CTEQ School on
QCD Analysis and Electroweak Phenomenology

LECTURE 4

Introduction to the Parton Model and Perturbative QCD

Fred Olness (SMU)

University of Pittsburgh, PA

18-28 July 2017

We already studies

DIS

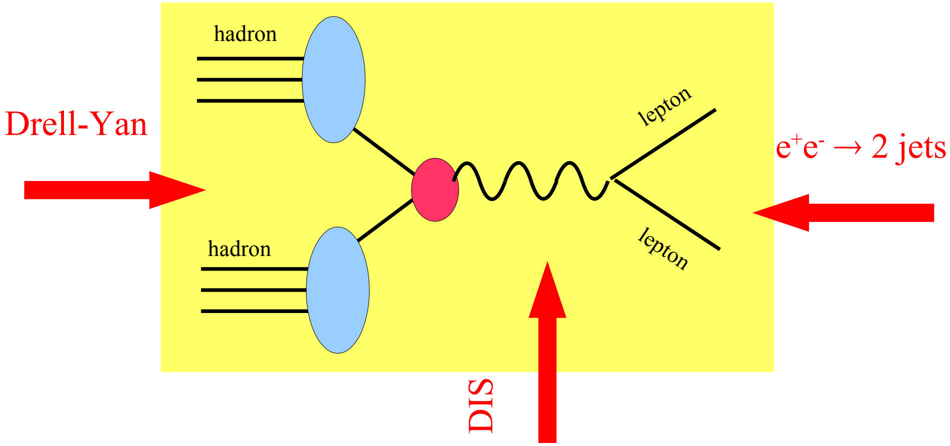
Now we consider

Drell-Yan Process

e^+e^-

Important for Tevatron and LHC

What is the Explanation

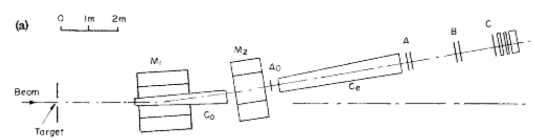


Drell-Yan and e^+e^- have an interesting historical relation

A Drell-Yan Example: Discovery of J/Psi

The Process: $p + Be \to e^+ e^- X$

very narrow width
⇒ long lifetime



at BNL AGS

VOLUME 33, NUMBER 23 PHYSICAL REVIEW LETTERS 2 DECE

Experimental Observation of a Heavy Particle J†

J. J. Aubert, U. Becker, P. J. Biggs, J. Burger, M. Chen, G. Everhart, P. Goldhagen
J. Leong, T. McCorriston, T. G. Rhoades, M. Rohde, Samuel C. C. Ting, and Sau Lan
Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technolog
Cambridge, Massachusetts 02139

and

Y. Y. Lee
Brookhaven National Laboratory, Upton, New York 11973
(Received 12 November 1974)

We report the observation of a heavy particle J, with mass $m = 3.1$ GeV and width approximately zero. The observation was made from the reaction $p + \mathrm{Be} \to e^+ + e^- + x$ by measuring the e^+e^- mass spectrum with a precise pair spectrometer at the Brookhaven National Laboratory's 30-GeV alternating-gradient synchrotron.

This experiment is part of a large program to

daily with a thin Al foil. The beam spot

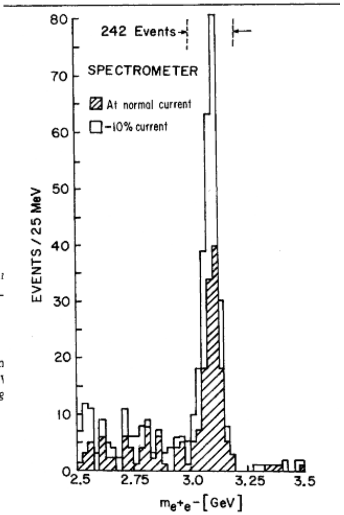


FIG. 2. Mass spectrum showing the existence of J. Results from two spectrometer settings are plotted showing that the peak is independent of spectrometer currents. The run at reduced current was taken two months later than the normal run.

The November Revolution: 1973

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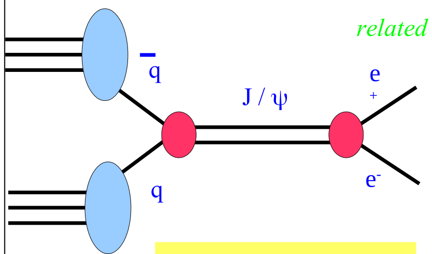
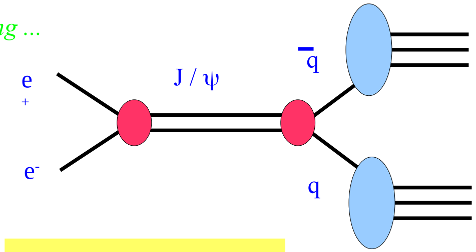
related by crossing ...





Drell-Yan
Brookhaven AGS

e^+e^- Production
SLAC SPEAR
Frascati ADONE

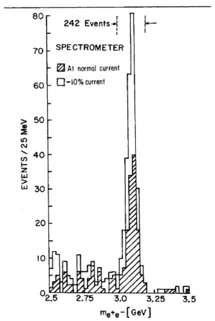
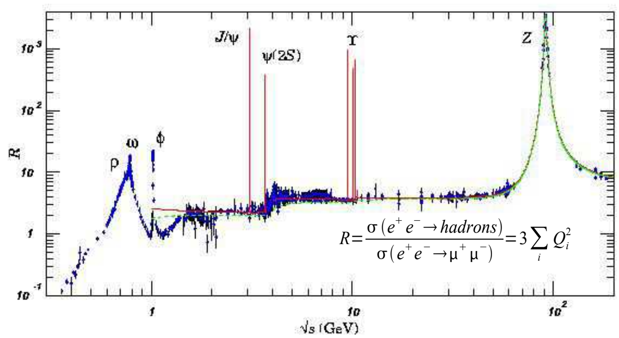


FIG. 2. Mass spectrum showing the existence of *J*. Results from two spectrometer settings are plotted showing that the peak is independent of spectrometer currents. The run at reduced current was taken two months later than the normal run.



