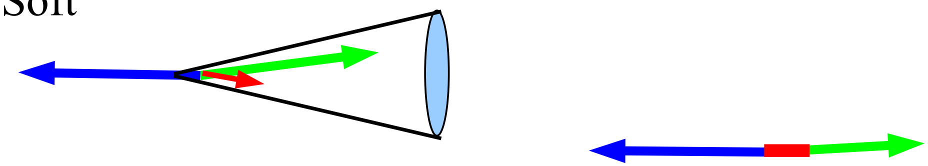
Un-Safe Infrared Observables:

Momentum of the hardest particle
(affected by collinear splitting)

100% isolated particles
(affected by soft emissions)

Particle multiplicity
(affected by both soft & collinear emissions)

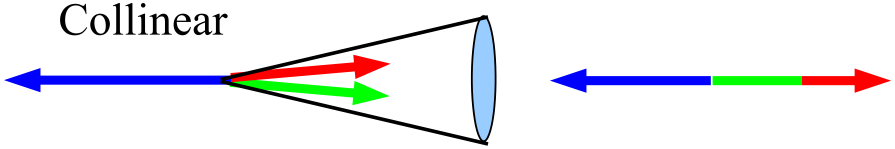
Soft



if $p_s \to 0$

$$\mathcal{O}_{n+1}(p_1, ..., p_n, p_s) \longrightarrow \mathcal{O}_n(p_1, ..., p_n)$$

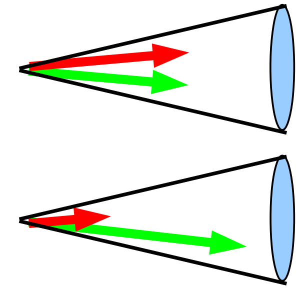
Collinear

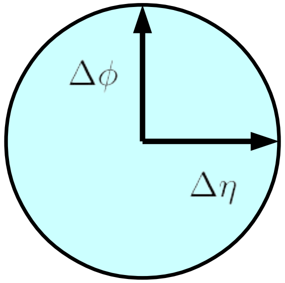


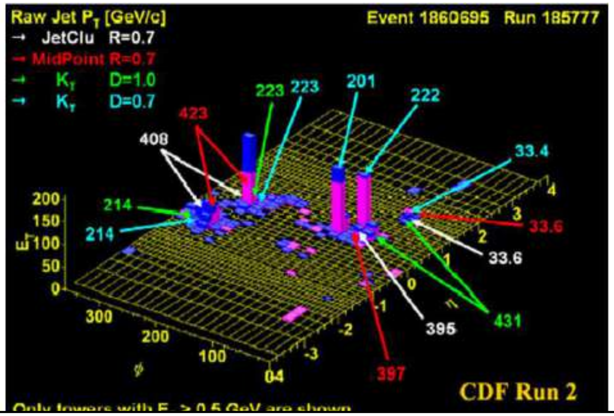
if $p_a \parallel p_b$

$$\mathcal{O}_{n+1}(p_1, ..., p_a, p_b, ..., p_n) \longrightarrow \mathcal{O}_n(p_1, ..., p_a + p_b, ..., p_n)$$

