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Pythagorean Tuning (1,2,3,4)
*All tones found by pure 5ths*

Relationship in ratios:

OCTAVE: 2/1
FIFTH: 3/2
FOURTH: 4/3

Or

| 8va | 5th | 4th |

1   2   3   4

4/3   3/2   2/1

C   **D**   **E**   **F**   **G**   **A**   **B**   **C**

9/8   81/64   9/8   256/243   9/8   9/8   9/8   256/243
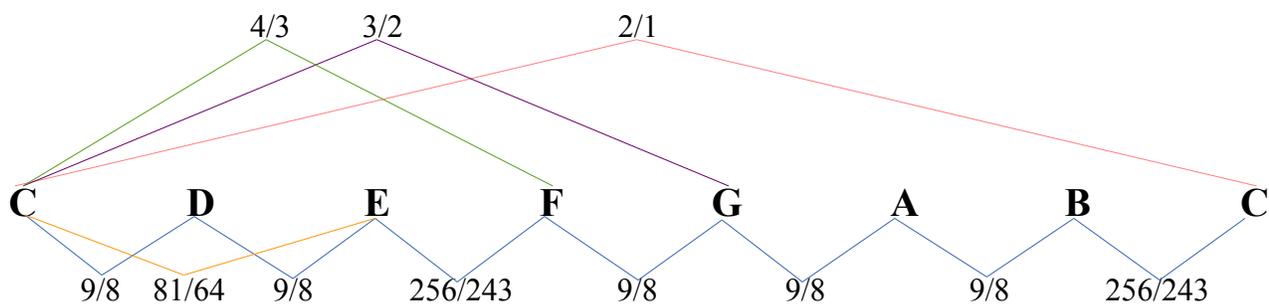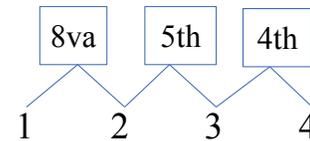
Pythagorean Third: C-E

C x 3/2 = G → G x 3/2 = D → D x 1/2 = D → D x 3/2 = A → A x 3/2 = E → E x 1/2 = E

Calculated out:

3/2 x 3/2 = 9/4 → 9/4 x ½ = 9/8 → 9/8 x 3/2 = 27/16 → 27/16 x 3/2 = 81/32 → 81/32 x 1/2 = 81/64

E = 81/64 *Note, this is close to 80/64 = 10/8 = 5/4 = ideal Major Third*

Relationship in cents:

1110
408   702
C 204 **D** 204 **E** 90 **F** 204 **G** 204 **A** 204 **B** 90 **C**
498
906
1200
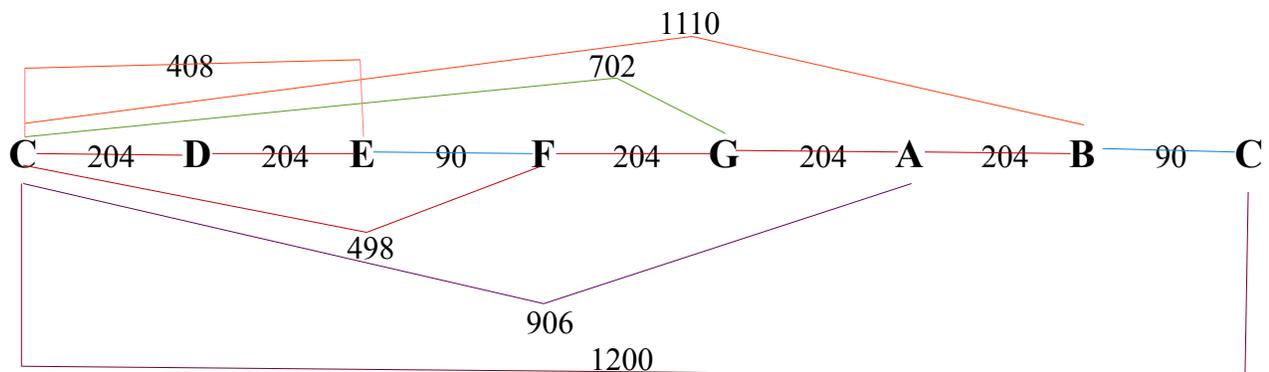
Pythagorean Comma:

Difference between B# arrived at by twelve 5ths (702¢) and C arrived at by seven octaves (1200¢).

Pythagorean Comma = (12×2¢) = 24¢

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Just Tuning (1,2,3,4,5)
Based on pure 5th plus pure 3rd

Relationship in ratios:

OCTAVE: 2/1
FIFTH: 3/2
FOURTH: 4/3
THIRD: 5/4
Major SECOND: 9/8 (best) *or* 10/9 (acceptable)