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We'll look at Drell-Yan

Specifically W/Z production

Side Note: From $pp \to \gamma$ / Z / W, we can obtain $pp \to \gamma$ / Z / $W \to l^+ l^-$

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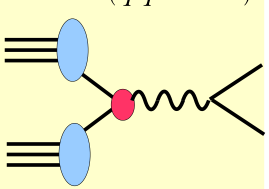
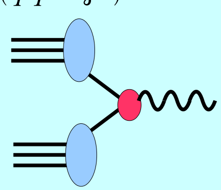
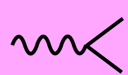
Schematically:

$$d\sigma(q\bar{q} \to l^+ l^-) = d\sigma(q\bar{q} \to \gamma^*) \times d\sigma(\gamma^* \to l^+ l^-)$$







For example:

$$\frac{d\sigma}{dQ^2\, d\hat{t}}(q\bar{q} \to l^+ l^-) = \frac{d\sigma}{d\hat{t}}(q\bar{q} \to \gamma^*) \times \frac{\alpha}{3\pi Q^2}$$

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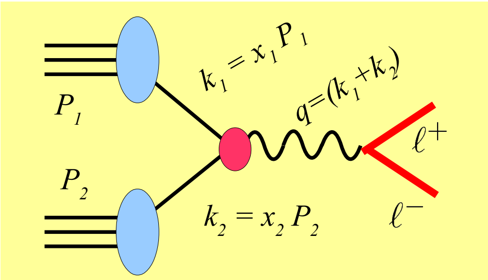
Kinematics in the hadronic CMS

Kinematics for Drell-Yan



$$P_1 = \frac{\sqrt{s}}{2}(1,0,0,+1) \qquad P_1^2=0$$
$$P_2 = \frac{\sqrt{s}}{2}(1,0,0,-1) \qquad P_2^2=0$$

$$k_1=x_1 P_1 \qquad k_1^2=0$$
$$k_2=x_2 P_2 \qquad k_2^2=0$$

$$\frac{d\sigma}{dx_1\, dx_2} = \sum_{q,\bar{q}} \{q(x_1)\bar{q}(x_2)+q(x_2)\bar{q}(x_1)\}\ \hat{\sigma}$$

Hadronic cross section

Parton distribution functions

Partonic cross section

Kinematics for Drell-Yan

Trade $\{x_1, x_2\}$ variables for $\{\tau, y\}$

$$x_{1,2} = \sqrt{\tau}\, e^{\pm y} \qquad y = \frac{1}{2}\ln\left(\frac{x_1}{x_2}\right) \qquad \tau = x_1 x_2$$

$$s = (P_1+P_2)^2 = \frac{\hat{s}}{x_1 x_2} = \frac{\hat{s}}{\tau}$$

Therefore $\tau = x_1 x_2 = \frac{\hat{s}}{s} \equiv \frac{Q^2}{s}$

Fractional energy² between partonic and hadronic system

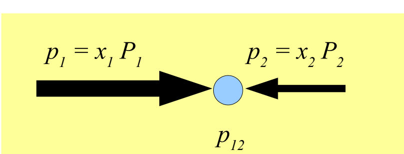
Using: $d x_1 d x_2 = d\tau\, dy$

$$\frac{d\sigma}{d\tau\, dy} = \sum_{q,\bar{q}} \{q(x_1)\bar{q}(x_2)+q(x_2)\bar{q}(x_1)\}\ \hat{\sigma}$$

Rapidity & Longitudinal Momentum Distributions

The rapidity is defined as: $y = \frac{1}{2}\ln\left\{\frac{E_{12}+p_L}{E_{12}-p_L}\right\}$

Partonic CMS has longitudinal momentum w.r.t. the hadron frame

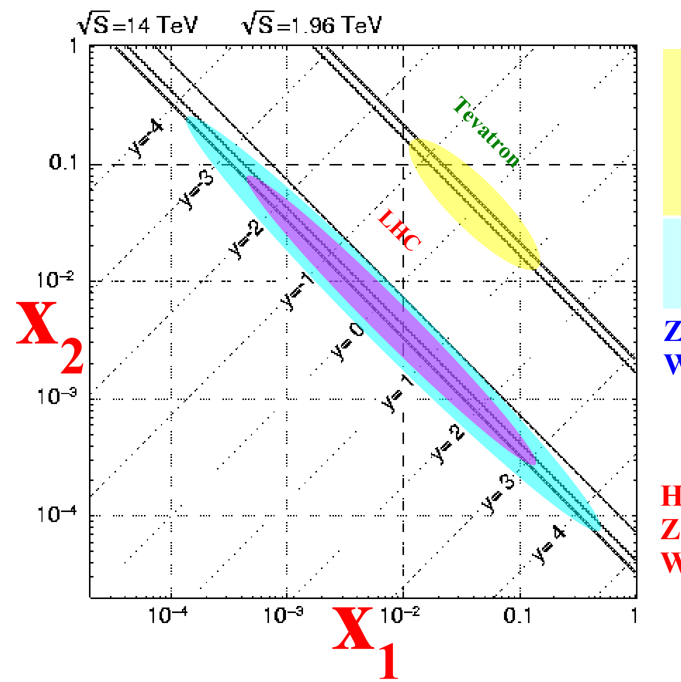


$$p_{12}=(p_1+p_2)=(E_{12},0,0,p_L)$$
$$E_{12}=\frac{\sqrt{s}}{2}(x_1+x_2)$$
$$p_L=\frac{\sqrt{s}}{2}(x_1-x_2) \equiv \frac{\sqrt{s}}{2} x_F$$

x_F is a measure of the longitudinal momentum

$$y = \frac{1}{2}\ln\left\{\frac{E_{12}+p_L}{E_{12}-p_L}\right\} = \frac{1}{2}\ln\left\{\frac{x_1}{x_2}\right\}$$