Jet Cone

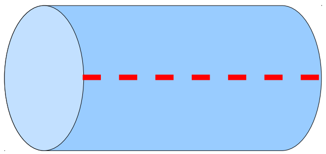




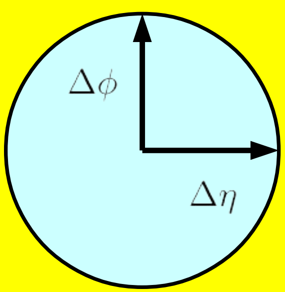


$$R^2 = (\Delta\eta)^2 + (\Delta\phi)^2$$

Let's examine this definition a bit more closely



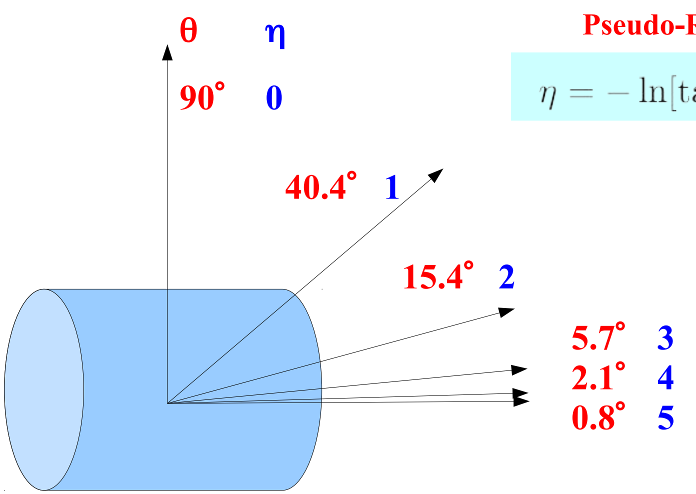
Jet Cone



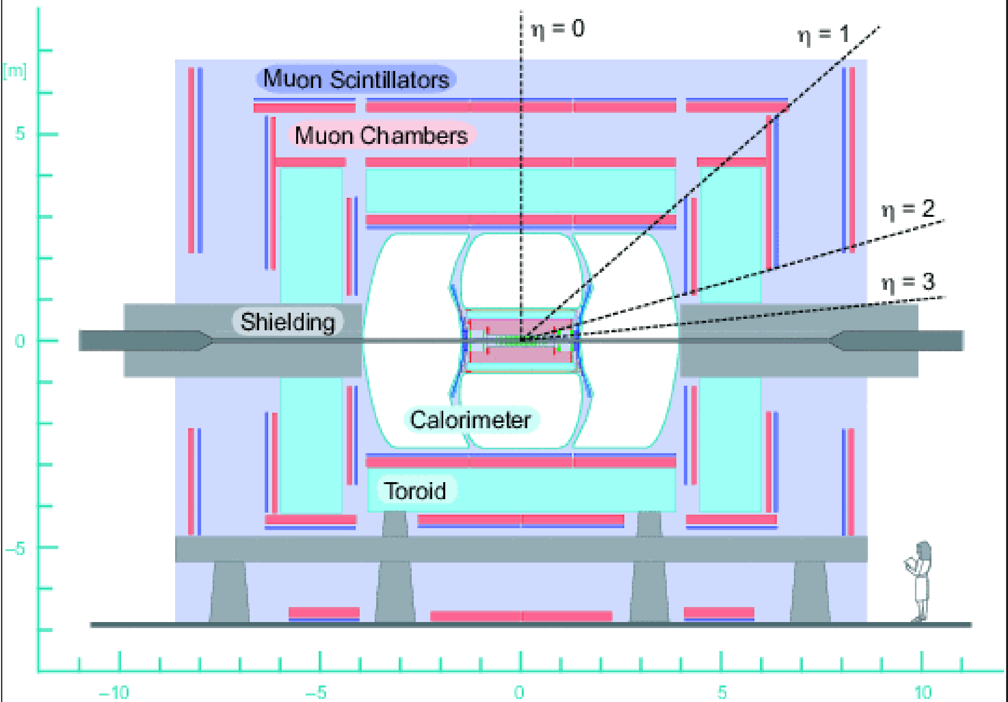
$$R^2 = (\Delta\eta)^2 + (\Delta\phi)^2$$