Major Second: D-E

$5/4 - 9/8 \rightarrow 5/4 \times 8/9 = $ 10/9

Minor second: E-F

$4/3 - 5/4 = 4/3 \rightarrow 4/3 \times 4/5 = $ 16/15

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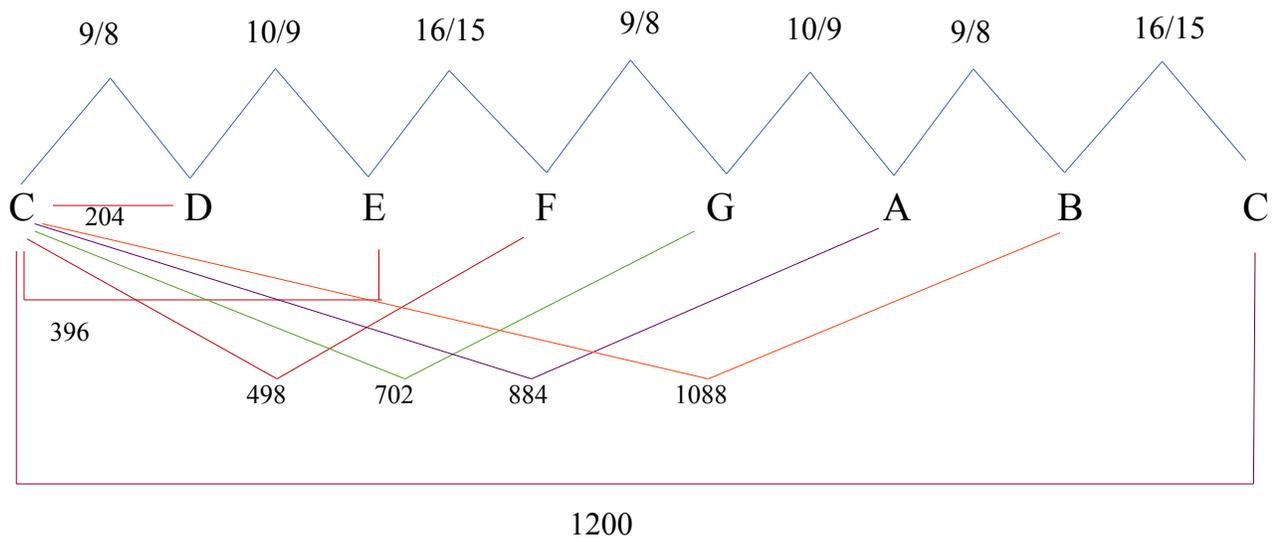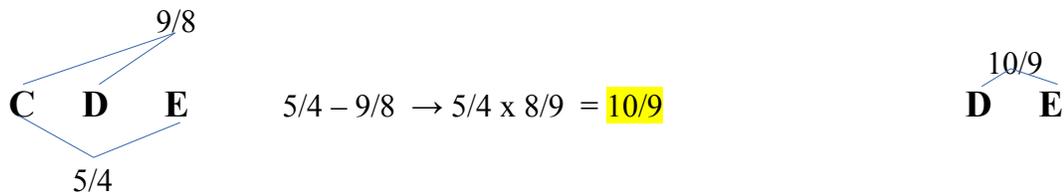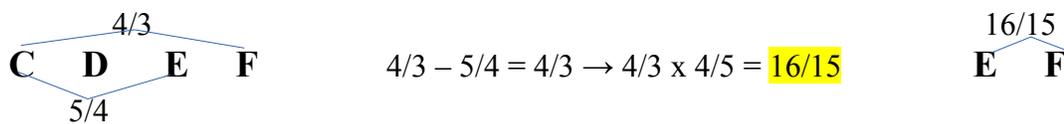
Mean Tone Temperament
### *Based on altering successive 5ᵗʰˢ to get pure 3rd*

Finding Pure 3ʳᵈ:

- Spreading 1/4 Didymean Comma (81/80 or 22 cents) over each of the four 5ᵗʰ
- 1/4 of 22 cents is 5½ cents
- So each 5ᵗʰ is 696½ cents instead of 702 cents

Pythagorean: 702   702   702   702   = 2808
         C   G   D   A   E
Mean tone: 696½   696½   696½   696 ½   = 2768
       2768 – 2400 (two octaves) = 386
          Pure 3ʳᵈ = 386¢

OCTAVE: 1200¢
FIFTH: 696½¢
THIRD: 386¢
SECOND: 386/2 = 193¢   (meantone: we divide the 3ʳᵈ evenly in half to find the second)
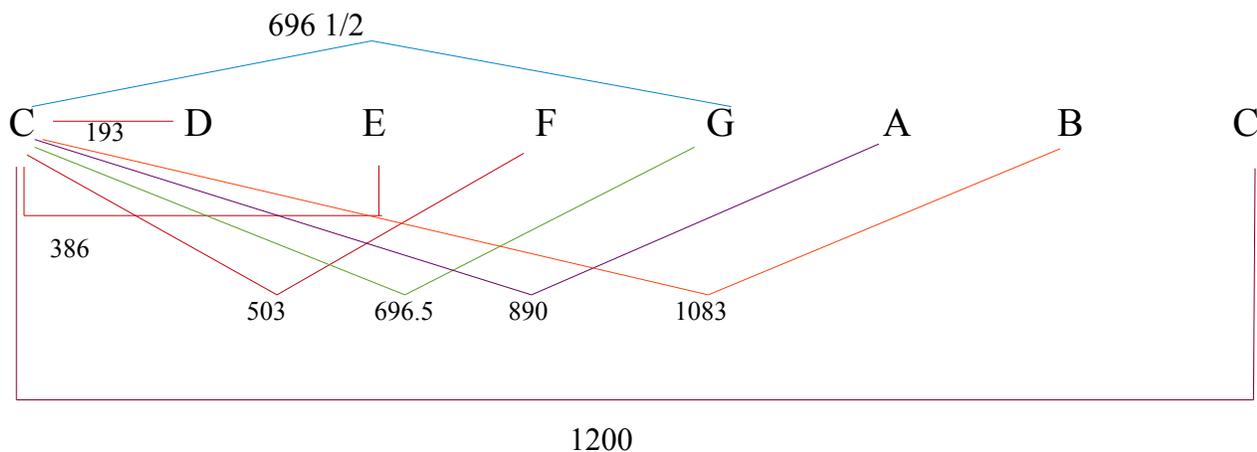
Temperament: Note, "Meantone" and "Equal" are compromises as they involve intervals that are not interger ratios such as 193¢ or $^{12}\sqrt{2}$. This is in contrast to the "Pythagorean" and "Just" which are **Tunings** and do have interger ratios, albeit not always nice ones such as 81/64.

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Equal Temperament

Based on …

_…_