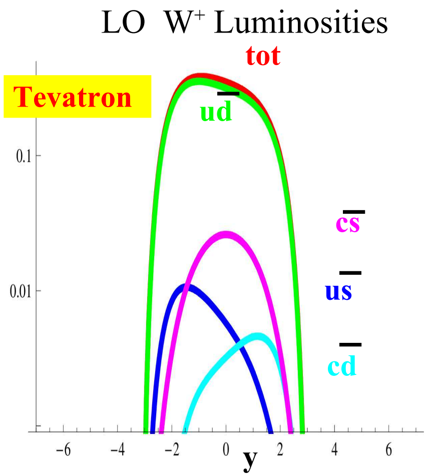
Kinematics for W / Z / Higgs Production

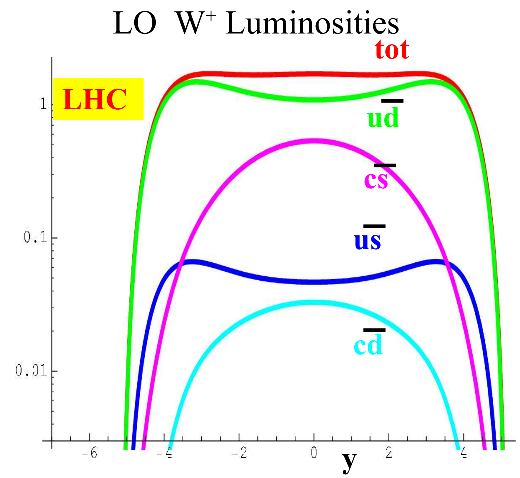


$$y = \frac{1}{2}\ln\left(\frac{x_1}{x_2}\right) \qquad \tau = x_1 x_2$$

$$x_{1,2} = \sqrt{\tau}\, e^{\pm y}$$

Kinematics for W production at Tevatron & LHC





$$d\sigma = \int dx_1 \int dx_2 \int d\tau \left\{ q(x_1)\bar{q}(x_2) + q(x_2)\bar{q}(x_1) \right\} \ \hat{\sigma}\ \delta\left(\tau - \frac{M^2}{S}\right)$$

$$\frac{d\sigma}{d\tau} = \frac{dL}{d\tau}\ \hat{\sigma}(\tau)$$

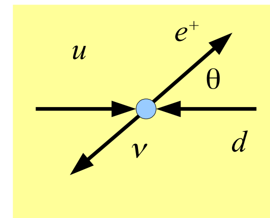
$$\frac{dL}{d\tau} = f \otimes f$$

Kinematics in a Hadron-Hadron Interaction:

The CMS of the parton-parton system is moving longitudinally relative to the hadron-hadron system

How do we measure the W-boson mass?

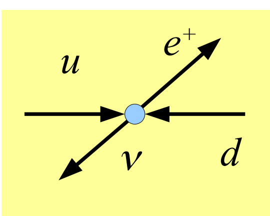
$$u + \bar{d} \rightarrow W^+ \rightarrow e^+ \nu$$



Can't measure W directly
Can't measure ν directly
Can't measure longitudinal momentum

We can measure the P_T of the lepton

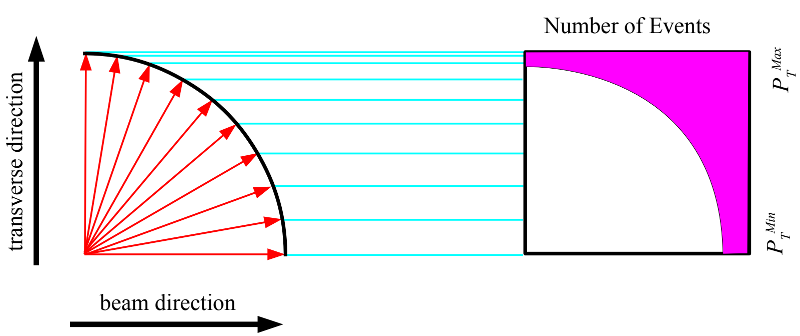
The Jacobian Peak



Suppose lepton distribution is uniform in θ

The dependence is actually $(1+\cos\theta)^2$, but we'll worry about that later

What is the distribution in P_T?



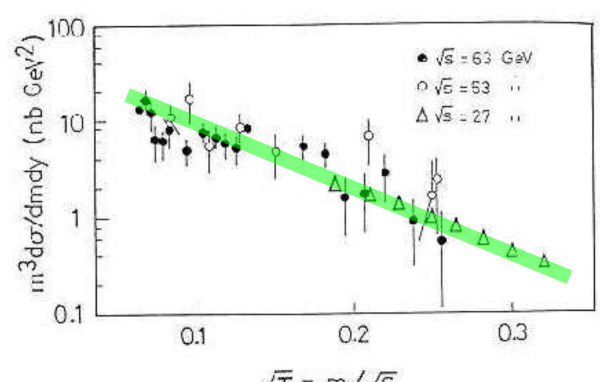
We find a peak at $P_T^{max} \approx M_W/2$

Drell-Yan Cross Section and the Scaling Form

Using: $\hat{\sigma}_0 = \dfrac{4\pi\alpha^2}{9\hat{s}} Q_i^2$ and $\delta(Q^2 - \hat{s}) = \dfrac{1}{s\,x_1}\,\delta\left(x_2 - \dfrac{\tau}{x_1}\right)$

we can write the cross section in the scaling form:

$$Q^4 \frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{9} \sum_{q,\bar{q}} Q_i^2 \int_\tau^1 \frac{dx_1}{x_1}\, \tau \left\{ q(x_1)\bar{q}(\tau/x_1) + \bar{q}(x_1) q(\tau/x_1) \right\}$$



Notice the RHS is a function of only τ, not Q.

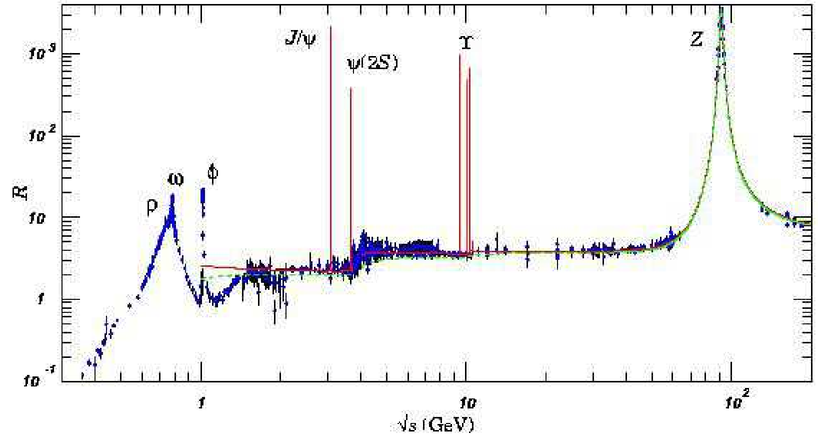
This quantity should lie on a universal scaling curve.