Pseudo-Rapidity

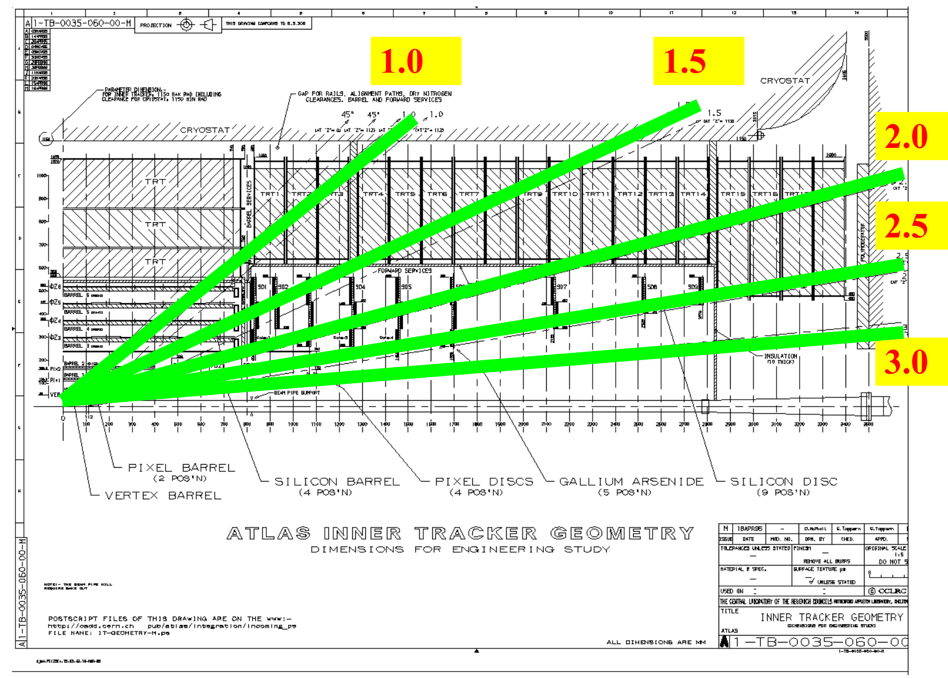
$$\eta = -\ln[\tan(\theta/2)]$$



D0 Detector Schematic

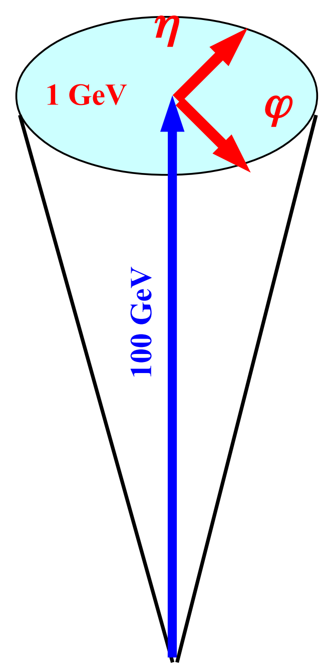


ATLAS Detector Schematic



homework

HOMEWORK: Jet Cone Definition



PROBLEM #2: In a Tevatron detector, consider two particles traveling in the transverse direction:

$$p_1^\mu = \{E, 100, 0, 1\}$$
$$p_2^\mu = \{E, 100, 1, 0\}$$

where the componets are expresedn in GeV units. E is defined such that the particles are massless.

a) Compute E.

b) For each particle, compute the pseudorapidity η and azimuthal angle ϕ.

c) Explain how the above exercise justifies the correct jet radius definition to be:

$$R = \sqrt{\eta^2 + \phi^2}$$

In particular, why is the above correct and $R = \sqrt{\eta^2 + 2\phi^2}$, for example, incorrect.

HOMEWORK: Light-Cone Coordinates & Boosts

$$P_\mu = \{P_t, P_x, P_y, P_z\}$$

$$P_\mu = \{P_+, \overrightarrow{P_\perp}, P_-\} \qquad \overrightarrow{P}_\perp = \{P_x, P_y\}$$

$$P_\pm = \frac{1}{\sqrt{2}}(P_t \pm P_z)$$

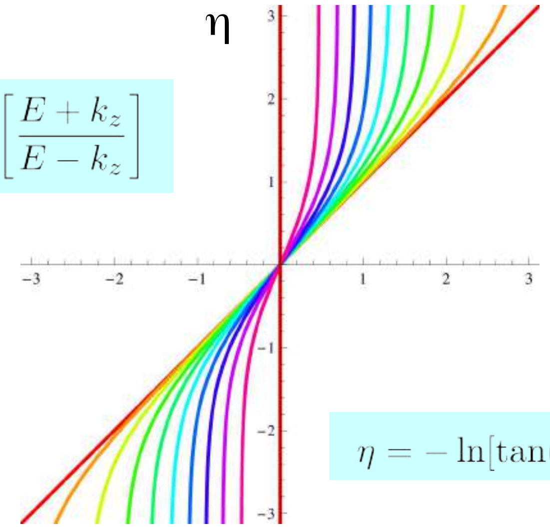
1) Compute the metric $g_{\mu\nu}$ in the light-cone frame, and compute $\overrightarrow{P}_1 \cdot \overrightarrow{P}_2$ in terms of the light-cone components.