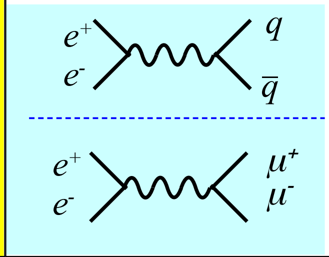
Cf., DIS case, & scattering of point-like constituents

e^+e^- R ratio

$$R=\frac{\sigma(e^+e^-\to hadrons)}{\sigma(e^+e^-\to\mu^+\mu^-)}$$

e^+e^- Ratio of hadrons to muons





3 quark colors

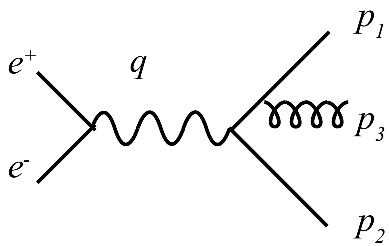
$$R=\frac{\sigma(e^+e^-\to hadrons)}{\sigma(e^+e^-\to\mu^+\mu^-)}=3\sum_i Q_i^2\left[1+\frac{\alpha_s}{\pi}\right]$$

NLO correction

e^+e^-

NLO corrections

e^+e^- to 3 particles final state



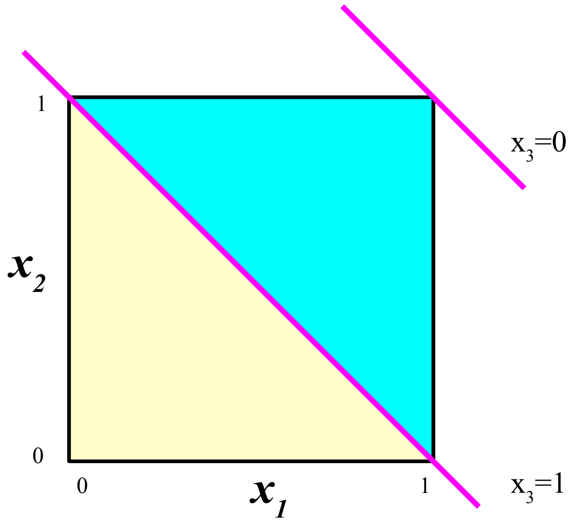
Define the energy fractions E_i:
$$x_i=\frac{E_i}{\sqrt{s}/2}=\frac{2p_i\cdot q}{s}$$

Energy Conservation:
$$\sum_i x_i=2$$

Range of x:
$$x_i\subset[0,1]$$

Exercise: show 3-body phase space is flat in dx_1dx_2

3-Particle Phase Space

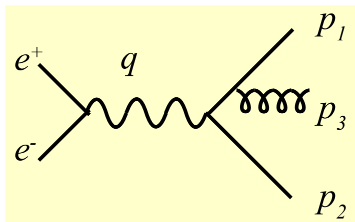


$$x_i = \frac{E_i}{\sqrt{s}/2}$$

$$x_i \subset [0,1]$$

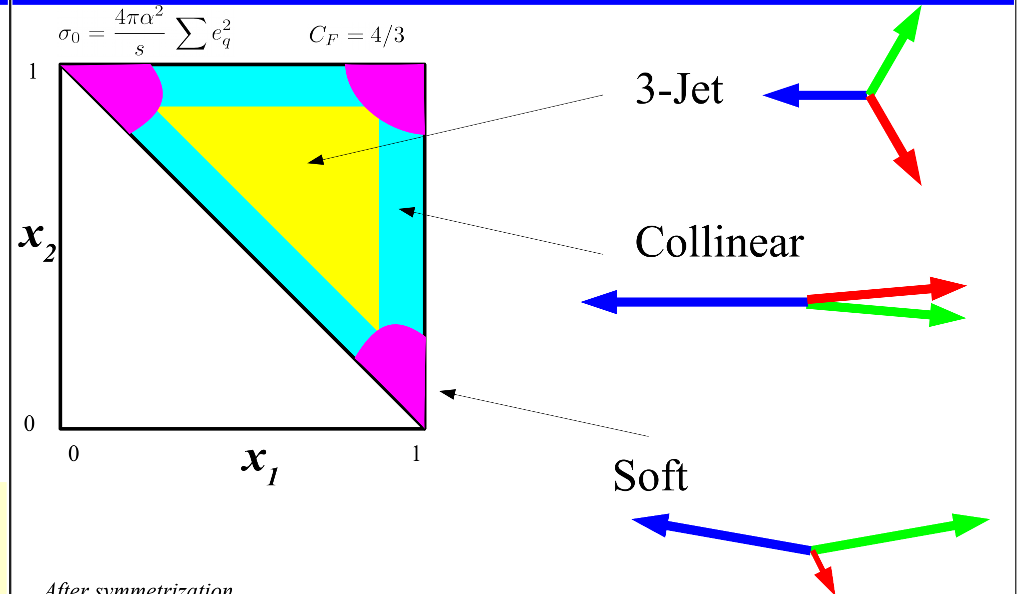
$$x_1+x_2+x_3=2$$

$$d\Gamma \sim dx_1\, dx_2$$



3-Particle Configurations

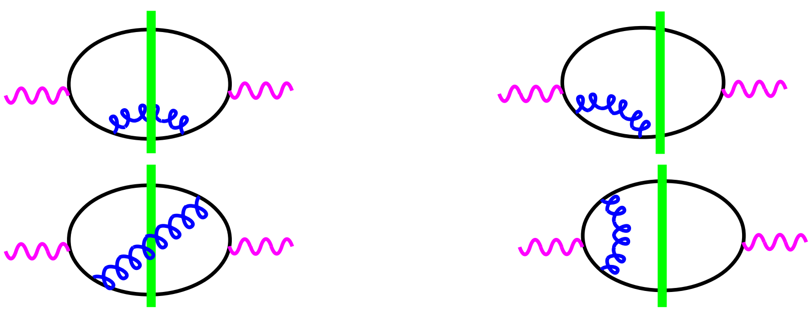
$$\sigma_0 = \frac{4\pi\alpha^2}{s}\sum e_q^2 \qquad C_F = 4/3$$



After symmetrization

$$\frac{1}{\sigma_0}\frac{d\sigma}{dx_1\, dx_2} = \frac{\alpha_s}{2\pi}C_F\frac{x_1^2+x_2^2+x_3^2}{(1-x_1)(1-x_2)(1-x_3)}$$

Singularities cancel between 2-particle and 3-particle graphs



$$\sigma_2^{(\epsilon)} = \sigma_0\, C_F\,\frac{\alpha_s}{\pi}\,(\ldots)\left[+\frac{-1}{\epsilon^2}+\frac{-3}{2\epsilon}+\frac{\pi^2}{2}-4\right]$$

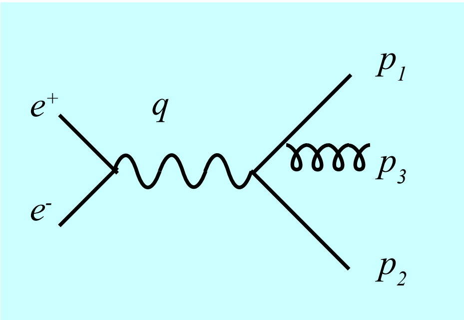
$$\sigma_3^{(\epsilon)} = \sigma_0\, C_F\,\frac{\alpha_s}{\pi}\,(\ldots)\left[+\frac{+1}{\epsilon^2}+\frac{+3}{2\epsilon}+\frac{-\pi^2}{2}-\frac{19}{4}\right]$$

$$\sigma_2^{(\epsilon)}+\sigma_3^{(\epsilon)} = \sigma_0\, C_F\,\frac{\alpha_s}{\pi}\,(\ldots)\left[0+0+0+-\frac{35}{4}\right]$$

Same result with gluon mass regularization

e^+e^- Differential Cross Sections

Differential Cross Section



What do we do about soft and collinear singularities????

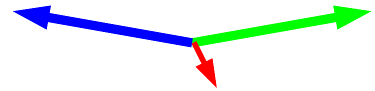
Introduce the concept of "Infrared Safe Observable"

The soft and collinear singularities will cancel
ONLY
if the physical observables are appropriately defined.

Infrared Safe Observables

Observables must satisfy the following requirements:

Soft



if $p_s \to 0$

$$\mathcal{O}_{n+1}(p_1, ..., p_n, p_s) \longrightarrow \mathcal{O}_n(p_1, ..., p_n)$$

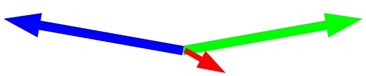
Collinear

if $p_a \parallel p_b$

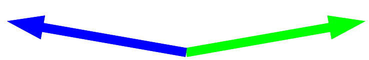
$$\mathcal{O}_{n+1}(p_1, ..., p_a, p_b, ..., p_n) \longrightarrow \mathcal{O}_n(p_1, ..., p_a + p_b, ..., p_n)$$

Infrared Safe Observables

Soft

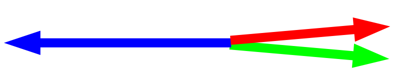


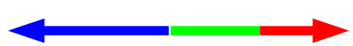
if $p_s \to 0$



$$\mathcal{O}_{n+1}(p_1, ..., p_n, p_s) \longrightarrow \mathcal{O}_n(p_1, ..., p_n)$$

Collinear





if $p_a \parallel p_b$

$$\mathcal{O}_{n+1}(p_1, ..., p_a, p_b, ..., p_n) \longrightarrow \mathcal{O}_n(p_1, ..., p_a + p_b, ..., p_n)$$

Examples: Infrared Safe Observables

Infrared Safe Observables:

Event shape distributions
Jet Cross sections

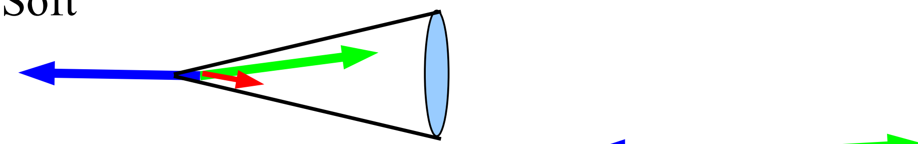
Un-Safe Infrared Observables:

Momentum of the hardest particle
(affected by collinear splitting)

100% isolated particles
(affected by soft emissions)

Particle multiplicity
(affected by both soft & collinear emissions)

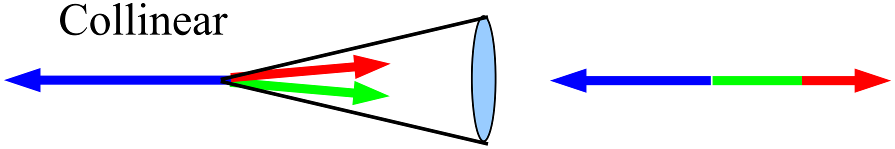
Soft



if $p_s \to 0$

$$\mathcal{O}_{n+1}(p_1, ..., p_n, p_s) \longrightarrow \mathcal{O}_n(p_1, ..., p_n)$$