2) Compute the boost matrix B for a boost along the z-axis, and show the light-cone vector transforms in a particularly simple manner.

3) Show that a boost along the z-axis uniformily shifts the rapidity of a vector by a constant amount.

Rapidity vs. Pseudo-Rapidity

$$y = \frac{1}{2}\ln\left[\frac{E+k_z}{E-k_z}\right]$$



$$\eta = -\ln[\tan(\theta/2)]$$

$$\frac{M}{E} = \{0, 0.1, 0.2...\}$$

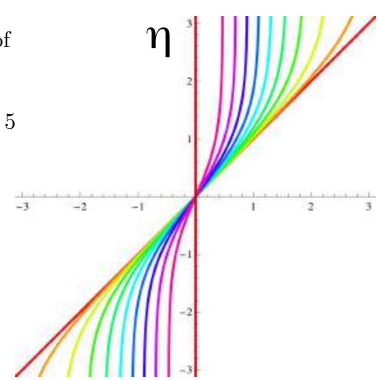
HOMEWORK: Rapidity vs. Pseudo-Rapidity

PROBLEM #1: Consider the rapidity y and the pseudo-rapidity η:

$$y = \frac{1}{2}\ln\left(\frac{E+P_z}{E-P_z}\right)$$

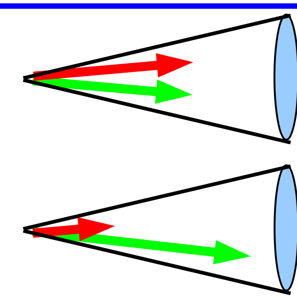
$$\eta = -\ln\left[\tan\left(\frac{\theta}{2}\right)\right]$$

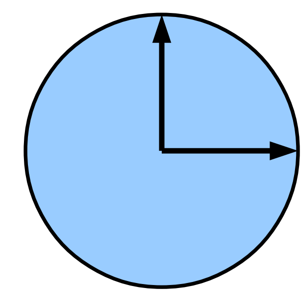
a) Make a parametric plot of $\{y, \eta\}$as a function of of the particle.
b) Show that in the limit $m \to 0$ that $y \to \eta$.
c) Make a table of η for $\theta = [0°, 180°]$ in steps of 5
d) Make a table of θ for $\eta = [0, 10]$ i



Infrared Safe Observables: Define Jets

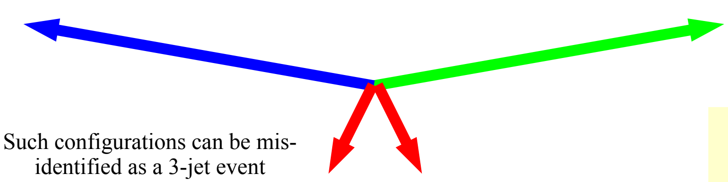
Jet Cone





Problem:
The cone definition is simple,
BUT
it is too simple

$$R^2 = (\Delta\eta)^2 + (\Delta\phi)^2$$



Such configurations can be mis-identified as a 3-jet event

See talk by
Dave Soper &
Andrew Larkoski

End of lecture 4: Recap

Drell-Yan: Tremendous discovery potential

- Need to compute 2 initial hadrons

e^+e^- processes:

- Total Cross Section:
- Differential Cross Section: singularities

Infrared Safe Observables

- Stable under soft and collinear emissions

Jet definition

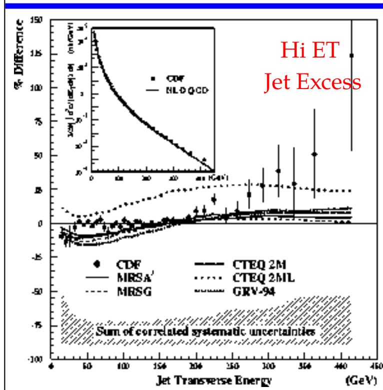
- Cone definition is simple:
- ... it is TOO simple