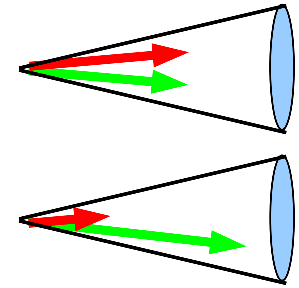
Collinear

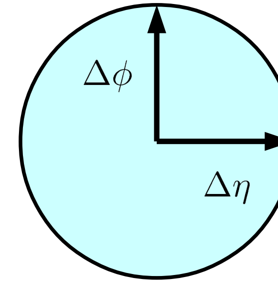


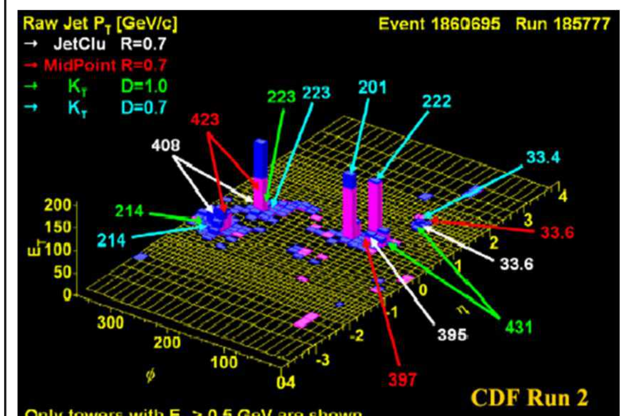
if $p_a \parallel p_b$

$$\mathcal{O}_{n+1}(p_1, ..., p_a, p_b, ..., p_n) \longrightarrow \mathcal{O}_n(p_1, ..., p_a + p_b, ..., p_n)$$

Jet Cone

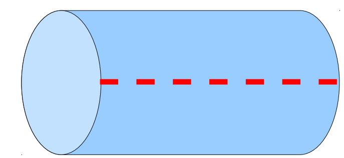




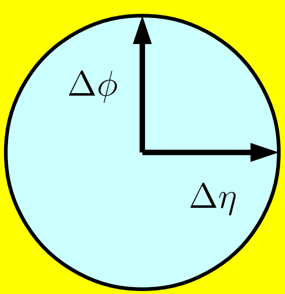


$$R^2 = (\Delta\eta)^2 + (\Delta\phi)^2$$

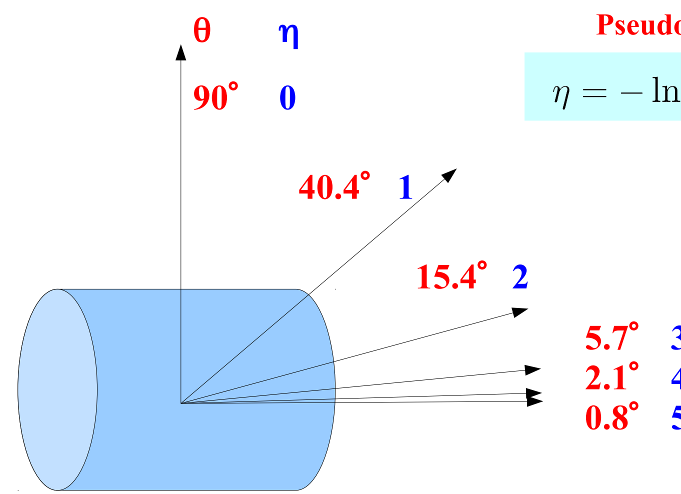
Let's examine this definition a bit more closely



Jet Cone



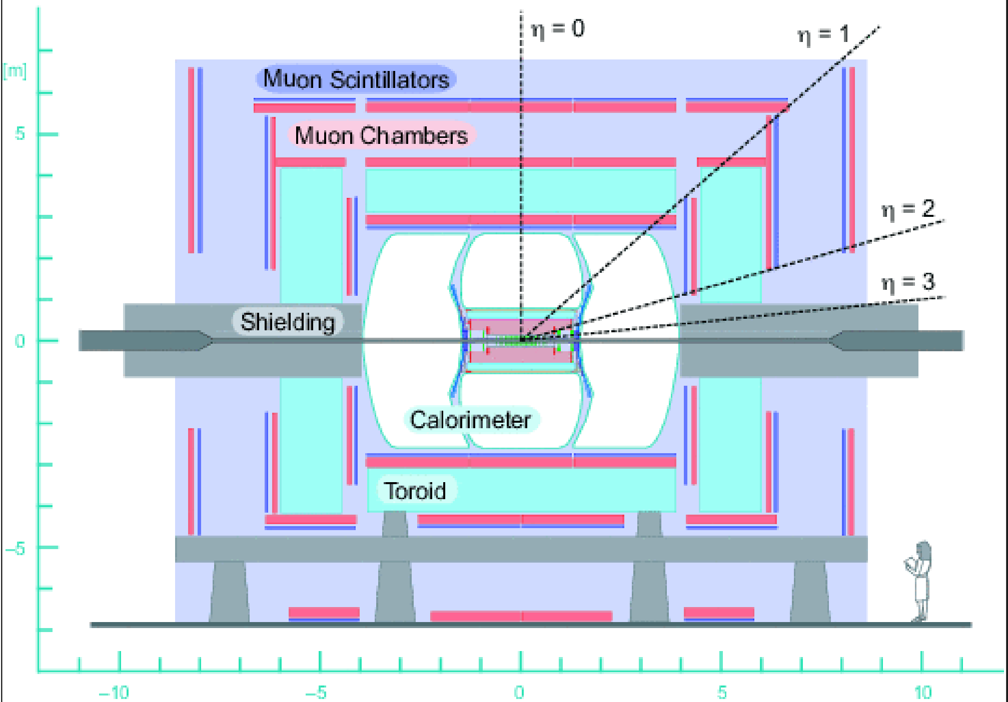
$$R^2 = (\Delta\eta)^2 + (\Delta\phi)^2$$