Final Thoughts

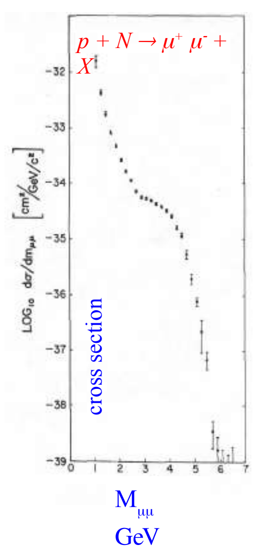
$$\mathcal{L}_{\rm QCD} = \bar\psi_i \left(i\gamma^\mu (D_\mu)_{ij} - m\,\delta_{ij}\right)\psi_j - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a$$
$$= \bar\psi_i(i\gamma^\mu\partial_\mu - m)\psi_i - gG^a_\mu\bar\psi_i\gamma^\mu T^a_{ij}\psi_j - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a ,$$

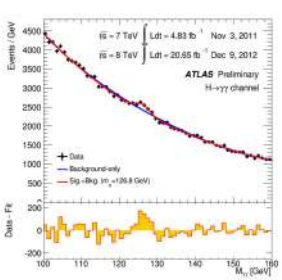
Scaling, Dimensional Analysis, Factorization, Regularization & Renormalization, Infrared Saftey ...

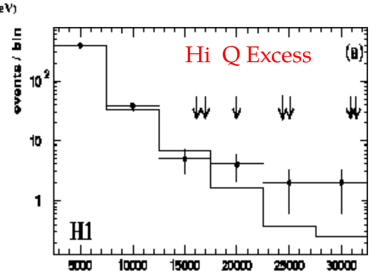
Can you find the Nobel Prize???



CDF Collaboration, PRL 77, 438 (1996)







H1 Collaboration, ZPC74, 191 (1997)
ZEUS Collaboration, ZPC74, 207 (1997)

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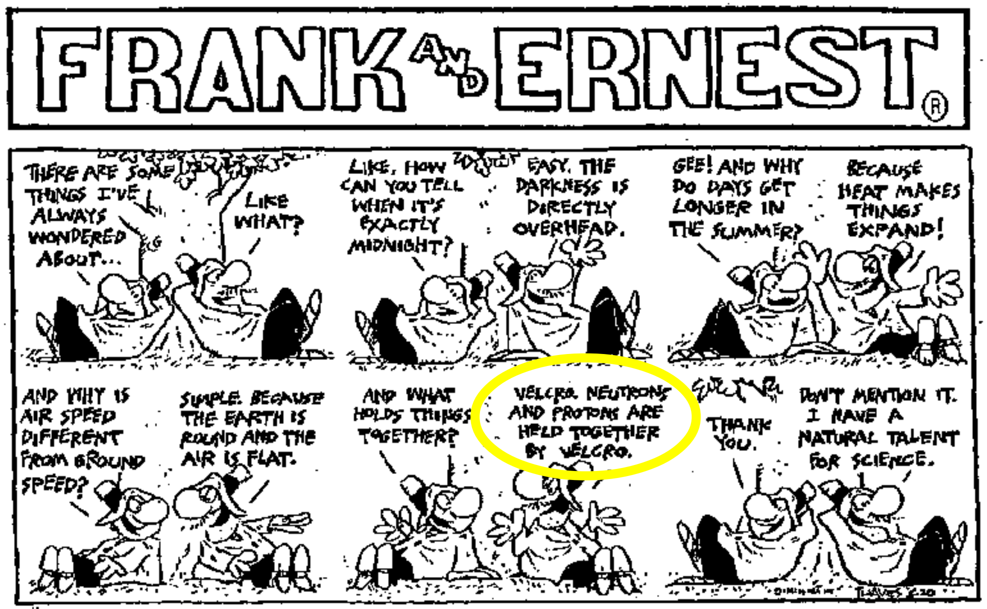
To the CTEQ and MCnet folks
for making all this possible.

LibreOffice The Document Foundation

and the many web pages where I borrowed my figures ...

PowerPoint

Keep an open mind!!!



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END OF LECTURE 4