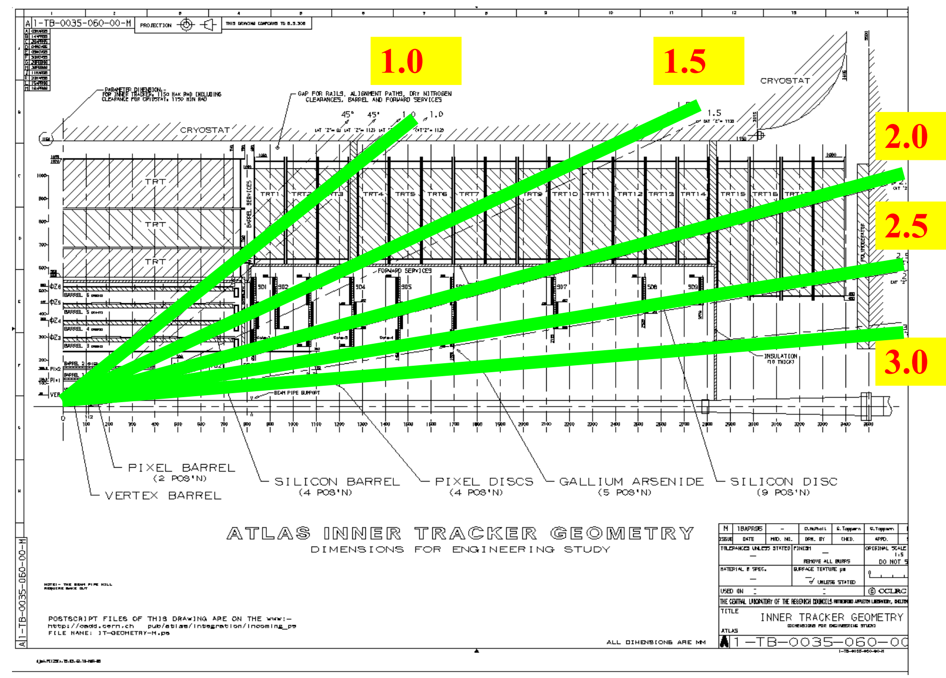
Pseudo-Rapidity

$$\eta = -\ln[\tan(\theta/2)]$$

D0 Detector Schematic

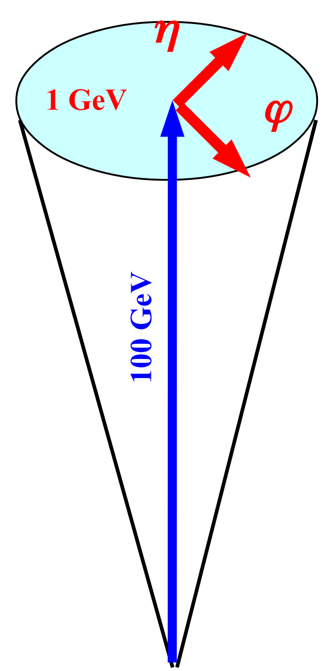


ATLAS Detector Schematic



homework

HOMEWORK: Jet Cone Definition



PROBLEM #2: In a Tevatron detector, consider two particles traveling in the transverse direction:

$$p_1^\mu = \{E, 100, 0, 1\}$$
$$p_2^\mu = \{E, 100, 1, 0\}$$

where the componets are expresedn in GeV units. E is defined such that the particles are massless.

a) Compute E.

b) For each particle, compute the pseudorapidity η and azimuthal angle ϕ.

c) Explain how the above exercise justifies the correct jet radius definition to be:

$$R = \sqrt{\eta^2 + \phi^2}$$

In particular, why is the above correct and $R = \sqrt{\eta^2 + 2\phi^2}$, for example, incorrect.

HOMEWORK: Light-Cone Coordinates & Boosts

$$P_\mu = \{P_t, P_x, P_y, P_z\}$$

$$P_\mu = \{P_+, \overrightarrow{P_\perp}, P_-\} \qquad \overrightarrow{P}_\perp = \{P_x, P_y\}$$

$$P_\pm = \frac{1}{\sqrt{2}}\,(P_t \pm P_z)$$

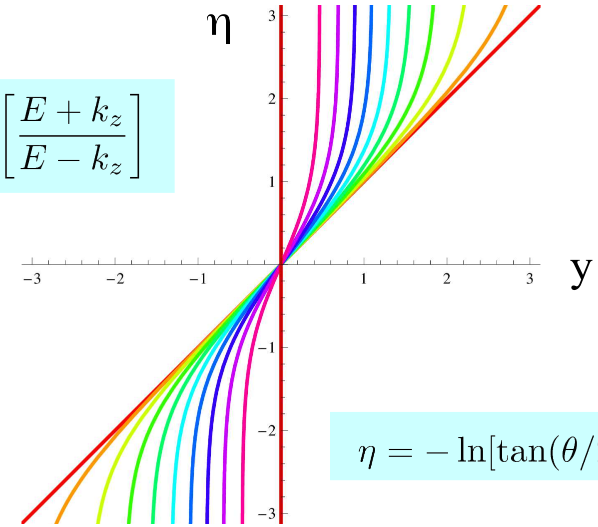
1) Compute the metric $g_{\mu\nu}$ in the light-cone frame, and compute $\overrightarrow{P}_1 \cdot \overrightarrow{P}_2$ in terms of the light-cone components.

2) Compute the boost matrix B for a boost along the z-axis, and show the light-cone vector transforms in a particularly simple manner.

3) Show that a boost along the z-axis uniformily shifts the rapidity of a vector by a constant amount.

Rapidity vs. Pseudo-Rapidity

$$y = \frac{1}{2}\,\ln\left[\frac{E + k_z}{E - k_z}\right]$$



$$\eta = -\ln[\tan(\theta/2)]$$

$$\frac{M}{E} = \{0, 0.1, 0.2 ...\}$$

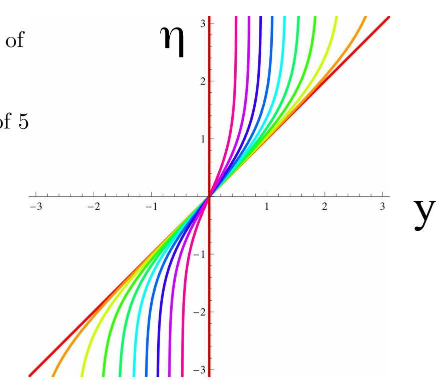
HOMEWORK: Rapidity vs. Pseudo-Rapidity

PROBLEM #1: Consider the rapidity y and the pseudo-rapidity η:

$$y = \frac{1}{2}\ln\left(\frac{E + P_z}{E - P_z}\right)$$

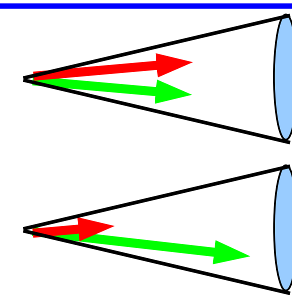
$$\eta = -\ln\left[\tan\left(\frac{\theta}{2}\right)\right]$$

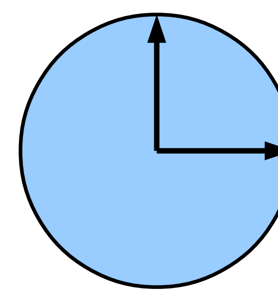
a) Make a parametric plot of $\{y, \eta\}$as a function of of the particle.
b) Show that in the limit $m \to 0$ that $y \to \eta$.
c) Make a table of η for $\theta = [0°, 180°]$ in steps of 5
d) Make a table of θ for $\eta = [0, 10]$ i



Infrared Safe Observables: Define Jets

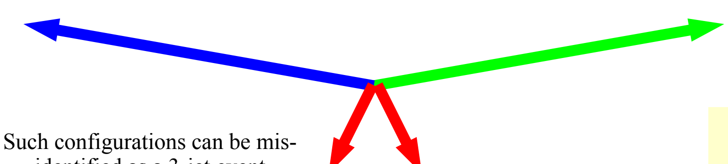
Jet Cone





Problem:
The cone definition is simple,
BUT
it is too simple

$$R^2 = (\Delta\eta)^2 + (\Delta\phi)^2$$



Such configurations can be mis-identified as a 3-jet event

See talk by
Dave Soper &
Andrew Larkoski

End of lecture 4: Recap

Drell-Yan: Tremendous discovery potential

- Need to compute 2 initial hadrons

e^+e^- processes:

- Total Cross Section:
- Differential Cross Section: singularities

Infrared Safe Observables

- Stable under soft and collinear emissions

Jet definition

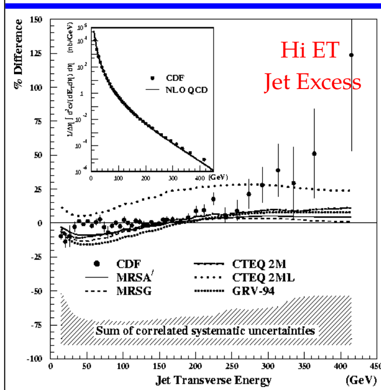
- Cone definition is simple:
- ... it is TOO simple

Final Thoughts

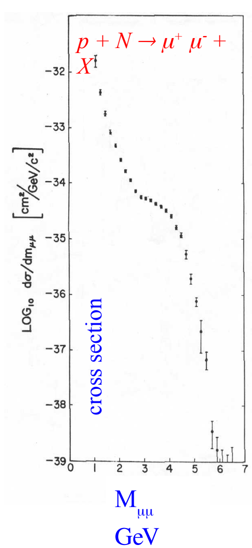
$$\boxed{\begin{aligned}\mathcal{L}_{\mathrm{QCD}} &= \bar{\psi}_i\left(i\gamma^\mu (D_\mu)_{ij} - m\,\delta_{ij}\right)\psi_j - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a \\ &= \bar{\psi}_i(i\gamma^\mu\partial_\mu - m)\psi_i - gG^a_\mu\bar{\psi}_i\gamma^\mu T^a_{ij}\psi_j - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a\,,\end{aligned}}$$

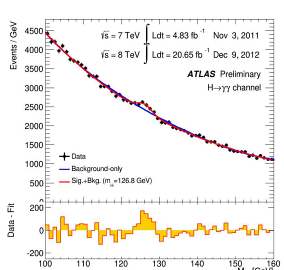
Scaling, Dimensional Analysis, Factorization, Regularization & Renormalization, Infrared Saftey ...

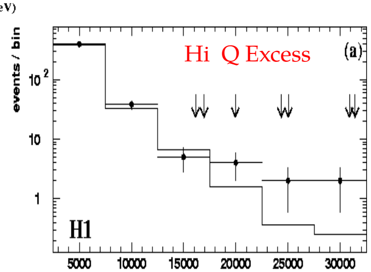
Can you find the Nobel Prize???



CDF Collaboration, PRL 77, 438 (1996)







H1 Collaboration, ZPC74, 191 (1997) Q^2_e (GeV^2)
ZEUS Collaboration, ZPC74, 207 (1997)

Thanks to ...

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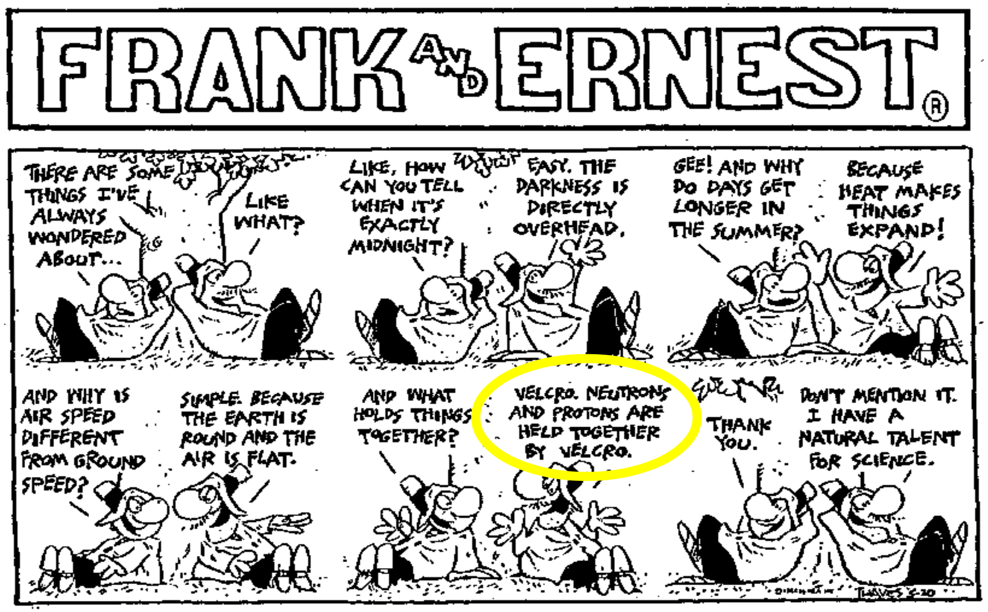
To the CTEQ and MCnet folks for making all this possible.

LibreOffice
The Document Foundation

and the many web pages where I borrowed my figures ...

PowerPoint

Keep an open mind!!!



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END OF LECTURE 4